Online Appendix

Systemic Banking Panics, Liquidity Risk, and Monetary Policy

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A Equilibrium definition and solution methods

This appendix formulates the definition of equilibrium, lists the equations that are used to solve for the equilibrium, and discusses the approach used to solve for the bad equilibria analyzed in the main part of the paper.

A.1 Bankless economy: equilibrium definition

In the bankless economy, given real lump-sum taxes T_2, an equilibrium is a collection of:

- Prices: prices of capital and consumption goods, Q_0, p_1, and p_2; and return on capital, r_2^K;
- Quantities: money and capital held by households, M_0^h and K_0^h for all h; consumption at $t = 1$, $c_1\left(\varepsilon^H\right)$ and $c_2\left(\varepsilon^L\right)$; and consumption at $t = 2$, c_2^h for all h;

such that households maximize (1) subject to (6)-(8) and the non-negativity constraints on the choice variables, the budget constraint (4) of the fiscal authority holds, and markets clear: money market at $t = 0$, $\int M_0^h dh = \overline{M}$; capital market at $t = 0$, $\int K_0^h dh = \overline{K}$; and goods market at $t = 1$, $\int c_1^h dh = 1$, and $t = 2$, $\int c_2^h dh = A_2\overline{K}$.

I solve for the equilibrium using the following equations:

- The definition of the return on capital, Equation (9);
- The households' first-order conditions for money and capital, which imply Equation (10), and for $c_1\left(\varepsilon^L\right)$, Equation (11);
- The cash-in-advance constraint for impatient households, $p_1 c_1\left(\varepsilon^H\right) = M_0^h$;
- The market clearing conditions at $t = 0$ and $t = 1$, which imply $M_0^h = \overline{M}$, $K_0^h = \overline{K}$, and $\kappa c_1\left(\varepsilon^H\right) + (1-\kappa)\, c_1\left(\varepsilon^L\right) = 1$;
- The feasibility constraint for time-2 consumption, Equation (8).
- The budget constraint of the fiscal authority, Equation (4).

A.2 Economy with banks and full information about banks' balance sheets: equilibrium definition and good equilibrium

In the economy with banks in Section 3, given real lump-sum taxes T_2, an equilibrium is a collection of:

- Prices: prices of capital and consumption goods, Q_0, p_1, and p_2; the return on capital, r_2^K; promised return on deposit, r_2^D; and actual return on deposits, r_2^b for all b;
- Quantities: net worth of banks and households, N_0^h for all h and N_0^b for all b; money, deposits, and capital held by banks, M_0^b, D_0^b, K_0^b for all b; money, deposits, and capital held by households, M_0^h, D_0^h, K_0^h for all h; the fraction γ_1^b of deposits that can be withdrawn at $t = 1$ by depositors of bank b, for all b; withdrawals by households at $t = 1$, w_1^h for all h, and withdrawals from bank b, w_1^b for all b; consumption at $t = 1$, c_1^h for all h; profits π_2^b for all b, and total profits paid to households π; and consumption at $t = 2$, c_2^h for all h;

such that

- Households maximize (1) subject to the budget constraint (16), the constraint on withdrawals (20), the cash-in-advance constraint (21), the resource constraint (24) for time-2 consumption, and the non-negativity constraints;
- Banks maximize (22) subject to the budget constraint (17) and the feasibility constraint (19) for withdrawals;
- The definition of the actual return on deposits, Equation (23), holds for all b;
- The budget constraint (4) of the fiscal authority holds;
- Markets clear: money market at $t = 0$, $\int M_0^h dh + \int M_0^b db = \overline{M}$; capital market at $t = 0$, $\int K_0^h dh + \int K_0^b db = \overline{K}$; deposit market at $t = 0$, $\int D_0^h dh = \int D_0^b db$; goods market at $t = 1$, $\int c_1^h dh = 1$; and consumption goods at $t = 2$, $\int c_2^h dh = A_2 \overline{K}$.

I focus on symmetric equilibria in which all banks have the same amount of deposits, provided that their non-negativity constraints are not binding.

A.2.1 Solving for the good equilibrium (Proof of Proposition 3.2)

The assumption $T_2 = 1$ implies $p_2 = \overline{M}$, as noted in Equation (5). I conjecture that $r_2^b = r_2^D$ for all b, that the limit on withdrawals is $\gamma_1^b = 1$ for all b, that withdrawals are $w_1^h = D_0^h$ for impatient households (i.e., if $\varepsilon_1^h = \varepsilon^H$) and $w_1^h = 0$ for patient households (i.e., if $\varepsilon_1^h = \varepsilon^L$), that consumption at $t = 1$ is the same for all impatient households, $c_1^h = c_1\left(\varepsilon^H\right)$ if $\varepsilon_1^h = \varepsilon^H$, and the same for all patient households, $c_1^h = c_1\left(\varepsilon^L\right)$ if $\varepsilon_1^h = \varepsilon^L$, and that the cash-in-advance constraint (21) holds with equality for all households. Using (18), these conjectures imply that total withdrawals from each bank b are $w_1^b = \kappa D_0^h$. Then, I solve for Q_0, p_1, r_2^K, r_2^D, M_0^h, D_0^h, M_0^b, D_0^b, $c_1\left(\varepsilon^H\right)$, $c_1\left(\varepsilon^L\right)$ (i.e., ten endogenous variables), using the following ten equations:[1]

- The definition of the return on capital, Equation (9);
- The households' first-order conditions for money, deposits, and capital, which imply Equation (25) and Equation (26);

[1] Recall that $P_2 = \overline{M}$ in the three-period model; see Section 2.1.

- The result of the banks' profit maximization problem, which implies $M_0^b = \kappa D_0^b$;
- The cash-in-advance constraints for impatient and patient households, which, combined with the conjecture about withdrawals, imply

$$\begin{aligned} p_1 c_1\left(\varepsilon^H\right) &= M_0^h + D_0^h, \\ p_1 c_1\left(\varepsilon^L\right) &= M_0^h; \end{aligned}$$

- The market clearing conditions for money at $t = 0$ and consumption goods at $t = 1$, which imply $M_0^h + M_0^b = \overline{M}$, and $\kappa c_1\left(\varepsilon^H\right) + (1-\kappa) c_1\left(\varepsilon^L\right) = 1$;
- The market clearing condition for deposits, $D_0^b = D_0^h$, which also implies $r_2^D = r_2^K$ using the results of Proposition 3.1.

Then, I use the results obtained so far to compute:

- The net worth of banks and the wealth of households, using Equation (14) and Equation (15);
- The amount of capital held by households and banks, which follows from the respective budget constraints, Equation (16) and Equation (17);
- Banks' profits, using Equation (22);
- Total profits paid to households at $t = 2$, using $\pi_2 = \int \pi_2^b db$.
- Households' consumption at $t = 2$, using Equation (24).

The market clearing condition for capital, $\int K_0^h dh + \int K_0^b db = \overline{K}$, holds by Walras' law. The conjectures about r_2^b can be verified using Equation (23). The conjecture about γ_1^b is verified because $w_1^b = M_0^b$ (which follows from $w_1^b = \kappa D_0^h$, as noted before, and the banks' profit maximization problem, which implies $M_0^b = \kappa D_0^b$), and thus Equation (19) is not violated. The optimality of the conjectures about withdrawals is verified as follows. For patient households, I check that withdrawing and spending an extra dollar delivers a lower marginal utility than not withdrawing:

$$\varepsilon^L u'\left[c_1\left(\varepsilon^L\right)\right] \frac{1}{p_1} \leq \beta \frac{1 + r_2^D}{p_2}.$$

For an impatient household, I check that if the household does not withdraw all of its deposits and leave a dollar in the bank, its marginal utility of withdrawing the last dollar is greater than the marginal benefit of leaving it in the bank:

$$\varepsilon^H u'\left[c_1\left(\varepsilon^H\right)\right] \frac{1}{p_1} \geq \beta \frac{1 + r_2^D}{p_2}.$$

I also check that it is optimal for households to consume all of their money at $t = 1$, rather than carrying it over to $t = 2$. For patient households, this is the case if

$$\varepsilon^L u' \left[c_1\left(\varepsilon^L\right)\right] \frac{1}{p_1} \geq \beta \frac{1}{p_2},$$

and for impatient households, this is the case if

$$\varepsilon^H u' \left[c_1\left(\varepsilon^H\right)\right] \frac{1}{p_1} \geq \beta \frac{1}{p_2}.$$

Finally, I check that all the non-negativity constraints for households and banks are satisfied.

A.3 Economy with banks and asymmetric information about banks' balance sheets: equilibrium definition and bad equilibria solution method

In the economy with banks and asymmetric information in Section 4, given real lump-sum taxes T_2, an equilibrium is a collection of:

- Prices: prices of capital and consumption goods, Q_0, p_1, and p_2; the return on capital, r_2^K; promised return on deposit, r_2^D; and actual return on deposits, r_2^b for all b;
- Quantities: net worth of banks and households, N_0^h for all h and N_0^b for all b; money, deposits, and capital held by banks, M_0^b, D_0^b, K_0^b for all b; money, deposits, and capital held by households, M_0^h, D_0^h, K_0^h for all h; the fraction γ_1^b of deposits that can be withdrawn at $t = 1$ by depositors of bank b, for all b; withdrawals by households at $t = 1$, w_1^h for all h, and withdrawals from bank b, w_1^b for all b; consumption $t = 1$, c_1^h for all h; profits π_2^b for all b, and total profits paid to households π_2; and consumption at $t = 2$, c_2^h for all h;
- Beliefs: for each household h, a belief over γ_1^b and r_2^b given D_0^b and M_0^b, that is, as a joint cumulative density function $F^h\left(\gamma_1^b, r_2^b | D_0^b, M_0^b\right)$ for all D_0^b, M_0^b and for all b;

such that:

- Households maximize (1) subject to the budget constraint (16), the constraint on withdrawals (20), the cash-in-advance constraint (21), the resource constraint (24) for time-2 consumption, and the non-negativity constraints;
- Banks maximize (22) subject to the budget constraint (17), the feasibility constraint (19) for withdrawals, and the relevant non-negativity constraints;
- The definition of the actual return on deposits, Equation (23), holds for all b;
- The budget constraint (4) of the fiscal authority holds;
- Markets clear: money market at $t = 0$, $\int M_0^h dh + \int M_0^b db = \overline{M}$; capital market at $t = 0$, $\int K_0^h dh + \int K_0^b db = \overline{K}$; deposit market at $t = 0$, $\int D_0^h dh = \int D_0^b db$; goods market at $t = 1$, $\int c_1^h dh = 1$; and consumption goods at $t = 2$, $\int c_2^h dh = A_2 \overline{K}$.

- The beliefs are rational.

I focus on symmetric equilibria in which all banks have the same amount of deposits, provided that their non-negativity constraints are not binding.

A.3.1 Solving for the bad equilibrium

The assumption $T_2 = 1$ implies $p_2 = \overline{M}$, as noted in Equation (5). I conjecture that $r_2^b = r_2^D > 0$ for the fraction $1-\alpha$ of banks with endowment of capital K^H, and $r_2^b = \hat{r} < 0$ for the fraction α of banks with endowment of capital K^L. This conjecture implies that all households with deposits at banks that pay a return $r_2^b = \hat{r} < 0$ withdraw as much as possible at $t = 1$, as discussed in Section 4.2. Thus, banks' feasibility constraint on withdrawals, (19), and households' withdrawal constraints, (20), hold with equality, implying $w_1^b = M_0^b$ and $w_1^h = \gamma_1^{b(h)} D_0^h$. Then, similar to Appendix A.2, I conjecture that withdrawals for households with deposits at banks that pay a return $r_2^b = r_2^D > 0$ are $w_1^h = D_1^h$ for impatient households (i.e., if $\varepsilon_1^h = \varepsilon^H$) and $w_1^h = 0$ for patient households (i.e., if $\varepsilon_1^h = \varepsilon^L$); this conjecture implies that total withdrawals from these banks are $w_1^b = \kappa D_1^h$. In addition, I impose the equilibrium requirement that beliefs are rational, that is, $Pr\left(r_2^b = r_2^D > 0\right) = 1-\alpha$ and $Pr\left(r_2^b = \hat{r} < 0\right) = \alpha$.

Then, I solve for Q_0, p_1, r_2^K, r_2^D, $\hat{r}$ (i.e., the return r_2^b for banks with endowment K^L), M_0^h, D_0^h, N_0^b and K_0^b for banks with endowment $K_{-1}^b = K^L$, N_0^b and K_0^b for banks with endowment $K_{-1}^b = K^H$, M_0^b (which is the same for all banks), D_0^b (which is the same for all banks), $\hat{\gamma}$ (i.e., the fraction of deposits γ_1^b that can be withdrawn from banks that pay a return $r_2^b = \hat{r} < 0$ and thus are subject to runs), and consumption $c_1\left(\varepsilon^H, r_2^b = r_2^D\right)$ (i.e., consumption of impatient households with deposits at banks that pay a return R_2^D), $c_1\left(\varepsilon^H, \hat{r}\right)$ (i.e., the consumption of impatient households with deposits at banks subject to runs), and $c_1\left(\varepsilon^L\right)$ (i.e., the consumption of patient households, which is the same for all patient households), using the following seventeen equations:

- The definition of the return on capital, Equation (9);
- The households' first-order conditions for portfolio choices at $t = 0$, given by (31) and (32), evaluated using the previous conjectures, which imply

$$\beta \frac{1+r_2^K}{p_2} = (1-\alpha)\left[\kappa \varepsilon^H u'\left(\frac{M_0^h + D_0^h}{p_1}\right)\frac{1}{p_1} + (1-\kappa)\beta\frac{1+r_2^D}{p_2}\right] \\ + \alpha\left[\left(\kappa \varepsilon^H u'\left(\frac{M_0^h + \hat{\gamma} D_0^h}{p_1}\right)\frac{\hat{\gamma}}{p_1} + (1-\kappa)\beta\frac{\hat{\gamma}}{p_2}\right) + \beta\frac{(1-\hat{\gamma})(1+\hat{r})}{p_2}\right]$$

and

$$\beta\frac{1+r_2^K}{p_2} = (1-\alpha)\left[\kappa\varepsilon^H u'\left(\frac{M_0^h+D_0^h}{p_1}\right)\frac{1}{p_1}+(1-\kappa)\beta\frac{1}{p_2}\right] + \alpha\left[\kappa\varepsilon^H u'\left(\frac{M_0^h+\hat{\gamma}D_0^h}{p_1}\right)\frac{1}{p_1}+(1-\kappa)\beta\frac{1}{p_2}\right] \quad (40)$$

and the first-order condition for $c_1\left(\varepsilon^L\right)$, Equation (11);

- The definition of banks' net worth, Equation (14), applied separately for banks with endowment $K_{-1}^b = K^L$ and for banks with endowment $K_{-1}^b = K^H$;
- The budget constraint of banks, Equation (17), separately for banks with endowment $K_{-1}^b = K^L$ and for banks with endowment $K_{-1}^b = K^H$;
- The result of the banks' profit maximization problem, which implies $M_0^b = \kappa D_0^b$;
- The cash-in-advance constraints for impatient households with deposits at banks that pay a return $r_2^b = r_2^D$, given by $p_1 c_1\left(\varepsilon^H, r_2^b = r_2^D\right) = M_0^h + D_0^h$, and for impatient households with deposits at banks that pay a return $r_2^b < 0$, given by $p_1 c_1\left(\varepsilon^H, r_2^b < 0\right) = M_0^h + \hat{\gamma} D_0^h$;
- The constraint on withdrawals $\hat{\gamma} = \kappa$, which is obtained by integrating Equation (20) over the set of households that hold deposits at an insolvent bank b, and then using (18) and (19) holding with equality;
- The definition of the actual return on deposits, r_2^b, evaluated at $r_2^b = \hat{r}$ and at the value of K_0^b chosen by banks with endowment K^L;
- The market clearing conditions for money at $t = 0$ and consumption goods at $t = 1$, which imply $M_0^h + M_0^b = \overline{M}$, and $\kappa\left[(1-\alpha)c_1\left(\varepsilon^H, r_2^b = r_2^D\right) + \alpha c_1\left(\varepsilon^H, r_2^b < 0\right)\right] + (1-\kappa)c_1\left(\varepsilon^L\right) = 1$;
- The market clearing condition for deposits, $D_0^b = D_0^h$, which also implies $r_2^D = r_2^K$ from Proposition 3.1.

Then, given the result for the variables solved in the first step, I compute:

- The wealth of households, using Equation (15);
- The amount of capital held by households, which follows from the budget constraints in Equation (16);
- Banks' profits, using Equation (22), and total profits paid to households at $t = 2$, using $\pi_2 = \int \pi_2^b db$.
- Households' consumption at $t = 2$, using Equation (24).

The market clearing condition for capital, $\int K_0^h dh + \int K_0^b db = \overline{K}$, holds by Walras' law. The conjectures about r_2^b can be verified using Equation (23). The optimality of the conjectures about the withdrawals of depositors of solvent banks (i.e., the banks that pay the promised return on deposits r_2^D) is verified as follows. First, patient depositors have a non-binding cash-in-advance constraint, and thus it is optimal for them not to withdraw and earn the return $r_2^D > 0$. Second, for an impatient household, I check that if the household does not withdraw all of its deposits and leave a dollar in the bank, its marginal utility of withdrawing the last dollar is greater than the marginal benefit of leaving it in the bank:

$$\varepsilon^H u' \left[c_1 \left(\varepsilon^H, r_2^b = r_2^D\right)\right] \frac{1}{p_1} \geq \beta \frac{1 + r_2^D}{p_2}.$$

Finally, I check that all the non-negativity constraints on the choices of households and banks are not violated.

A.3.2 Numerical approach to solving for the bad equilibria

Two methods can be used to compute the bad equilibria with $D_0^h > 0$, and they both deliver the same results. One approach is to use the same method employed to construct Figure 3. The equilibria are then at the intersection of the best response with the 45-degree line. The second approach is based on finding all solutions to the non-linear equations that describe the equilibrium using a numerical procedure based on Gröbner basis methods. This second approach is both computationally simpler, as it is embedded in a simple routine in Mathematica, and, under a theoretical point of view, guarantees that all solutions to the system of equations can be found.

The application of Gröbner-basis methods to economic models is discussed by Kubler and Schmedders (2010). Even if the usage of these methods has been limited in economics, Kubler and Schmedders (2010) argue that these methods are useful in tackling multiple equilibria, in contrast to more standard numerical methods that are often designed to find one equilibrium only. The basic intuition of these methods is as follows. The equilibrium in many economic models solves a system of possibly non-linear polynomial equations. A result from algebraic geometry, commonly referred to as Shape Lemma, states that under mild conditions, a given equilibrium system has the same solution set as a much simpler system. The process by which this simpler system is computed is similar to that of Gaussian elimination in linear algebra. In particular, the set of solutions is the same as that of a system in which (i) one equation is a (possibly) high-degree polynomial in a single unknown, say x_1, and (ii) each of the other equations is a polynomial in the unknown x_1 and in only one other unknown. Thus, it is possible to find all the solutions of x_1 using the first equation, and then, for each solution, to use the remaining equations to compute all the other variables. Numerically, I exploit a routine embedded in the command NSolve of Mathematica.[2] More precisely, I conjecture that $r_2^b < 0$ for banks with endowment K^L, I compute all solutions to the system of equations that describe the equilibrium, and then I ignore the solutions that do not

[2]See Wolfram Language & System Documentation Center, "Some Notes on Internal Implementation," available at https://reference.wolfram.com/language/tutorial/SomeNotesOnInternalImplementation.html.

satisfy the appropriate non-negativity constraints or in which the conjecture $r_2^b < 0$ is not verified.

From a theoretical point of view, Gröbner-basis methods allow one to find, in principle, all solutions of a system of polynomial equations. This is particularly important in my models in which the number of equilibria that exist cannot be established in general, similar to many models with multiple equilibria.

A.4 Monetary injections: equilibrium definition

I now consider the economy with banks and asymmetric information in which the central bank implements temporary monetary injections, as discussed in Section 5. The equilibrium concept is the same as in Appendix A.3. However, the equilibrium definition is slightly different because it includes the variables associated with the monetary injections and the related equilibrium requirements.

Given real lump-sum taxes T_2 and a policy of temporary monetary injections $\left\{\mu_0, L_0^{CB}, K_0^{CB}\right\}$ that satisfies the budget constraint of the central bank, $L_0^{CB} + Q_0 K_0^{CB} \leq \overline{M}\mu_0$, an equilibrium is a collection of:

- Prices: prices of capital and consumption goods, Q_0, p_1, and p_2; the return on capital, r_2^K; promised return on deposit, r_2^D; returns on loans from the central bank, r_2^{CB}; and actual return on deposits, r_2^b for all b;
- Quantities: net worth of banks and households, N_0^h for all h and N_0^b for all b; money, deposits, and capital held by banks, M_0^b, D_0^b, K_0^b for all b, and loans from the central bank L_0^b for all b; money, deposits, and capital held by households, M_0^h, D_0^h, K_0^h for all h; the fraction γ_1^b of deposits that can be withdrawn at $t = 1$ by depositors of bank b, for all b; withdrawals by households at $t = 1$, w_1^h for all h, and withdrawals from bank b, w_1^b for all b; consumption $t = 1$, c_1^h for all h; profits π_2^b for all b, and total profits paid to households π_2; transfers from the central bank to households, π_2^{CB}; and consumption at $t = 2$, c_2^h for all h;
- Beliefs: for each household h, a belief over γ_1^b and r_2^b given D_0^b and M_0^b, that is, as a joint cumulative density function $F^h\left(\gamma_1^b, r_2^b | D_0^b, M_0^b\right)$ for all D_0^b, M_0^b and for all b;

such that:

- Households maximize (1) subject to the budget constraint (16), the constraint on withdrawals (20), the cash-in-advance constraint (21), the resource constraint for time-2 consumption, which is similar to (24) but also includes the transfers π_2^{CB} from the central bank,

$$p_2 c_2^h = M_0^h + w_1^h - p_1 c_1^h\left(\varepsilon_1^h\right) + Q_0 K_0^h\left(1 + r_2^K\right) + \left(D_0^h - w_1^h\right)\left(1 + r_2^{b(h)}\right) + \pi_2 + \pi_2^{CB} - p_2 T_2,$$

  and the non-negativity constraints on all choice variables;
- Banks maximize (37) subject to the budget constraint (36), the feasibility constraint (19) for

withdrawals, and the non-negativity constraints on all choice variables;

- The definition of the actual return on deposits, Equation (39), holds for all b;
- The budget constraint (4) of the fiscal authority holds;
- Markets clear: money market at $t = 0$, $\int M_0^h dh + \int M_0^b db = \overline{M}(1 + \mu_0)$; capital market at $t = 0$, $\int K_0^h dh + \int K_0^b db + K_0^{CB} = \overline{K}$; deposit market at $t = 0$, $\int D_0^h dh = \int D_0^b db$; market for loans from the central bank, Equation (34); goods market at $t = 1$, $\int c_1^h dh = 1$; and consumption goods at $t = 2$, $\int c_2^h dh = A_2\overline{K}$.
- The beliefs are rational.

I focus on symmetric equilibria in which all banks have the same amount of deposits, provided that their non-negativity constraints are not binding.

B Friedman rule

This appendix discusses the Friedman rule in the bankless economy of Section 2. The central bank can implement the Friedman rule by driving to zero the opportunity cost of holding money, which is represented by the nominal return r_2^K on physical capital.

As in standard cash-in-advance models, many paths of the money supply implement the Friedman rule (see, e.g., Cole and Kocherlakota, 1998). Here, for simplicity, I focus on one specific path so that (i) the central bank changes the money supply from $\overline{M}$ to $\overline{M}(1 + \mu_0)$ at $t = 0$, and then it brings it back to $\overline{M}$ at $t = 2$, and (ii) at $t = 1$, impatient households spend all the money M_0^h held from period $t = 0$.

The central bank changes the money supply using nominal lump-sum transfers F_0 and F_2. Thus, Equations (6) and (8) are replaced by

$$M_0^h + Q_0K_0^h \leq \overline{M} + Q_0\overline{K} + F_2,$$

$$p_2c_2^h = \left(M_0^h - p_1c_1^h\right) + K_0^h\left(p_1A_1 + p_2A_2\right) + F_2,$$

respectively. Since the change in the money supply is given by $\mu_0\overline{M}$ and the policy is reverted at $t = 2$ (similar to Section 5), the budget constraints of the central bank are

$$F_0 = \overline{M}\mu_0, \quad F_2 = -\overline{M}\mu_0,$$

at $t = 0$ and $t = 2$, respectively.

Given a monetary policy μ_0, the equilibrium definition is analogous to that of the bankless economy with fixed money supply, described in Appendix A.1. To solve for the equilibrium, I solve for the monetary injection μ_0 that achieves $r_2^K = 0$ in equilibrium. Since the opportunity cost of holding money is zero, households' cash-in-advance constraints are never binding in equilibrium (i.e., their Lagrange multiplier is zero). As mentioned above, I solve for the level of μ_0 such that impatient households spend all the money M_0^h.

Under the above premises, the central bank implements the Friedman rule by setting $\mu_0 = \varepsilon^H/\beta - 1 > 0$. The entire money supply is held by households at $t = 0$, and thus $M_0^h = \overline{M}\varepsilon^H/\beta$. This policy implements the first-best allocation described in Section 2.2, and prices are given by

$$Q_0 = \frac{\overline{M}}{\beta}\left(A_1 + \beta A_2\right), \quad p_1 = \frac{\overline{M}}{\beta}.$$

Note that the price level at $t = 2$ is unchanged at $p_2 = \overline{M}$, and thus the Friedman rule gives rise to a deflation between $t = 2$ and $t = 1$ at the rate of time preferences, as in standard monetary models.

Stepping outside of the model, some extensions could provide a justification for why the central bank does not follow this policy. For instance, Berentsen et al. (2007) show that a growth rate of money above that suggested by the Friedman rule can be welfare improving because it represents a punishment for agents that are excluded from the credit market and forced to use money following a default on their debt. Andolfatto (2013) discusses the limit to the implementation of the Friedman rule when agents can default on lump-sum tax payments.

C The bad equilibrium with insolvency of all banks

This appendix shows that there exists a bad equilibrium in which all banks are insolvent at $t = 0$ in the economies with both full information about banks' balance sheets (Section 3) and asymmetric information about banks' balance sheets (Section 4). Since all banks fail at $t = 0$, the equilibrium is the same as that in the bankless economy.

To study the bad equilibrium in which all banks are insolvent, I need to slightly amend the model in the main part of the paper. Equations (14) and (15), which define households' and banks' wealth and net worth, respectively, are replaced by

$$N_0^b = Q_0 K_{-1}^b + M_{-1}^b - \chi_0^b D_{-1}^b$$

and

$$N_0^h = Q_0 K_{-1}^h + M_{-1}^h + \chi_0^{b(h)} D_{-1}^h.$$

The term $\chi_0^b \in [0,1]$ is the recovery rate on the preexisting deposits D_{-1}^b of banks b; similarly, $\chi_0^{b(h)} \in [0,1]$ is the recovery rate for household h that has preexisting deposits at bank $b(h)$. The main part of the paper considers only equilibria in which the recovery rate is $\chi_0^b = 1$ for all banks

b. But now I allow the recovery rate χ_0^b to be less than one; that is, I allow banks to impose haircuts on the preexisting deposits at $t = 0$.

The recovery rate χ_0^b is less than one only if the non-negativity constraint on capital and money, $K_0^b \geq 0$ and $M_0^b \geq 0$, is binding. In the bad equilibrium of this appendix, in which households withdraw all of their preexisting deposits and thus choose $D_0^h = 0$, all banks are insolvent and the recovery rate χ_0^b is less than one. If χ_0^b were equal to one, the budget constraint (17) would imply $Q_0 K_0^b + M_0^b < 0$ because $N_0^b < 0$ and $D_0^b = 0$. However, this is not possible because of the non-negativity constraints $M_0^b \geq 0$ and $K_0^b \geq 0$. As a result, the non-negativity constraints on M_0^b and K_0^b are binding, and the recovery rate is $\chi_0^b < 1$ for all b to ensure that the budget constraint (17) holds. Note that I do not allow for strategic default, and thus a recovery rate $\chi_0^b < 1$ is necessarily associated, in equilibrium, with $N_0^b \leq 0$.

In the bad equilibrium in this appendix, all households choose $D_0^h = 0$; that is, they withdraw all of their preexisting deposits. If a household deviates and chooses $D_0^h > 0$, the deposits $D_0^h > 0$ are lost. This is because all banks have $M_0^b = 0$ and $K_0^b = 0$, and thus deposits cannot be withdrawn at $t = 1$ (i.e., $\gamma_0^b = 0$ for all b) and pay a zero gross return at $t = 2$ (i.e., $r_2^b = -1$).

It is worth emphasizing that the bad equilibrium with insolvency of all banks exists under the assumption that no new banks can enter the economy at $t = 0$. If instead new banks enter the market to replace all banks liquidated at the beginning of $t = 0$, households would simply move their deposits from old failed banks to new banks. But if this were the case, prices would be the same as in the good equilibrium, and thus old banks would not fail in the first place. While the assumption that many new banks do not enter during a crisis is reasonable, it is imposed in an ad hoc way; extending the model to study the frictions that prevent entry is, however, outside the scope of this paper. The bad equilibrium in Carapella (2012) and Gertler and Kiyotaki (2015) is similar to the one derived here because all banks in their model fail, and they also impose similar restrictions on the entry of new banks.[3]

Proposition C.1. *(Bad equilibrium with insolvency of all banks) Assume that*

$$K_0^b \left[\overline{M} \, \frac{A_1 \left(1 - \kappa \left(\varepsilon^H - \beta\right)\right) + \beta A_2}{\left(1 - \kappa\right) \beta + \kappa \varepsilon^H} \right] + M_0^b - D_0^b < 0 \tag{41}$$

for all b. Then, in the economies with both full information about banks' balance sheets (Section 3) and asymmetric information about banks' balance sheets (Section 4), there exists an equilibrium in which $D_0^h = 0$ for all h, Q_0, p_1, and time-1 consumption allocations are the same as in the bankless economy, and $\chi_0^b = \frac{Q_0 K_{-1}^b + M_{-1}^b}{D_{-1}^b}$ for all b, $\gamma_0^b = 0$ and $r_2^b = -1$ for all b, and $r_2^D = r_2^K$.

Proof. Given $r_2^D = r_2^K$, banks are indifferent about their choice of D_0^b, and thus $D_0^b = 0$ is optimal,

[3] In Gertler and Kiyotaki (2015), some banks enter the economy during a crisis, but the entry rate is exogenous.

as shown by Proposition 3.1. Under the bankless-economy prices, (14) and (41) imply that all banks would have negative net worth if the recovery rate were $\chi_0^b = 1$. However, given $\chi_0^b = \frac{Q_0 K_{-1}^b + M_{-1}^b}{D_{-1}^b}$, banks' net worth is $N_0^b = 0$, and thus the resources available to be invested in money and capital are $N_0^b + D_0^b = 0$. As a result, the non-negativity constraints $M_0^b \geq 0$ and $K_0^b \geq 0$ imply $M_0^b = 0$ and $K_0^b = 0$. The restriction on withdrawals $\gamma_0^b = 0$ and the actual return on deposits $r_2^b = -1$ follow from the fact that banks have no resources invested in money and capital at $t = 0$, and thus they have no money or capital to pay withdrawals at $t = 1$ or returns at $t = 2$. Households' choice of deposits is optimal as described above and follows from the non-negativity constraint on deposits.

Since $D_0^h = 0$ and prices Q_0 and p_1 are the same as in the bankless economy, households' first-order conditions are the same as in the bankless economy, implying the same allocation of consumption at $t = 1$, and thus, from the cash-in-advance constraint of impatient households $p_1 c_1 \left(\varepsilon^H\right) = M_0^h$, the same money holding at $t = 0$. □

D Other proofs

D.1 Proof of Proposition 3.1

I conjecture that profits are $\pi_2^b \geq 0$. Thus, with non-negative profits, the bank must have enough resources to repay the promised return r_2^D on deposits not withdrawn at $t = 1$, which implies that the actual return on deposits is $r_2^b = r_2^D$. The first-order conditions imply that the marginal contribution to profits of an extra dollar of deposit is $r_2^K - r_2^D$. As a result, if $r_2^D < r_2^K$, then $D_0^b = +\infty$ and $M_0^b = 0$ because the bank can invest all of the deposits in capital and earn a positive profit on every dollar of deposit. If $r_2^D > r_2^K$, then $D_0^b = 0$ because the bank would earn negative profits for each dollar of deposit (and, thus, the non-negativity constraint $D_0^b \geq 0$ is binding) and $M_0^b = 0$ because profits are maximized by investing the bank's net worth into capital, which pays a positive return, rather than money, which pays a zero return. And if $r_2^D = r_2^K$, then any $D_0^b \geq 0$ is optimal and $M_0^b = \kappa D_0^b$ in order to have enough money to repay withdrawals at $t = 1$ to the mass κ of impatient depositors (note that patient households have no incentive to withdraw because $r_2^D > 0$); that is, withdrawals are given by $w_1^b = \kappa D_1^b$, and thus, they satisfy the feasibility constraint $w_1^b \leq M_0^b$. The choice of K_0^b is residually determined by the budget constraint (17) and the non-negativity constraint $K_0^b \geq 0$. For all values of $r_2^D, r_2^K \geq 0$, the conjecture $\pi_2^b \geq 0$ is then verified.

D.2 Proof of Proposition 5.1

Under the asset purchase and partial deposit insurance policy, the transfers from the central bank to households at $t = 2$, (35), become

$$\pi_2^{CB} = Q_0 K_0^{CB} \left(1 + r_2^K\right) - \alpha \tau_2 \left(\mu_0\right) - \mu_0 \overline{M} \tag{42}$$

because the mass of banks subject to runs is α. In addition, the actual return on deposits (39) for banks subject to runs becomes

$$1 + r_2^b = \frac{Q_0 K_0^b \left(1 + r_2^K\right) + \tau_2 \left(\mu_0\right)}{D_0^b - w_1^b}.$$

Fixing prices at the level of the equilibrium that arises under loans to banks, the same households' choices are trivially optimal in the economy with asset purchase and partial deposit insurance. The same choices of deposits and money are optimal for banks as well, given the result in Proposition 3.1. However, the choice of capital, which follows from the banks' budget constraint, is different; let $K_0^b\left(LB\right)$ and $K_0^b\left(AI\right)$ be the holdings of capital of bank b under the loans-to-banks and asset purchase policies, respectively.

Let $r_2^b\left(LB\right)$ and $r_2^b\left(AI\right)$ be the actual return of bank b under the loans-to-banks and asset purchase policies, respectively. Let $\tau_2\left(\mu_0\right)$ such that $r_2^b\left(LB\right) = r_2^b\left(AI\right)$ for a bank b subject to runs at $t = 1$:

$$1 + r_2^b\left(LB\right) = \frac{Q_0 K_0^b\left(AI\right)\left(1 + r_2^K\right) + \tau_2\left(\mu_0\right)}{D_0^b - w_1^b}.$$

Then, using Equation (39) and the market clearing from loans, $L_0^b = L_0^{CB}$, we have

$$\frac{Q_0 K_0^b\left(LB\right)\left(1 + r_2^K\right)}{D_0^b - w_1^b + L_0^{CB}} = \frac{Q_0 K_0^b\left(AI\right)\left(1 + r_2^K\right) + \tau_2\left(\mu_0\right)}{D_0^b - w_1^b}.$$

I can further simplify this expression using the budget constraint of banks to replace $Q_0 K_0^b\left(LB\right)$ and $Q_0 K_0^b\left(AI\right)$. Rearranging:

$$\tau_2\left(\mu_0\right) = \frac{-N_0^b L_0^{CB}}{D_0^b - w_1^b + L_0^{CB}} \left(1 + r_2^K\right). \tag{43}$$

Thus, Equation (42) implies that transfers from the central bank under asset purchases and partial insurance, $\pi_2^{CB}(AI)$, are the same as under loans to banks, $\pi_2^{CB}(LB)$,

$$\begin{aligned}\pi_2^{CB}(AI) &= Q_0 K_0^{CB}\left(1+r_2^K\right) - \alpha\tau_2(\mu_0) - \mu_0\overline{M} \\ &= L_0^{CB}\left(1+\alpha\frac{N_0^b}{D_0^b - w_1^b + L_0^{CB}}\right)\left(1+r_2^K\right) - \mu_0\overline{M} \\ &= \pi_2^{CB}(LB)\,.\end{aligned}$$

The second line uses the budget constraint of the central bank, the assumption that the size of the monetary injection is the same under the two policies, and (43). The third line uses (35) and the fact that, under loans to banks, a mass α of banks are subject to runs, paying the return r_2^b defined in (39), and the remaining mass $1-\alpha$ pays r_2^K.

E Extension: Choices of insolvent banks

In this appendix, I provide an extension that sharpens the results about the choices of insolvent banks in the bad equilibria analyzed in Section 4. In the main part of the paper, I do not allow insolvent banks to announce their identity. Here, I relax this assumption, and I also allow banks to invest in a high-risk, low-return technology. A key result of this extension is that insolvent banks strictly prefer not to announce their identity and thus prefer to remain active at $t=0$. Indeed, these banks invest in the risky technology, thereby gambling for resurrection. If we impose some restrictions on the risky technology, the outcome can be made arbitrarily close to that analyzed in the main part of the paper while maintaining the strict preference of insolvent banks to remain active.

To clarify the analysis, let me first analyze the implications of allowing insolvent banks to announce their identity, without adding any other features to the model. To do so, I first state and prove the following lemma.

Lemma E.1. *If $N_0^b < 0$ and $r_2^D \geq r_2^K \geq 0$, then $\pi_2^b = 0$ for any feasible choice $\left\{D_0^b, M_0^b, K_0^b\right\}$.*

Proof. To prove the lemma, I reformulate the definition of profits in (22) as

$$\pi_2^b = \max\left\{0,\ Q_0 K_0^b\left(1+r_2^K\right) + \left(M_0^b - w_1^b\right) - \left(D_0^b - w_1^b\right)\left(1+r_2^D\right)\right\} \tag{44}$$

to account for the possibility that $M_0^b - w_1^b$ is positive, that is, that only a fraction of the money M_0^b

is withdrawn at $t = 1$. Then we have

$$\begin{aligned}
\pi_2^b &= \max\left\{0,\ Q_0 K_0^b \left(1 + r_2^K\right) + \left(M_0^b - w_1^b\right) - \left(D_0^b - w_1^b\right)\left(1 + r_2^D\right)\right\} \\
&= \max\left\{0,\ N_0^b \left(1 + r_2^K\right) + \left(D_0^b - M_0^b\right)\left(1 + r_2^K\right) + \left(M_0^b - w_1^b\right) - \left(D_0^b - w_1^b\right)\left(1 + r_2^D\right)\right\} \\
&= \max\left\{0,\ N_0^b \left(1 + r_2^K\right) + R_2^K \left(D_0^b - M_0^b\right) - r_2^D \left(D_0^b - w_1^b\right)\right\} = 0
\end{aligned}$$

where the first line uses (44), the second line uses the budget constraint (17), and the third line rearranges. The final result that $\pi_2^b = 0$ follows from $N_0^b < 0$, $r_2^D \geq r_2^K$, and the feasibility of withdrawals (19). □

Lemma E.1 shows that insolvent banks are indifferent about taking any feasible choice because their profit will always be zero, as a result of the binding limited liability constraint. A bank that announces its identity is equivalent to a bank that chooses $D_0^b = 0$; given the bad-equilibrium prices, such a bank will be liquidated immediately. Since this choice is feasible, it yields the same (zero) payoff as any other choice, including that of setting D_0^b and M_0^b to the same values chosen by solvent banks. Thus, insolvent banks are indifferent between pretending to be solvent or announcing their identity and being liquidated immediately.

Next, I also introduce a high-risk, low-return technology. Assume that capital purchased at $t = 0$ can be invested either in the riskless technology that produces A_1 and A_2 at $t = 1, 2$, as in the main part of the paper, or in a risky technology that produces $A_1\xi_1$ and $A_2\xi_1$ at $t = 1, 2$, where ξ_1 is an idiosyncratic shock that is independent of households' preference shocks and is realized at $t = 1$. The shock ξ_1 takes the values

$$\xi_1 = \begin{cases} \xi^H & \text{with probability } \pi \\ \xi^L & \text{with probability } 1 - \pi, \end{cases}$$

with $0 < \xi^L < 1 < \xi^H$ and $\mathbb{E}\left\{\xi_1\right\} < 1$. This last assumption implies that the risky technology is less productive than the riskless technology. Note that the return of the risky technology is

$$1 + r_2^{K,risky} = \xi_1 \left(1 + r_2^K\right),$$

where r_2^K is still defined by (9). The assumption $\mathbb{E}\left\{\xi_1\right\} < 1$ implies

$$\mathbb{E}\left\{r_2^{K,risky}\right\} < r_2^K.$$

That is, the expected return of the risky technology is lower in comparison to the riskless technology. The choice to adopt the safe or the risky technology is private information of each agent, at

$t = 0$, and becomes common knowledge at $t = 1$.

Next, I turn to households' and banks' choices when both the riskless and risky technology are available. Because of the low return generated by the risky technology, households and solvent banks are strictly better off by using the safe technology; that is, their choices are not affected by the extension. In contrast, insolvent banks strictly prefer to use the risky technology under some conditions. I conjecture that an insolvent bank b (i.e., a bank b with net worth $N_0^b < 0$) uses the risky technology. When $\xi_1 = \xi^H$, and using the equivalent of (22) for a bank that uses the risky technology, its profits are strictly positive if

$$Q_0 K_0^b \xi^H \left(1 + r_2^K\right) - \left(D_0^b - w_1^b\right)\left(1 + r_2^D\right) > 0.$$

If $M_0^b = \kappa D_0^b$, $w_1^b = M_0^b$, and $r_2^D = r_2^K$, as in the bad equilibrium of the main part of the paper, the last expression becomes

$$\xi^H N_0^b + \left(\xi^H - 1\right) D_0^b \left(1 - \kappa\right) > 0. \tag{45}$$

Under this restriction, insolvent bank b strictly prefers to use the risky technology because it obtains strictly positive profits with positive probability (i.e., with the same probability that $\xi_1 = \xi^H$, that is, π) and, thus, strictly positive expected profits.[4] In contrast, an insolvent bank that uses the safe technology obtains zero profits for any feasible choice (i.e, its limited liability constraint is binding), as shown in Lemma E.1. With respect to the choice of deposits and money holdings, insolvent banks make the same choices as solvent banks in order to exploit their private information and pool with such banks. If an insolvent bank made a different choice, households would be able to identify such a bank and thus would not make deposits at it. As in the main part of the paper, I assume that if a household observes a bank with values for deposits or money that differ from the values chosen by solvent banks, the household assigns probability one that such a bank is insolvent.

Finally, I discuss how the outcome of this extension can be made arbitrarily close to that in the main part of the paper while maintaining the result that insolvent banks strictly prefer to remain active. Before doing that, note that the fact that insolvent banks use the risky technology affects (i) the amount of consumption goods available at $t = 1$ and $t = 2$ and (ii) the actual return on deposits r_2^b of insolvent banks because the risky technology affects the amount of resources that such banks have available at $t = 2$ to repay deposits not withdrawn.

The outcome of the model in the main part of the paper can be obtained by choosing ξ^L arbitrarily close to one and π arbitrarily close to zero; formally, this is obtained by considering the appropriate limiting economy in which $\left\{\pi, \xi^L\right\} \to \{0, 1\}$ while still assuming $\xi^L < 1$, $\pi > 0$, and $\mathbb{E}\left\{\xi_1\right\} < 1$ along the sequence over which the limit is taken. Under these limits, the amount

[4] Using the parameter values of Section 4.3, I verify numerically that there exists a $\xi^H > 1$ that satisfies the restriction in (45) evaluated at N_0^b and D_0^b of insolvent banks.

of consumption goods available at $t = 1$ converges to one, and thus (3) approximately holds, $\mathbb{E}\left\{r_2^{K,risky}\right\} \to r_2^K$, and the actual return on deposits of an insolvent bank converges to that in Equation (23).

F Quantitative example: Data sources

This appendix presents more details about the data that are used in the quantitative example of Section 4.3.

Price level and GDP. The price-level data are from the NBER Macrohistory Database. The GDP data are from Jordà et al. (2017).

Deposits, money held by banks, and money held by households. Data about deposits and money held by households are from Friedman and Schwartz (1970), Table 1. Deposits are the sum of demand deposits and time deposits at commercial banks, and money held by households is defined as currency held by the public. For 1929, I use the data in January ($42.7 billion for deposits and $3.9 billion for currency held by the public), although the numbers are approximately the same during the other months of that year. For 1933, I use the data in March, that is, at the peak of the last wave of bank runs ($25.7 billion for deposits and $5.6 billion for currency held by the public).

The figure for money held by banks comes from Friedman and Schwartz (1963), Table A-2, and is the sum of vault cash and bank deposits at Federal Reserve banks (this sum is denoted as bank reserves). For 1929, I use the data in January ($3.3 billion); for 1933, I use the data in March ($2.9 billion). Note that the sum of currency held by the public and money held by banks is $7.2 billion, which corresponds to what Friedman and Schwartz (1963) refer to as "high-powered money" in 1929 (Friedman and Schwartz, 1963, Table B.3).

Capital held by banks. I use data from *All-Bank Statistics* (Board of Governors of the Federal Reserve System, 1959), and I focus on commercial banks for comparison with the data in Friedman and Schwartz (1970).[5] Table A-1a groups banks' non-money assets into three broad categories: loans; securities issued by the US government, states, and other governmental entities; and other securities and assets. I consider assets issued by the private non-financial sector as the empirical counterpart of capital in the model, and thus I exclude securities issued by government entities. In 1929, the empirical counterpart of capital held by banks is thus the sum of loans ($36.1 billion), other securities ($6.9 billion), and other assets ($3.6 billion), for a total of $46.6 billion. In 1933,

[5] The data in *All-Bank Statistics* are reported at a yearly frequency only.

it is the sum of loans (\$16.5 billion), other securities (\$4.3 billion), and other assets (\$2.6 billion), for a total of \$23.4 billion.

Returns. I use data on yields of AAA and Baa corporate bonds from *Banking and Monetary Statistics, 1914-1941*, pages 469-470 (Board of Governors of the Federal Reserve System, 1943).

In January 1929, the yield on AAA corporate bonds was 4.62% per year, which corresponds to a yield of 0.31% per period of 3.5 weeks. The yield on Baa corporate bonds was 5.63% per year or 0.38% per period. In March 1933, the yield on AAA corporate bonds was 4.68% per year or 0.31% per period. The yield on Baa corporate bonds was 8.91% per year or 0.59% per period, after peaking at 11.6% per year or 0.75% per period in May 1932.

Leverage. The leverage of banks is based on the dataset of Koch et al. (2016) and is defined as the ratio of total assets over owners' equity (i.e., paid-in capital plus surplus and other retained earnings). The leverage ratio for the lowest decile in 1929 is 12.18, and in the text I refer to this number as the leverage of the most levered banks in 1929.[6]

G On the source(s) of multiple equilibria: additional discussion

This appendix provides more details on the sources of the multiple equilibria discussed in Section 4.4. In particular, I discuss the sources of the concavity of the best response in Figure 2. Similar to Section 4.4, I focus on the equilibria that arise if the model is calibrated using the parameter values discussed in Section 4.3.

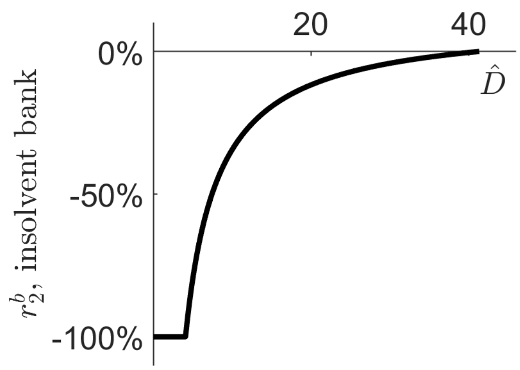
The concavity of the best response in Figure 2 can be understood by decomposing the slope of the best response, $\partial D_0^h\left(\hat{D}; Q_0, p_1\right)/\partial \hat{D}$, into two components:

$$\frac{\partial D_0^h\left(\hat{D}; Q_0, p_1\right)}{\partial \hat{D}} = \frac{\partial D_0^h}{\partial r_2^b} \times \frac{\partial r_2^b}{\partial \hat{D}}.$$

The actual return r_2^b in Equation (33) is a concave function of $\hat{D}$, and this result follows mechanically from the formulation of Equation (33). This is represented in Figure 6. In addition, the households' first-order conditions (31) and (32) imply that $\partial D_0^h/\partial r_2^b$ is approximately constant when evaluated at the parameter values used to construct the quantitative example in Section 4.3 and when $D_0^h > 0$. This second result is represented in Figure 7. Thus, the slope of the best response

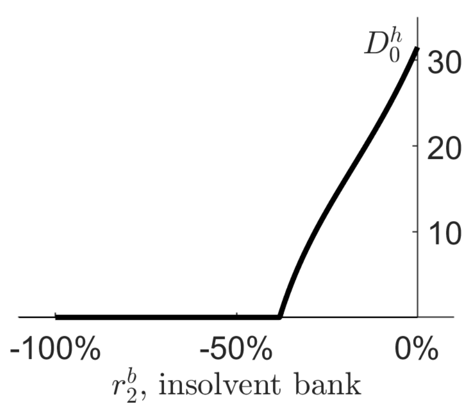
[6] I thank Patrick Van Horn for sharing these data.

Figure 6: Return r_2^b of insolvent banks



The solid line represents the actual return on deposits in Equation (33), as a function of $\hat{D}$. Prices Q_0 and p_1 are fixed at the mild-crisis equilibrium values. For parameter values, see Section 4.3.

Figure 7: Deposits D_0^h and return r_2^b of insolvent banks



The solid line represents the choice of deposits, D_0^h, of a household that faces the mild-crisis equilibrium prices as a function of r_2^b, and is computed by solving the first-order conditions (31) and (32) with respect to D_0^h and M_0^h. For parameter values, see Section 4.3.

$D_0^h\left(\hat{D};Q_0,p_1\right)$ is proportional to that of r_2^b, which implies that the concavity of $D_0^h\left(\hat{D};Q_0,p_1\right)$ is inherited from the concavity of r_2^b in $\hat{D}$. Finally, in the flat region in Figure 7, the value of r_2^b corresponds to deposits $\hat{D}$ such that the best response is $D_0^h\left(\hat{D};Q_0,p_1\right)=0$.

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