Appendix for Online Publication

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Title: Relationships in the Interbank Market

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Appendix

A Reserve Distributions In The Core Market

The distribution of positive reserves is as follows. The distribution of positive reserves is as follows. Define

$$\begin{aligned} N^+ &= nN_L(1-G(0)) + N_S(1-F(\hat{R}^+)) + \\ &\quad n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[1-G(-m_1-\xi)]d\xi \\ &\quad +(1-N_S)\int_{-\infty}^{\infty} f(\xi)[1-G(\bar{R}^+-\xi)]d\xi \end{aligned}$$

we have for $R^+ \in [0, \hat{R}^+)$:

$$\begin{aligned} \Omega^+(R^+) &= \frac{1}{N^+}\{nN_L[G(R^+)-G(0)] \\ &\quad +n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(R^+-\xi)-G(-\xi)]d\xi\}. \end{aligned}$$

For $R^+ \in [\hat{R}^+, \bar{R}^+/2)$:

$$\begin{aligned} \Omega^+(R^+) &= \frac{1}{N^+}\{nN_L[G(R^+)-G(0)] \\ &\quad +N_S[F(R^+)-F(\hat{R}^+)] \\ &\quad +n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(R^+-\xi)-G(-\xi)]d\xi\}. \end{aligned}$$

For $R^+ \in [\bar{R}^+/2, \bar{R}^+)$:

$$\begin{aligned} \Omega^+(R^+) &= \frac{1}{N^+}\{nN_L[G(R^+)-G(0)] \\ &\quad +N_S[F(R^+)-F(\hat{R}^+)] \\ &\quad +n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(R^+-\xi)-G(-\xi)]d\xi \\ &\quad +n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(2R^+-\xi)-G(\bar{R}^+-\xi)]d\xi \\ &\quad +(1-N_S)\int_{-\infty}^{\infty} f(\xi)[G(2R^+-\xi)-G(\bar{R}^+-\xi)]d\xi\}. \end{aligned}$$

For $R^+ \in [\bar{R}^+, +\infty)$:

$$\begin{aligned}
\Omega^+(R^+) &= \frac{1}{N^+}\{nN_L[G(R^+) - G(0)] \\
&+N_S[F(R^+) - F(\hat{R}^+)] \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(\bar{R}^+ - \xi) - G(-\xi)]d\xi \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(2R^+ - \xi) - G(\bar{R}^+ - \xi)]d\xi \\
&+(1-N_S)\int_{-\infty}^{\infty} f(\xi)[G(2R^+ - \xi) - G(\bar{R}^+ - \xi)]d\xi\}.
\end{aligned}$$

Similarly, the distribution of negative reserves is as follows. With

$$\begin{aligned}
N^- = {} & nN_LG(0) + N_SF(\hat{R}^-) \\
& + n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(-\xi)]d\xi \\
& + (1-N_S)\int_{-\infty}^{\infty} f(\xi)[G(\bar{R}^- - \xi)]d\xi
\end{aligned}$$

we have, for $R^- \in [\hat{R}^-, 0)$:

$$\begin{aligned}
\Omega^-(R^-) &= \frac{1}{N^-}\{nN_LG(R^-) \\
&+N_SF(\hat{R}^-) \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(\bar{R}^- - \xi)]d\xi \\
&+(1-N_S)\int_{-\infty}^{\infty} f(\xi)[G(\bar{R}^- - \xi)]d\xi \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(R^- - \xi) - G(\bar{R}^- - \xi)]d\xi\}.
\end{aligned}$$

For $R^- \in [\bar{R}^-/2, \hat{R}^-)$:

$$\begin{aligned}
\Omega^-(R^-) &= \frac{1}{N^-}\{nN_LG(R^-) \\
&+N_SF(R^-) \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(\bar{R}^- - \xi)]d\xi \\
&+(1-N_S)\int_{-\infty}^{\infty} f(\xi)[G(\bar{R}^- - \xi)]d\xi + \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(R^- - \xi) - G(\bar{R}^- - \xi)]d\xi\}.
\end{aligned}$$

For $R^- \in [\bar{R}^-, \bar{R}^-/2)$:

$$\begin{aligned}
\Omega^-(R^-) &= \frac{1}{N^-}\{nN_L G(R^-) \\
&+N_S F(R^-) \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(2R^- - \xi)]d\xi \\
&+(1-N_S)\int_{-\infty}^{\infty} f(\xi)[G(2R^- - \xi)]d\xi \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(R^- - \xi) - G(\bar{R}^- - \xi)]d\xi\}.
\end{aligned}$$

For $R^- \in (-\infty, \bar{R}^-)$:

$$\begin{aligned}
\Omega^-(R^-) &= \frac{1}{N^-}\{nN_L G(R^-) \\
&+N_S F(R^-) \\
&+n(1-N_L)\int_{-\infty}^{\infty} f(\xi)[G(2R^- - \xi)]d\xi \\
&+(1-N_S)\int_{-\infty}^{\infty} f(\xi)[G(2R^- - \xi)]d\xi\}.
\end{aligned}$$

To understand how we construct the distributions of reserves, Figure 1 shows the possible reserve holdings for the different types of banks in the core market.

Panels (iii) and (iv) of Figure 1 show the possible reserves holdings of single banks when they are active in the core market. Panel (iii) shows the one for single banks L: As they always enter the market, the possible reserve holdings is simply the liquidity shock ε they received. Panel (iv) shows the possible reserves holdings of single banks S in the core market. Single banks S only enter the core market if their reserves are sufficiently different from 0, i.e. below $\hat{R}^-$ and above $\hat{R}^+$, otherwise they do not enter the core market. Therefore, there cannot be any single S banks in the core market with reserves $R \in [\hat{R}^-, \hat{R}^+]$.

Panels (i) and (ii) of Figure 1 show the possible reserve holdings of banks with a business partner. There, the total amount of reserves that this bank and her partner hold, $\varepsilon + \xi$, plays an important role. Whenever this amount of reserves is small, bank S trades away her reserves surplus or deficit with bank L and does not enter the core market. Otherwise the S and L banks split the total amount of reserves equally and the bank S enters the core market. Therefore, there is no bank S with a partner holding reserves $R \in [\bar{R}^-, \bar{R}^+]$ in the core market. Similarly, banks L with a business partner will bring the sum of their reserves to the core market whenever the sum is relatively small, but otherwise he will

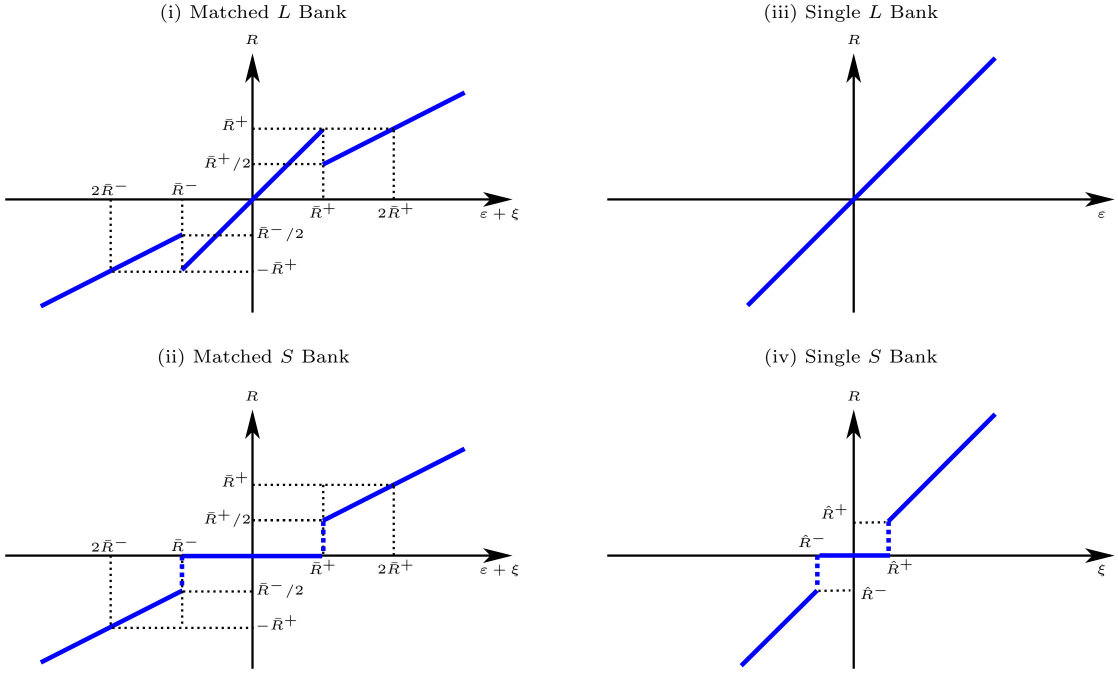

Figure 1: Reserve balances held by different types

only bring half of it (hence the different slopes). The discontinuity is due to the need for sufficient gains from trade before S banks enter with half of the total reserves available in the pair of partner banks.

B Proof of Lemma 1

Proof. Using the constraint, the objective function can be rewritten as maximizing the total surplus by choosing x:

$$\max_x R^+[i(R^+ - x) - i_d] + R^-[i(R^- + x) - i_\ell] + x[i(R^- + x) - i(R^+ - x)]. \tag{B.1}$$

Note that X does not enter the objective at all and is determined solely by the constraint, which implies

$$\begin{aligned} X &= \Theta(R^- + x)[1 + i(R^- + x)] - (1 - \Theta)(R^+ - x)[1 + i(R^+ - x)] \\ &\quad + (1 - \Theta)R^+[1 + i_d] - \Theta R^-[1 + i_\ell]. \end{aligned} \tag{B.2}$$

Consider two possible cases of R^+ and R^- :

Case 1: $R^+ > 0 > R^- > -R^+$.

Total surplus $S = R^+[i(R^+-x)-i_d]+R^-[i(R^-+x)-i_\ell]+x[i(R^-+x)-i(R^+-x)]$. Any $x \in (-R^-, R^+)$ implies $S = R^-(i_d-i_\ell) > 0$. Any $x < -R^-$ implies $S = x(i_\ell-i_d) < R^-(i_d-i_\ell)$, thus dominated. Any $x > R^+$ implies $S = (R^+-x)(i_\ell-i_d)-R^-(i_\ell-i_d) < R^-(i_d-i_\ell)$, thus dominated. Therefore, $x = (R^+ - R^-)/2 \in (-R^-, R^+)$ is optimal.

Case 2: $R^+ > 0 > -R^+ > R^-$.

Total surplus $S = R^+[i(R^+-x)-i_d]+R^-[i(R^-+x)-i_\ell]+x[i(R^-+x)-i(R^+-x)]$. Any $x \in (R^+, -R^-)$ implies $S = R^+(i_\ell - i_d) > 0$. Any $x < R^+ < -R^-$ implies $S = x(i_\ell - i_d) < R^+(i_\ell - i_d)$, thus dominated. Any $x > -R^- > R^+$ implies $S = R^+(i_\ell - i_d) - (R^- + x)(i_\ell - i_d) < R^+(i_\ell - i_d)$, thus dominated. Therefore, $x = (R^+ - R^-)/2 \in (R^+, -R^-)$ is optimal.

Therefore, $x = (R^+ - R^-)/2$ is a solution and (B.2) implies

$$X = (2\theta - 1)(\frac{R^- + R^+}{2})[1 + i(\frac{R^i + R^+}{2})] + (1 - \theta)R^+[1 + i_d] - \theta R^i[1 + i_\ell].$$

□

C Proof of Proposition 4

Proof. The total surplus is

$$\begin{aligned}
TS(z, \hat{i}, c) &= TS_L(z, \hat{i}, c) + TS_S(z, \hat{i}, c) \\
&= \max\{\Pi(R_S - z) - \gamma, 0\} + \Pi(R_L + z) \\
&\quad +\beta\left[(R_S - z)(1 + i(R_S - z)) + (R_L + z)(1 + i(R_L + z))\right] \\
&\quad - \max\{\Pi(R_S) - \gamma, 0\} - \Pi(R_L) - \beta R_S(1 + i(R_S)) - \beta R_L(1 + i(R_L)) \\
&\quad +\beta c[\bar{v}_1(1) - \beta\bar{v}_1(0)]
\end{aligned}$$

and the choice z only affects the second line above. Now, suppose the optimal choice z^* is such that bank S enters in the core market. Then z^* has to maximize

$$\Pi(R_S - z) + \beta(R_S - z)(1 + i(R_S - z)) + \Pi(R_L + z) + \beta(R_L + z)(1 + i(R_L + z)),$$

and so $z^* = (R_S - R_L)/2$. However, entry implies $\Pi(R_S + z^*) = \Pi\left(\frac{R_S+R_L}{2}\right) \geq \gamma$.

Now, suppose the optimal choice is such that bank S does not enter the core market. Then z^* has to maximize

$$\Pi(R_L + z) + \beta(R_L + z)(1 + i(R_L + z)) + \beta(R_S - z)(1 + i(R_S - z)),$$

subject to the constraint that $\Pi(R_S - z^*) \leq \gamma$. Notice that $\Pi(R) + \beta R(1 + i(R))$ is increasing in R. We have four cases to consider.

First, suppose $R_S, R_L > 0$ so that $i(R_S) = i_d$. Then for all $z \in [0, R_S]$ we get

$$\beta(R_L + z)(1 + i(R_L + z)) + \beta(R_S - z)(1 + i(R_S - z)) \quad = \quad \beta(R_L + R_S)(1 + i_d)$$

so that the interest "bill" at the central bank is the same for the pair of banks, independent of who holds reserves. Since $\Pi(R_L + z)$ is increasing in z, we get that for $z \in [0, R_S]$ the solution is $z = R_S$ and the total surplus then is

$$\Pi(R_L + R_S) + \beta(R_L + R_S)(1 + i_d).$$

Should bank S lend ε more to bank L and go negative? Then the total surplus would be

$$\begin{aligned} \Pi(R_L + R_S + \varepsilon) + \beta(R_S + R_L + \varepsilon)(1 + i_d) - \beta\varepsilon(1 + i_\ell) \quad &= \\ \Pi(R_L + R_S + \varepsilon) + \beta(R_S + R_L)(1 + i_d) - \beta\varepsilon(i_\ell - i_d) & \end{aligned}$$

So the two banks would prefer this iff

$$\Pi(R_L + R_S + \varepsilon) - \beta\varepsilon(i_\ell - i_d) > \Pi(R_L + R_S)$$

Arranging and taking the limit as $\varepsilon \to 0$, the two banks would prefer that the S bank borrows ε more iff

$$\Pi'(R_L + R_S) > \beta(i_\ell - i_d)$$

However we know that $\Pi'(R) < \beta(i_\ell - i_d)$ for all R – as the right side is the gains from trade *given* there is trade. Hence, given $R_L, R_S > 0$ *and* bank S does not enter the core market, then the optimal z is $z^* = R_S$.

Second, suppose $R_S, R_L < 0$ so that $i(R_S) = i_\ell$. Then for all $z \in [0, R_S]$ we get

$$\beta(R_L + z)(1 + i(R_L + z)) + \beta(R_S - z)(1 + i(R_S - z)) \quad = \quad \beta(R_L + R_S)(1 + i_\ell)$$

so that the interest "bill" at the central bank is the same for the pair of banks, independent of who holds reserves. Since $\Pi(R_L + z)$ is decreasing in z whenever $R_L + z < 0$ we get that for $z \in [R_S, 0]$ the solution is $z = R_S$ (bank L lends R_S to bank S) and the total surplus then is

$$\Pi(R_L + R_S) + \beta(R_L + R_S)(1 + i_\ell).$$

Should bank L lend ε more to bank S, so that it goes positive? Then the total surplus would be

$$\begin{aligned}\Pi(R_L+R_S-\varepsilon)+\beta(R_S+R_L-\varepsilon)(1+i_\ell)+\beta\varepsilon(1+i_d) &= \\ \Pi(R_L+R_S-\varepsilon)+\beta(R_S+R_L)(1+i_\ell)-\beta\varepsilon(i_\ell-i_d)\end{aligned}$$

So the two banks would prefer this iff

$$\Pi(R_L+R_S-\varepsilon)-\beta\varepsilon(i_\ell-i_d)>\Pi(R_L+R_S)$$

Arranging and taking the limit as $\varepsilon\to 0$, the two banks would prefer that bank S borrows ε more iff

$$-\Pi'(R_L+R_S)>\beta(i_\ell-i_d)$$

However we know that for $R<0$ $-\Pi'(R)<\beta(i_\ell-i_d)$. Hence, given $R_L,R_S<0$ *and* bank S does not enter the core market, then the optimal z is $z^*=R_S$.

Third, suppose $R_S>0$ while $R_L<0$. Should bank L borrow ε from bank S given $\Pi(R_L+z)$ is decreasing in z whenever $R_L+z<0$? If bank L borrows ε the total surplus becomes

$$\begin{aligned}\Pi(R_L+\varepsilon)+\beta(R_L+\varepsilon)(1+i_\ell)+\beta(R_S-\varepsilon)(1+i_d) &= \\ \Pi(R_L+\varepsilon)+\beta R_L(1+i_\ell)+\beta R_S(1+i_d)+\beta\varepsilon(i_\ell-i_d)\end{aligned}$$

So the two banks prefer this iff

$$\Pi(R_L+\varepsilon)+\beta\varepsilon(i_\ell-i_d)>\Pi(R_L)$$

again arranging and taking the limit as $\varepsilon\to 0$, we get that bank L prefers to borrow from bank S whenever

$$\Pi'(R_L)>-\beta(i_\ell-i_d)$$

which is always satisfied. Therefore, bank L will borrow up to R_S if $R_L+R_S<0$ and until $R_L+z=0$ if $R_L+R_S>0$. Will bank L borrow more? If $R_L+R_S<0$, the second case above shows that bank L will not borrow more. If $R_L+R_S>0$, however, since $\Pi(R_L+z)$ is increasing in z, whenever $R_L+z>0$, we get that for $z\in[-R_L,R_S]$ the solution is $z=R_S$ and the total surplus then is

$$\Pi(R_L+R_S)+\beta(R_L+R_S)(1+i_d).$$

Again, as in the first case, bank S should not lend more to bank L. Hence, given $R_S > 0$ *and* $R_L < 0$, *and* bank S does not enter the core market, then the optimal z is $z^* = R_S$.

Fourth, suppose $R_L > 0$ while $R_S < 0$. Should bank L lend ε to bank S given $\Pi(R_L + z)$ is increasing in z whenever $R_L + z > 0$? If bank L lends ε the total surplus becomes

$$\begin{aligned}\Pi(R_L - \varepsilon) + \beta(R_L - \varepsilon)(1 + i_d) + \beta(R_S + \varepsilon)(1 + i_\ell) &= \\ \Pi(R_L - \varepsilon) + \beta R_L(1 + i_d) + \beta R_S(1 + i_\ell) + \beta\varepsilon(i_\ell - i_d)\end{aligned}$$

So the two banks prefer this iff

$$\Pi(R_L - \varepsilon) + \beta\varepsilon(i_\ell - i_d) > \Pi(R_L)$$

again arranging and taking the limit as $\varepsilon \to 0$, we get that bank L prefers to borrow from bank S whenever

$$-\Pi'(R_L) > -\beta(i_\ell - i_d)$$

which is always satisfied. Therefore, bank L will lend up to R_S if $R_L + R_S > 0$ or until $R_L + z = 0$ if $R_L + R_S < 0$. Will bank L lend more? If $R_L + R_S > 0$, the first case above shows that bank L will not lend more. If $R_L + R_S < 0$, the second case above shows that bank L will not borrow more. Hence, given $R_L > 0$ *with* $R_S < 0$, *and* bank S does not enter the core market, then the optimal z is $z^* = R_S$.

Therefore, if bank S enters the core market, both banks L S enter the core market with $(R_L + R_S)/2$ and $z^* = (R_S - R_L)/2$. If bank S does not enter the core market, then bank L enters with $R_L + R_S$ and $z^* = R_S$. It remains to find the conditions under which bank S enters the core market. Bank S enters whenever

$$\begin{aligned}2\Pi\left(\frac{R_S + R_L}{2}\right) - \gamma + 2\beta\left(\frac{R_S + R_L}{2}\right)\left[1 + i\left(\frac{R_S + R_L}{2}\right)\right] \\ \geq \Pi\left(R_S + R_L\right) + \beta\left(R_S + R_L\right)\left[1 + i\left(R_S + R_L\right)\right]\end{aligned}$$

However, notice that $i\left(\frac{R_S+R_L}{2}\right) = i\left(R_S + R_L\right)$ as only the sign of $R_S + R_L$ matters for $i(.)$. Therefore, bank S enters whenever

$$2\Pi\left(\frac{R_S + R_L}{2}\right) - \gamma \geq \Pi\left(R_S + R_L\right).$$

Define $\bar{R}^-$ as the reserve balance that uniquely satisfies

$$2\Pi^-(\frac{\bar{R}^-}{2}) - \gamma = \Pi^-(\bar{R}^-),$$

and $\bar{R}^+$ as the balance that uniquely satisfies

$$2\Pi^+(\frac{\bar{R}^+}{2}) - \gamma = \Pi^+(\bar{R}^+).$$

Then bank S enters whenever $R_S + R_L > \bar{R}^+$ or $R_S + R_L < \bar{R}^-$. Note that

$$\Pi(\frac{\bar{R}^-}{2}) - \gamma = \Pi^-(\bar{R}^-) - \Pi^-(\frac{\bar{R}^-}{2}) > 0,$$
$$\Pi^+(\frac{\bar{R}^+}{2}) - \gamma = \Pi^+(\bar{R}^+) - \Pi^+(\frac{\bar{R}^+}{2}) > 0.$$

Therefore, $\bar{R}^-/2 < \hat{R}^- < 0 < \hat{R}^+ < \bar{R}^+/2$. This implies that whenever a bank S shares its balance with a bank L (i.e., $R_S + R_L \notin [\bar{R}^-, \bar{R}^+]$), it will indeed have an incentive to enter the core market in the following sub-period. □

D Proof of Lemma 5

D.1 Total Surplus in Relationship

Recall that

$$v_3(R, L, n) = \max\{\Pi(R) - \gamma, 0\} + \beta\{L + R(1 + i(R))\} + \beta\bar{v}_1(n).$$

If bank S lends z to bank L, the relationship will be continued. His payoff is

$$\begin{aligned} & v_3(R_S - z, z(1+\hat{i}), 1) \\ = \; & \max\{\Pi(R_S - z) - \gamma, 0\} + \beta\left\{z(1+\hat{i}) + (R_S - z)(1 + i(R_s - z))\right\} + \beta\bar{v}_1(1). \end{aligned}$$

If bank S does not lend, the relationship will be terminated if the bank L has a backup partner ($c_L = 1$). In that case, the bank S becomes single in the next if $c_S = 0$. Therefore, his payoff is

$$\begin{aligned} & v_3(R_S, 0, c_S, c_L) \\ = \; & \max\{\Pi(R_S) - \gamma, 0\} + \beta\{R_S(1 + i(R_S))\} + (1 - c_S)c_L\beta\bar{v}_1(0) + [1 - (1 - c_S)c_L]\beta\bar{v}_1(1). \end{aligned}$$

As a result, the bank S's trade surplus is

$$\begin{aligned} & TS_S(z, \hat{i}; c_S, c_L) \\ = & v_3(R_S - z, 0, 0) - v_3(R_S, 0, 0) + \beta z(1+\hat{i}) + \beta(1 - c_S)c_L[\bar{v}_1(1) - \bar{v}_1(0)] \\ = & \max\{\Pi(R_S - z) - \gamma, 0\} - \max\{\Pi(R_S) - \gamma, 0\} \\ & + \beta\left\{z(1+\hat{i}) + (R_S - z)(1 + i(R_S - z)) - R_S(1 + i(R_S))\right\} + \beta(1 - c_S)c_L[\bar{v}_1(1) - \bar{v}_1(0)]. \end{aligned}$$

Similarly, the surplus for a bank L with reserves R_L is

$$V_3(R_L + z, z(1 + \hat{i}), 1) - V_3(R_L, 0, 1),$$

where

$$\begin{aligned} V_3(R_L + z, z(1 + \hat{i}), 1) &= \Pi(R_L + z) + \beta\left\{-z(1 + \hat{i}) + (R_L + z)(1 + i(R_L + z))\right\} + \beta\bar{V}_1(1) \\ V_3(R_L, 0, 1) &= \Pi(R_L) + \beta[R_L(1 + i(R_L))] + (1 - c_L)c_S\beta\bar{V}_1(0) + [1 - (1 - c_L)c_S]\beta\bar{V}_1(1). \end{aligned}$$

Hence, the surplus is

$$\begin{aligned} & TS_L(z, \hat{i}; c_S, c_L) \\ =& V_3(R_L + z, 0, 1) - V_3(R_L, 0, 0) - \beta z(1 + \hat{i}) \\ =& \Pi(R_L + z) - \Pi(R_L) \\ & + \beta\left\{-z(1 + \hat{i}) + (R_L + z)(1 + i(R_L + z)) - R_L(1 + i(R_L))\right\} \\ & + (1 - c_L)c_S\beta[\bar{V}_1(1) - \bar{V}_1(0)] \end{aligned}$$

Then, the total surplus from a loan of size z is

$$\begin{aligned} & TS(z; c_S, c_L) \\ = \; & v_3(R_S - z, 0, 0) - v_3(R_S, 0, 0) + V_3(R_L + z, 0, 0) - V_3(R_L, 0, 0) \\ & + \beta(1 - c_S)c_L[\bar{v}_1(1) - \bar{v}_1(0)] + (1 - c_L)c_S\beta[\bar{V}_1(1) - \bar{V}_1(0)] \\ = \; & \max\{\Pi(R_S - z) - \gamma, 0\} - \max\{\Pi(R_S) - \gamma, 0\} + \Pi(R_L + z) - \Pi(R_L) \\ & + \beta\{(R_S - z)(1 + i(R_S - z)) - R_S(1 + i(R_S)) + (R_L + z)(1 + i(R_L + z)) - R_L(1 + i(R_L))\} \\ & + \beta(1 - c_S)c_L[\bar{v}_1(1) - \bar{v}_1(0)] + (1 - c_L)c_S\beta[\bar{V}_1(1) - \bar{V}_1(0)]. \end{aligned}$$

D.2 Relationship Rates and Expected Payoffs

Given the result above, when $R_S + R_L \in [\bar{R}^-, \bar{R}^+]$, the relationship rate is

$$\begin{aligned} & 1 + \hat{i}(R_S, R_L; c_S, c_L) \\ =& (1 - \theta)\frac{v_3(R_S, 0, 0) - v_3(0, 0, 0)}{\beta R_S} + \theta\frac{V_3(R_L + R_s, 0, 0) - V_3(R_L, 0, 0)}{\beta R_S} + \frac{\mathcal{V}(c_S, c_L)}{R_S}, \end{aligned}$$

where

$$\mathcal{V}(c_S, c_L) \equiv (1 - \sigma)\{\theta(1 - c_L)c_S\,[V_1(1) - V_1(0)] - (1 - \theta)(1 - c_S)c_L[v_1(1) - v_1(0)]\}.$$

Otherwise, it is

$$\begin{aligned}
&1+\hat{i}(R_S,R_L;c_S,c_L)\\
=&(1-\theta)\frac{v_3(R_S,0,0)-v_3((R_S+R_L)/2,0,0)}{\beta(R_S-R_L)/2}+\theta\frac{V_3((R_S+R_L)/2,0,0)-V_3(R_L,0,0)}{\beta(R_S-R_L)/2}\\
&+\frac{\mathcal{V}(c_S,c_L)}{(R_S-R_L)/2}.
\end{aligned}$$

We now derive the expected payoffs of different types *after* their liquidity shock and search. The payoff of the S bank for $R_S+R_L\in[\bar{R}^-,\bar{R}^+]$ is

$$\begin{aligned}
&v_3(0,D(R_S,R_L),1;c_S,c_L)\\
=\;&\beta D(R_S,R_L)+\beta\bar{v}_1(1)\\
=\;&(1-\theta)v_3(R_S,0,0)+\theta[V_3(R_L+R_s,0,0)-V_3(R_L,0,0)]+\beta\bar{v}_1(1)+\beta\mathcal{V}(c_S,c_L).
\end{aligned}$$

Otherwise, it is

$$\begin{aligned}
&v_3(\frac{R_S+R_L}{2},D(R_S,R_L),1;c_S,c_L)\\
=\;&\Pi(\frac{R_S+R_L}{2})-\gamma+\beta\left\{D(R_S,R_L)+\frac{R_S+R_L}{2}(1+i(\frac{R_S+R_L}{2}))\right\}+\beta\bar{v}_1(1)\\
=\;&\Pi(\frac{R_S+R_L}{2})-\gamma+\beta\frac{R_S+R_L}{2}(1+i(\frac{R_S+R_L}{2}))\\
&+(1-\theta)[v_3(R_S,0,0)-v_3(\frac{R_S+R_L}{2},0,0)]+\theta[V_3(\frac{R_S+R_L}{2},0,0)-V_3(R_L,0,0)]\\
&+\beta\bar{v}_1(1)+\beta\mathcal{V}(c_S,c_L).
\end{aligned}$$

The payoff of bank L is

$$\begin{aligned}
&V_3(R_L+z(R_S,R_L),-L(R_S,R_L),1;c_S,c_L)\\
=\;&\Pi(R_L+z(R_S,R_L))\\
&+\beta\{-D(R_S,R_L)+(R_L+z(R_S,R_L))(1+i(R_L+z(R_S,R_L)))\}+\beta\bar{V}_1(1)\\
=\;&\Pi(R_L+z(R_S,R_L))\\
&+\beta(R_L+z(R_S,R_L))(1+i(R_L+z(R_S,R_L)))\\
&-(1-\theta)[v_3(R_S,0,0)-v_3(R_S-z(R_S,R_L),0,0)]\\
&-\theta[V_3(R_L+z(R_S,R_L),0,0)-V_3(R_L,0,0)]-\beta\mathcal{V}(c_S,c_L)+\beta\bar{V}_1(1).
\end{aligned}$$

The payoff of bank L with no partner is

$$V_3(R_L, 0, 0) = \Pi(R_L) + \beta[R_L(1 + i(R_L))] + \beta\bar{V}_1(0).$$

Finally, the payoff of a single bank S is

$$v_3(R_S, 0, 0) = \max\{\Pi(R_S) - \gamma, 0\} + \beta\{R_S(1 + i(R_S))\} + \beta\bar{v}_1(0).$$

E Proof of propositions 7 and 8

Proof. We will need to know the gain from a relationship for S and L banks, that is $v_1(1) - v_1(0)$ and $V_1(1) - V_1(0)$. Notice that when a bank S with an amount of reserves R_S and contact c_S, matched with a bank L holding reserves R_L and contact c_L, we can write the value of bank S with $R = R_S - z(R_S, R_L)$, as

$$v_3(R, \hat{D}(R_S, R_L), 1) \quad = \quad \max\{\Pi(R) - \gamma, 0\} + \beta\{\Phi(R_S, R_L) + R(1 + i(R))\} + \beta\mathcal{V}(c_S, c_L) + \beta\bar{v}_1(1),$$

while the same bank S, but with no partner, has

$$v_3(R_S, 0, 0) \quad = \quad \max\{\Pi(R_S) - \gamma, 0\} + \beta R_S(1 + i(R_S)) + \beta\bar{v}_1(0),$$

Hence, given $\mathbf{c} = (c_S, c_L)$,

$$\begin{aligned} v_3(R, \hat{D}(R_S, R_L), 1) - v_3(R_S, 0, 0) \quad = \quad & \max\{\Pi(R) - \gamma, 0\} + \beta\{\Phi(R_S, R_L) + R(1 + i(R))\} \\ & - \max\{\Pi(R_S) - \gamma, 0\} - \beta R_S(1 + i(R_S)) \\ & + \beta\mathcal{V}(c_S, c_L) + \beta[\bar{v}_1(1) - \bar{v}_1(0)] \end{aligned}$$

Notice that the first two lines are independent of the contact $\mathbf{c}$, and taking expectations of the first two lines with respect to R_S and R_L we define the expected per-period benefit of having a business partner for the S bank as

$$\Gamma_S \quad \equiv \quad \int_{R_S}\int_{R_L} \left[\begin{array}{c} \max\{\Pi(R_S - z(R_S, R_L)) - \gamma, 0\} \\ +\beta\{\Phi(R_S, R_L) + (R_S - z(R_S, R_L))(1 + i(R_S - z(R_S, R_L)))\} \\ - \max\{\Pi(R_S) - \gamma, 0\} - \beta R_S(1 + i(R_S)) \end{array} \right] dF(R_L)dG(R_S)$$

Then

$$v_1(1) - v_1(0) \quad = \quad \Gamma_S + \beta E_{\mathbf{c}}[\mathcal{V}(\mathbf{c})] + \beta\bar{v}_1(1) - v(0)$$

where

$$\mathcal{V}(\mathbf{c}) = (1-\sigma)\{\theta(1-c_L)c_S\,[V_1(1)-V_1(0)] - (1-\theta)(1-c_S)c_L[v_1(1)-v_1(0)]\}$$

and

$$v(0) = \max\{\rho_{S1}\beta\bar{v}_1(1) + \beta(1-\rho_{S1})v_1(0) - \kappa_S, \rho_{S0}\beta\bar{v}_1(1) + \beta(1-\rho_{S0})v_1(0)\}$$

so the term $v(0)$ is not discounted because it includes the decision to search for a partner which is done today. In the equilibrium where $\alpha_L = 0$ we have $c_S = 0$, $\rho_{S0} = 0$, $\rho_{S1} = N_L \min\{1, \frac{n}{\alpha_S N_S}\}$ and $\rho_{L0} = \min\{1, \frac{\alpha_S N_S}{n}\}$. So $\mathcal{V}(0, c_L) = -(1-\sigma)(1-\theta)c_L[v_1(1)-v_1(0)]$ and $v(0) = \max\{\rho_{S1}\beta\bar{v}_1(1) + \beta(1-\rho_{S1})v_1(0) - \kappa_S, \beta v_1(0)\}$. Hence, using $E(c_L) = \rho_{L0}$ we obtain

$$\begin{aligned} v_1(1) - v_1(0) &= \Gamma_S - \beta(1-\sigma)(1-\theta)\rho_{L0}[v_1(1)-v_1(0)] + \beta(1-\sigma)\,[v_1(1)-v_1(0)] \\ &\quad - \max\{\rho_{S1}\beta(1-\sigma)\,[v_1(1)-v_1(0)] - \kappa_S, 0\} \end{aligned}$$

Since we concentrate on the case where S banks search, we get $\rho_{S1}\beta(1-\sigma)\,[v_1(1)-v_1(0)] \geq \kappa_S$ so that

$$v_1(1) - v_1(0) = \frac{\Gamma_S + \kappa_S}{1 + \beta(1-\theta)\rho_{L0}(1-\sigma) + \rho_{S1}\beta(1-\sigma) - \beta(1-\sigma)} \tag{E.1}$$

Similarly,

$$V_1(1) - V_1(0) = \frac{\Gamma_L - \beta(1-\theta)\rho_{L0}(1-\sigma)[v_1(1)-v_1(0)]}{1 - (1-\rho_{L0})\beta\,(1-\sigma)} \tag{E.2}$$

Since $\alpha_L = 0$ and $\alpha_S \in [0,1]$, we now need to check conditions for which

$$\begin{aligned} [\rho_{S1} - \rho_{S0}](1-\sigma)\beta[v_1(1)-v_1(0)] &\lesseqqgtr \kappa_S \\ [\rho_{L1} - \rho_{L0}](1-\sigma)\beta[V_1(1)-V_1(0)] &< \kappa_L \end{aligned}$$

where

$$\begin{aligned} \rho_{L0} &= \min\{1, \frac{\alpha_S N_S}{n(1-\alpha_L N_L)}\} = \min\{1, \alpha_S \frac{N_S}{n}\} \qquad \text{(E.3)} \\ \rho_{S0} &= \min\{1, \frac{n\alpha_L N_L}{1-\alpha_S N_S}\} = 0 \\ \rho_{L1} &= \frac{(1-\alpha_S)N_S}{1-\alpha_S N_S}\min\{1, \frac{1-\alpha_S N_S}{n\alpha_L N_L}\} = \frac{(1-\alpha_S)N_S}{1-\alpha_S N_S} \\ \rho_{S1} &= \frac{nN_L(1-\alpha_L)}{n(1-\alpha_L N_L)}\min\{1, \frac{n(1-\alpha_L N_L)}{\alpha_S N_S}\} = N_L \min\{1, \frac{n}{\alpha_S N_S}\} \end{aligned}$$

In any equilibrium the number of banks S with a business partner is equal to the number of banks L also with a business partner, so that

$$1 - N_S = n(1 - N_L)$$

so using 9 and 10 we obtain

$$\begin{aligned} &\sigma \left\{ [\rho_{S0} + \alpha_S(\rho_{S1} - \rho_{S0})] - n \left[\rho_{L0} + \alpha_L \left(\rho_{L1} - \rho_{L0} \right) \right] \right\} \\ = &(n-1)(1-\sigma) \left[\rho_{L0} + \alpha_L \left(\rho_{L1} - \rho_{L0} \right) \right] \left[\rho_{S0} + \alpha_S(\rho_{S1} - \rho_{S0}) \right] \end{aligned}$$

using the above expression for ρ's using (E.3), we obtain that when $\alpha_L = 0$,

$$\sigma \left(N_L - N_S \right) = (n-1)(1-\sigma) \min\{1, \alpha_S \frac{N_S}{n}\} N_L \tag{E.4}$$

which we can write as

$$\sigma(1 - N_L) = (1-\sigma) \min\{1, \alpha_S \frac{1 - n(1 - N_L)}{n}\} N_L$$

We now concentrate on the three sub-cases of interest. Suppose

$$\min\{1, \frac{1 - n(1 - N_L)}{n}\} = \frac{1 - n(1 - N_L)}{n}$$

Then in this case setting $\alpha_S = 1$,

$$N_L = \frac{-\left[\sigma n + (1-\sigma)(1-n) \right] + \sqrt{\left[\sigma n + (1-\sigma)(1-n) \right]^2 + 4(1-\sigma)\sigma n^2}}{2(1-\sigma)n}$$

And we can check that

$$\frac{1 - n(1 - N_L)}{n} < 1$$

whenever $n > 1/(2 - \sigma)$. So in this case, whenever $n > 1/(2 - \sigma)$, we get $\alpha_S \frac{1-n(1-N_L)}{n} < 1$ and the solution is given by

$$\sigma \left(N_L - N_S \right) = (n-1)(1-\sigma) \alpha_S \frac{N_S}{n} N_L,$$

or using $N_S = 1 - n(1 - N_L)$, we get $N_L = N_L^*(n, \alpha_S)$. This is an equilibrium whenever

$$\begin{aligned} N_L^* \min\{1, \frac{n}{\alpha_S N_S^*}\}(1-\sigma)\beta[v_1(1) - v_1(0)] &\geq \kappa_S \\ N_L^*(1-\sigma)\beta[v_1(1) - v_1(0)] &\geq \kappa_S \end{aligned}$$

so using (E.1) and (E.3)

$$N_L^*(1-\sigma)\beta \frac{\Gamma_S}{1 + \beta(1-\theta)\alpha_S \frac{N_S^*}{n}(1-\sigma) - \beta(1-\sigma)} \geq \kappa_S$$

($\alpha_S = 1$ if $>$, $\alpha_S \in (0,1)$ if $=$ and $\alpha_S = 0$ if $<$). Then since $c_S = 0$, we get

$$
\begin{aligned}
E\left[\mathcal{V}(\mathbf{c})\right] &= -(1-\sigma)(1-\theta)E\left[c_L\right]\left[v_1(1)-v_1(0)\right] \\
&= -(1-\sigma)(1-\theta)\rho_{L0}[v_1(1)-v_1(0)] \\
&= \frac{-(1-\sigma)(1-\theta)\rho_{L0}\left(\Gamma_S+\kappa_S\right)}{1+\beta(1-\theta)\rho_{L0}(1-\sigma)+\rho_{S1}\beta(1-\sigma)-\beta(1-\sigma)} \\
&= \frac{-(1-\sigma)(1-\theta)\min\{1,\alpha_S\frac{N_S}{n}\}\left(\Gamma_S+\kappa_S\right)}{1+\beta(1-\theta)\min\{1,\alpha_S\frac{N_S}{n}\}(1-\sigma)+N_L\min\{1,\frac{n}{\alpha_S N_S}\}\beta(1-\sigma)-\beta(1-\sigma)} \\
&= \frac{-(1-\sigma)(1-\theta)(\Gamma_S+\kappa_S)}{1+\beta(1-\theta)\alpha_S\frac{N_S^*}{n}(1-\sigma)+N_L^*\beta(1-\sigma)-\beta(1-\sigma)}
\end{aligned}
$$

We now concentrate on the sub-case when $n < 1/(2-\sigma)$.

Case 1. $\alpha_S = 0$**.**

For $\alpha_S = 0$, single banks S do not search and it is easy to see that all banks are single, that is $N_L = N_S = 1$. Then this is an equilibrium iff

$$
\begin{aligned}
(1-\sigma)\beta[v_1(1)-v_1(0)] &< \kappa_S \\
(1-\sigma)\beta[V_1(1)-V_1(0)] &< \kappa_L
\end{aligned}
$$

Using (E.1) and (E.2), as well as (E.3), these conditions become

$$
\begin{aligned}
(1-\sigma)\beta\frac{\Gamma_S}{1-\beta(1-\sigma)} &< \kappa_S \\
(1-\sigma)\beta\frac{\Gamma_L}{1-\beta\left(1-\sigma\right)} &< \kappa_L
\end{aligned}
$$

Then since $c_S = 0$, we get

$$
\begin{aligned}
E\left[\mathcal{V}(\mathbf{c})\right] &= \frac{-(1-\sigma)(1-\theta)\min\{1,\alpha_S\frac{N_S}{n}\}\left(\Gamma_S+\kappa_S\right)}{1+\beta(1-\theta)\min\{1,\alpha_S\frac{N_S}{n}\}(1-\sigma)+N_L\min\{1,\frac{n}{\alpha_S N_S}\}\beta(1-\sigma)-\beta(1-\sigma)} \\
&= 0
\end{aligned}
$$

Case 2. $\alpha_S \in (0,1]$**.**

From (E.4), we have to distinguish two cases: $n \gtrless 1-n(1-\sigma)$.

First, we consider the case where $n < 1-n(1-\sigma)$.

There are two sub-cases. First there are less banks L than the number of single banks S searching: $n \leq \alpha_S N_S$. Then

$$
\sigma\left(N_L-N_S\right) = (n-1)(1-\sigma)N_L
$$

and since $N_S = 1 - n(1 - N_L)$,

$$\begin{aligned} \sigma(1 - N_L) &= (1-\sigma)N_L \\ N_L &= \sigma \end{aligned}$$

L banks are only single because they are hit by the separation shock. It is easy to check that the no-search condition for banks L is always satisfied, while the search condition for banks S is

$$\begin{aligned} [\rho_{S1} - \rho_{S0}](1-\sigma)\beta[v_1(1) - v_1(0)] &\geq \kappa_S \\ N_L \min\{1, \frac{n}{\alpha_S N_S}\}(1-\sigma)\beta[v_1(1) - v_1(0)] &\geq \kappa_S \\ \frac{\sigma n}{(1-n(1-\sigma))}(1-\sigma)\beta[v_1(1) - v_1(0)] &\geq \alpha_S \kappa_S \end{aligned}$$

$$\frac{\sigma n}{(1-n(1-\sigma))}(1-\sigma)\beta \frac{\Gamma_S + \kappa_S}{1 + \beta(1-\theta)\rho_{L0}(1-\sigma) + \rho_{S1}\beta(1-\sigma) - \beta(1-\sigma)} \geq \alpha_S \kappa_S$$

Using (E.3) and the fact that $n \leq \alpha_S N_S$, this condition is simply

$$\frac{\sigma n(1-\sigma)\beta}{(1-n(1-\sigma))} \frac{\Gamma_S + \kappa_S}{\alpha_S[1 - \beta\theta(1-\sigma)] + \sigma \frac{n}{1-(1-\sigma)n}\beta(1-\sigma)} \geq \kappa_S$$

Hence, when $n \leq \alpha_S N_S$, we obtain $\alpha_S = 1$ whenever

$$\frac{\sigma n(1-\sigma)\beta}{[1-n(1-\sigma)][1-\beta\theta(1-\sigma)]}\Gamma_S > \kappa_S$$

and since $n \leq \alpha_S N_S$, this requires

$$n < \frac{1}{2-\sigma}$$

while when κ_S is above the threshold above, then $\alpha_S \in (0,1)$ it is given by

$$\alpha_S = \frac{\sigma n(1-\sigma)\beta}{[1-n(1-\sigma)][1-\beta\theta(1-\sigma)]} \frac{\Gamma_S}{\kappa_S}$$

Since $n \leq \alpha_S N_S$, this requires

$$\frac{\sigma n(1-\sigma)\beta}{[1-n(1-\sigma)][1-\beta\theta(1-\sigma)]}\Gamma_S \leq \kappa_S \leq \frac{\sigma(1-\sigma)\beta}{[1-\beta\theta(1-\sigma)]}\Gamma_S \tag{E.5}$$

At the upper bound, notice that $\alpha_S = \frac{n}{1-(1-\sigma)n}$. In this case,

$$\begin{aligned} E[\mathcal{V}(\mathbf{c})] &= \frac{-(1-\sigma)(1-\theta)\min\{1, \alpha_S \frac{N_S}{n}\}(\Gamma_S + \kappa_S)}{1 + \beta(1-\theta)\min\{1, \alpha_S \frac{N_S}{n}\}(1-\sigma) + N_L \min\{1, \frac{n}{\alpha_S N_S}\}\beta(1-\sigma) - \beta(1-\sigma)} \\ E[\mathcal{V}(\mathbf{c})] &= \frac{-(1-\sigma)(1-\theta)(\Gamma_S + \kappa_S)}{1 + \beta(1-\theta)(1-\sigma) + N_L \frac{n}{\alpha_S N_S}\beta(1-\sigma) - \beta(1-\sigma)} \\ E[\mathcal{V}(\mathbf{c})] &= \frac{-(1-\sigma)(1-\theta)(\Gamma_S + \kappa_S)}{1 + \beta(1-\theta)(1-\sigma) + \sigma \frac{n}{\alpha_S(1-(1-\sigma)n)}\beta(1-\sigma) - \beta(1-\sigma)} \end{aligned}$$

The second sub-case is when there are more banks L than the number of single banks S searching: $n \geq \alpha_S N_S$. From (E.4), we obtain

$$\sigma\left(N_L - N_S\right) = (n-1)(1-\sigma)\alpha_S \frac{N_S}{n} N_L$$

and since $N_S = 1 - n(1 - N_L)$, we can solve for N_L as

$$N_L = \frac{-\left[\sigma n + (1-\sigma)\alpha_S(1-n)\right] + \sqrt{\left[\sigma n + (1-\sigma)\alpha_S(1-n)\right]^2 + 4(1-\sigma)\sigma\alpha_S n^2}}{2(1-\sigma)\alpha_S n}$$

When $\alpha_S = \frac{n}{1-(1-\sigma)n}$ then $N_L = \sigma$.

$$\begin{aligned}
[\rho_{S1} - \rho_{S0}](1-\sigma)\beta[v_1(1) - v_1(0)] &= \kappa_S \\
N_L(1-\sigma)\beta[v_1(1) - v_1(0)] &= \kappa_S
\end{aligned}$$

So we need to check

$$N_L(1-\sigma)\beta \frac{\Gamma_S + \kappa_S}{1 + \beta(1-\theta)\rho_{L0}(1-\sigma) + \rho_{S1}\beta(1-\sigma) - \beta(1-\sigma)} \geq \kappa_S$$

$$\begin{aligned}
N_L(1-\sigma)\beta \frac{\Gamma_S + \kappa_S}{1 + \beta(1-\theta)(1-\sigma)\alpha_S\frac{N_S}{n} + N_L\beta(1-\sigma) - \beta(1-\sigma)} &\geq \kappa_S \\
\frac{N_L(1-\sigma)\beta\Gamma_S}{\left(1 + \beta(1-\theta)(1-\sigma)\alpha_S\frac{N_S}{n} - \beta(1-\sigma)\right)} &\geq \kappa_S
\end{aligned}$$

with equality when $\alpha_S \in (0,1)$. When κ_S is at the upper bound of (E.5), then α_S is given by

$$\begin{aligned}
\frac{N_L}{\left(1 - \beta(1-\sigma) + \beta(1-\theta)(1-\sigma)\alpha_S\frac{N_S}{n}\right)} &= \frac{\sigma}{[1 - \beta\theta(1-\sigma)]} \\
N_L\left[1 - \beta\theta(1-\sigma)\right] &= \sigma\left(1 - \beta(1-\sigma) + \beta(1-\theta)(1-\sigma)\alpha_S\frac{N_S}{n}\right) \\
N_L\left[1 - \beta\theta(1-\sigma)\right] &= \sigma\left(1 - \beta(1-\sigma) + \beta(1-\theta)(1-\sigma)\alpha_S\frac{1 - n + nN_L}{n}\right) \\
\alpha_S &= \frac{nN_L\left[1 - \beta\theta(1-\sigma)\right] - \sigma\left(1 - \beta(1-\sigma)\right)}{\sigma\left(\beta(1-\theta)(1-\sigma)(1 - n + nN_L)\right)} \qquad \text{(E.6)}
\end{aligned}$$

and using the expression for N_L in this case and some algebra,

$$\alpha_S = \frac{n}{1 - n + n\sigma}.$$

The condition for the L banks not searching is

$$\begin{aligned}
[\rho_{L1} - \rho_{L0}](1-\sigma)\beta[V_1(1) - V_1(0)] &< \kappa_L \\
\left[\frac{(1-\alpha_S)N_S}{1 - \alpha_S N_S} - \min\{1, \alpha_S\frac{N_S}{n}\}\right](1-\sigma)\beta[V_1(1) - V_1(0)] &< \kappa_L
\end{aligned}$$

$$\left[\frac{(1-\alpha_S)N_S}{1-\alpha_S N_S}-\alpha_S\frac{N_S}{n}\right](1-\sigma)\beta[V_1(1)-V_1(0)] \quad < \quad \kappa_L$$

$$\left[\frac{(1-\alpha_S)N_S}{1-\alpha_S N_S}-\alpha_S\frac{N_S}{n}\right](1-\sigma)\beta\frac{\Gamma_L-\beta(1-\theta)\rho_{L0}(1-\sigma)[v_1(1)-v_1(0)]}{1-(1-\rho_{L0})\beta\,(1-\sigma)} \quad < \quad \kappa_L$$

$\alpha_S \to 0$ could be an equilibrium. Indeed, by L'Hopital's rule $N_L \to 1$, Hence $N_S \to 1$ and the L bank's condition for no search becomes

$$(1-\sigma)\beta\frac{\Gamma_L}{1-\beta\,(1-\sigma)} < \kappa_L$$

while the S bank's condition for search becomes

$$(1-\sigma)\beta\frac{\Gamma_S}{1-\beta(1-\sigma)} = \kappa_S$$

To conclude, when $n \geq \alpha_S N_S$, $\alpha_S < n/(1-n+n\sigma) \leq 1$ is given by (E.6) whenever

$$\frac{\sigma(1-\sigma)\beta}{[1-\beta\theta(1-\sigma)]}\Gamma_S \leq \kappa_S < (1-\sigma)\beta\frac{\Gamma_S}{1-\beta\,(1-\sigma)}$$

Suppose now $1-n(1-\sigma) > n$. Then $N_S = 1-n+nN_L$ and so

$$N_L^*(n,\alpha_S) = \frac{-\left[\sigma n+(1-\sigma)\alpha_S(1-n)\right]+\sqrt{\left[\sigma n+(1-\sigma)\alpha_S(1-n)\right]^2+4(1-\sigma)\sigma\alpha_S n^2}}{2(1-\sigma)\alpha_S n}$$

and

$$N_S^*(n,\alpha_S) = 1-n+nN_L^*(n,\alpha_S)$$

while $\alpha_S \in (0,1)$ is implicitly given by

$$\frac{N_L^*(n,\alpha_S)(1-\sigma)\beta\Gamma_S}{\left(1+\beta(1-\theta)(1-\sigma)\alpha_S\frac{N_S^*(n,\alpha_S)}{n}-\beta(1-\sigma)\right)} = \kappa_S$$

while $\alpha_S = 1$ whenever

$$\frac{N_L^*(n,1)(1-\sigma)\beta\Gamma_S}{\left(1+\beta(1-\theta)(1-\sigma)\frac{N_S^*(n,1)}{n}-\beta(1-\sigma)\right)} > \kappa_S$$

and $\alpha_S = 0$ whenever

$$(1-\sigma)\beta\frac{\Gamma_S}{1-\beta(1-\sigma)} \leq \kappa_S$$

$$E\left[\mathcal{V}(\mathbf{c})\right] \quad = \quad \frac{-(1-\sigma)(1-\theta)\min\{1,\alpha_S\frac{N_S}{n}\}\left(\Gamma_S+\kappa_S\right)}{1+\beta(1-\theta)\min\{1,\alpha_S\frac{N_S}{n}\}(1-\sigma)+N_L\min\{1,\frac{n}{\alpha_S N_S}\}\beta(1-\sigma)-\beta(1-\sigma)}$$

Since $n \leq 1/(2-\sigma)$ and $n \geq \alpha_S N_S$,

$$\begin{aligned} E\left[\mathcal{V}(\mathbf{c})\right] &= \frac{-(1-\sigma)(1-\theta)\alpha_S \frac{N_S^*(n,\alpha_S)}{n}\left(\Gamma_S+\kappa_S\right)}{1+\beta(1-\theta)\alpha_S \frac{N_S^*(n,\alpha_S)}{n}(1-\sigma)+N_L^*(n,\alpha_S)\beta(1-\sigma)-\beta(1-\sigma)} \\ &= \frac{-(1-\sigma)(1-\theta)\alpha_S \frac{N_S^*(n,\alpha_S)}{n}\left(\Gamma_S+\kappa_S\right)}{1-\beta(1-\sigma)\left[1-N_L^*(n,\alpha_S)-(1-\theta)\alpha_S \frac{N_S^*(n,\alpha_S)}{n}\right]} \end{aligned}$$

It is clear that the parameter space defines three distinct and non-overlapping but connected regions. In each of these regions the equilibrium is well defined and unique. So the equilibrium with $\alpha_L = 0$ is unique. $\square$

F Numerical Algorithm

Below we briefly present the numerical algorithm developed for finding steady state equilibria $\{\alpha_S, \alpha_L, N_S, N_L, \mathcal{V}, \bar{\mathbf{R}}, \hat{\mathbf{R}}, \mathbf{N}, \boldsymbol{\Omega}\}$ of the model. The basic strategy of the algorithm is to start from an initial guess and solve backward from subperiod 4 to subperiod 1 in order to obtain an update.

A brief description of the algorithm follows:

- **Step 1**. Given an initial guess for $\{\alpha_S, \alpha_L, N_S, N_L, \mathcal{V}, \bar{\mathbf{R}}, \hat{\mathbf{R}}\}$, follow Appendix A to derive $(\mathbf{N}, \boldsymbol{\Omega})$. Follow the conditions in Lemma 3 and 4 to update $(\bar{\mathbf{R}}, \hat{\mathbf{R}})$. Repeat step 1 until $(\bar{\mathbf{R}}, \hat{\mathbf{R}})$ converge.

- **Step 2**. Use Proposition 7 and 8 to obtain $(\alpha_S, \alpha_L, N_S, N_L, \mathcal{V})$. Repeat steps 1 to 2 until $(\alpha_S, \alpha_L, N_S, N_L, \mathcal{V})$ converge.

- **Step 3**. Follow Lemma 1 and 5 to derive the implied distributions of rates in the core and periphery markets.

G Search for a Backup Partner

In the model, only single banks can search for a partner. Here, we briefly consider an extension in which banks with one partner can search for a new one to build a backup relationship. Having a backup relationship may be desirable because it improves a bank's bargaining position in the periphery market. We assume that at the end of a period a bank with two partners needs to choose one of the two potential partners to continue the relationship in the next period. The other one will be broken. Hence, banks always start a period with either 0 or 1 partners. Before the periphery market opens, banks S can pay a cost κ_S to contact a bank L. Similarly, banks L can pay a cost κ_L to contact a banks S. We use N_{L0} to denote the measure of banks L with no partner, and N_{L1} to denote the measure of banks L with 1 partner. We define N_{S0}, N_{S1} for banks S similarly. So we denote by α_{Sm} the measure of banks S with $m = 0, 1$ partner that search for a business partner and by α_{Lm} for banks L with m partner. Hence, for banks not searching, we have:

Banks not searching	Prob. of getting a new partner
L	$\rho_{L0} = \min\{1, \frac{\alpha_{S0}N_{S0}}{n(1-\alpha_{L0}N_{L0}-\alpha_{L1}N_{L1})}\}$
S	$\rho_{S0} = \min\{1, \frac{n\alpha_{L0}N_{L0}}{1-\alpha_{S0}N_{S0}-\alpha_{S1}N_{S1}}\}$

ρ_{L0} is the probability that a non-searching bank L can get a new partner. Note that there are no voluntary breakups in equilibrium. Hence, a bank L can get a new partner only by meeting a single bank S. We define ρ_{S0} for banks S similarly. For searching banks, we have:

Searching banks	Prob. of finding a new partner
L	$\rho_{L1} = \frac{(1-\alpha_{S0})N_{S0}}{1-\alpha_{S0}N_{S0}-\alpha_{S1}N_{S1}} \min\{1, \frac{1-\alpha_{S0}N_{S0}-\alpha_{S1}N_{S1}}{n(\alpha_{L0}N_{L0}+\alpha_{L1}N_{L1})}\}$
S	$\rho_{S1} = \frac{nN_{L0}(1-\alpha_{L0})}{n(1-\alpha_{L0}N_{L0}-\alpha_{L1}N_{L1})} \min\{1, \frac{n(1-\alpha_{L0}N_{L0}-\alpha_{L1}N_{L1}}{\alpha_{S0}N_{S0}+\alpha_{S1}N_{S1}}\}$

The fraction of single banks S searching is still

$$\alpha_{S0} = \begin{cases} 1 & \text{if } [\rho_{S1} - \rho_{S0}](1-\sigma)\beta[v_1(1) - v_1(0)] > \kappa_S \\ 0 & \text{if } [\rho_{S1} - \rho_{S0}](1-\sigma)\beta\,[v_1(1) - v_1(0)] < \kappa_S \\ [0,1] & \text{otherwise} \end{cases}$$

and the fraction of single L banks searching is

$$\alpha_{L0} = \begin{cases} 1 & \text{if } [\rho_{L1} - \rho_{L0}](1-\sigma)\beta[V_1(1) - V_1(0)] > \kappa_L \\ 0 & \text{if } [\rho_{L1} - \rho_{L0}](1-\sigma)\beta[V_1(1) - V_1(0)] < \kappa_L \\ [0,1] & \text{otherwise} \end{cases}$$

The fraction of banks S with a partner searching is

$$\alpha_{S1} = \begin{cases} 1 & \text{if } [\rho_{S1} - \rho_{S0}](1-\sigma)\{\theta(1-P_L)\,[V_1(1) - V_1(0)] + (1-\theta)P_L[v_1(1) - v_1(0)]\} > \kappa_S \\ 0 & \text{if } [\rho_{S1} - \rho_{S0}](1-\sigma)\{\theta(1-P_L)\,[V_1(1) - V_1(0)] + (1-\theta)P_L[v_1(1) - v_1(0)]\} < \kappa_S \\ [0,1] & \text{otherwise} \end{cases}$$

where $P_L = \alpha_{L1}\rho_{L1} + (1-\alpha_{L1})\rho_{L0}$ is the probability of a bank L having a backup partner. Here, the value of getting a backup partner is to improve the bargaining position in the periphery market if the counterparty cannot find one. The fraction of banks L with a partner searching is

$$\alpha_{L1} = \begin{cases} 1 & \text{if } [\rho_{L1} - \rho_{L0}](1-\sigma)\{(1-\theta)(1-P_S)[v_1(1) - v_1(0)] + \theta P_S\,[V_1(1) - V_1(0)]\} > \kappa_L \\ 0 & \text{if } [\rho_{L1} - \rho_{L0}](1-\sigma)\{(1-\theta)(1-P_S)[v_1(1) - v_1(0)] + \theta P_S\,[V_1(1) - V_1(0)]\} < \kappa_L \\ [0,1] & \text{otherwise} \end{cases}$$

where $P_S = \alpha_{S1}\rho_{S1} + (1-\alpha_{S1})\rho_{S0}$ is the probability of a bank S having a backup partner.

Again, the pairwise periphery market rate for a loan between two partner banks is given by

$$1 + \hat{\imath}(R_S, R_L, \mathbf{c}) = \frac{\Phi(R_S, R_L)}{z(R_S, R_L)} + \frac{\mathcal{V}(\mathbf{c})}{z(R_S, R_L)},$$

where

$$\begin{aligned} \beta\Phi(R_S, R_L) \quad &\equiv \quad (1-\theta)\,[v_3(R_S, 0, 0) - v_3(R_S - z(R_S, R_L), 0, 0)] \\ &\quad\quad +\theta\,[V_3(R_L + z(R_S, R_L), 0, 0) - V_3(R_L, 0, 0)] \end{aligned}$$

and

$$\mathcal{V}(\mathbf{c}) \quad \equiv \quad (1-\sigma)\{\theta(1-c_L)c_S\,[V_1(1) - V_1(0)] - (1-\theta)(1-c_S)c_L[v_1(1) - v_1(0)]\}.$$

Therefore, there is a non-zero relationship premium involved in trades in the periphery market if only one side of the transaction has a backup partner. In particular, for a fraction $(1-P_S)P_L$ of loans from banks S to banks L, the relationship premium lowers the loan rate.