A Online Appendix

A.1 Oil and other energy inputs

Figure 6 plots consumption of the energy inputs that are used in the U.S. over the extended period 1949-2015. The top graph reveals that oil has consistently been the dominant energy source during this period. Oil is followed, in quantitative importance, by the fossil-energy sources of natural gas and coal. The use of nuclear power and renewable energy has been growing over time, but these sources have been of relatively marginal quantitative importance during the considered period.

Even though oil is the dominant energy input, a potential worry could be that alternative energy sources frequently substitute for oil in the data, whereas these alternatives are not available in the model. This is clearly not the case for the non-fossil energy sources. The amounts of energy generated by these sources have only been marginal, and the correlation between usage of renewable energy and oil is close to zero (at -0.14). It should therefore be without loss of generality to abstract from non-fossil energy inputs to understand the period of consideration, however important these inputs may become in the future.

What about the fossil-energy sources of coal and natural gas? The two bottom graphs in Figure 6 plot the relationships between oil and total fossil-fuel use.Specifically, the middle graph shows annual changes in percent, whereas the lower graph focuses on annual changes in levels. Both graphs display highly correlated series; the correlation for annual changes in percent is 0.9 and the correlation for levels is 0.85. This implies that changes in oil use correspond with changes in total fossil-fuel use on an almost one-to-one basis, and that there is limited substitution between the fossil fuels.[46] In fact, the lower graph illustrates that positive (negative) changes in oil use are generally accompanied by positive (negative) changes in gas and coal.

I therefore conclude that it is not restrictive to focus on oil as the sole energy input for the United States. An alternative would be to model en-

[46] One reason for this is that the prices of the fossil fuels are positively correlated. The correlation between the price of oil and the price of gas is as high as 0.82.

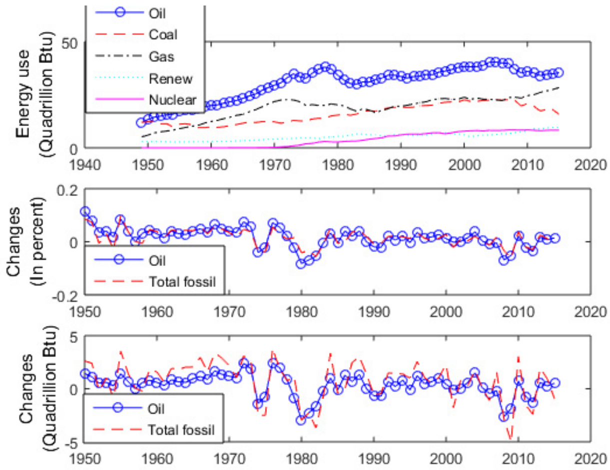

Figure 6: Oil and other energy inputs in the United States. Top graph: annual energy use of oil, coal, natural gas, renewable and nuclear energy. Middle graph: annual changes in percent of oil use and total fossil-energy use. Bottom graph: annual changes in levels of oil and total fossil-energy use. Source: the U.S. Energy Information Administration.

ergy as a fossil-fuel composite as in Hassler, Krusell, and Olovsson (2017).[47] Since it is more straightforward to calibrate the model to oil, however, I follow the previous literature and model the energy input as oil.[48] Setting up a model that takes coal and gas in addition to oil, and that also can replicate the properties in Figure 6, is an interesting extension, but it is left for future research.

A.2 First-order conditions to the social planning problem

The first-order conditions to (11) deliver an Euler equation for capital, an equation that governs the return to storing oil and an intra-temporal equation for labor supply. For notational simplicity, I omit here the fact that all variables are functions of the state variables. The equations are

[47]In fact, the correlation between usage of oil and a fossil-fuel composite that is constructed as in Hassler, Krusell and Olovsson, is 0.99 and the correlation between the oil price and the fossil-fuel composite price is 0.94.

[48]Basically, the difference between oil and the composite is found in the calibration of the supply shortfall and in the resulting volatility of the price. Using oil has the advantage that there are empirical estimates that can be used in the calibration, whereas this is not the case for the composite.

then, respectively, given by

$$\frac{1}{c} = \beta \mathbb{E}_t \frac{1}{c'} \left(F_{k'} + 1 - \delta \right), \tag{17}$$

$$\frac{1}{c} = \beta \mathbb{E}_t \frac{1}{c'} \frac{F_{o'} - \kappa'}{F_o - \kappa} + \frac{\lambda}{F_o - \kappa}, \tag{18}$$

and

$$\frac{F_l}{c} = \frac{1}{1-l} \frac{\mu}{1-\mu}, \tag{19}$$

where F_k, F_l and F_e respectively refer to the derivative of the production function with respect to k, l, and o, and I used the fact that $R'' = R' - o'_s$.

Equations (17) and (18) can be combined to derive the stochastic Hotelling equation. With $\kappa = 0$, this equation reads

$$\beta \mathbb{E}_t \frac{1}{c'} \frac{F_{o'}}{F_o} + \frac{\lambda}{F_o} = \beta \mathbb{E}_t \frac{1}{c'} \left(F_{k'} + 1 - \delta \right). \tag{20}$$

A.3 Hotelling pricing with a storage technology?

To derive some interpretable expressions, I abstract here from uncertainty. In addition, the expressions in this section are derived for a two-period model to enhance transparency, but the results generalize straightforward to an infinite horizon.

Combining (17) and (18) in Appendix A.2 delivers the following modified Hotelling formula:

$$\frac{p'_o}{p_o} = r' - \lambda \frac{c'}{\beta F_o}. \tag{21}$$

The above equation reveals that the oil price should grow at the rate of the interest minus a correction term that depends on the multiplier on (9). The standard Hotelling formula, thus, only holds when λ is exactly zero. If the multiplier is instead positive, the oil price should grow at a rate that is lower than the interest rate. Price growth can, in fact, be negative in this setting.

But when will λ be binding? It is not possible to give a general answer to this question, but if we are willing to assume that capital depreciates fully between periods and the elasticity of substitution between capital/labor and oil equals one, then the optimal depletion rate for stored oil can be shown to be governed by the following expression:

$$\frac{o_{g,1} + R_0 - o_{s,0}}{o_{g,0} + o_{s,0}} - \frac{\lambda\left(\eta_1 + R_0 - o_{s,0}\right)}{(1-\alpha)\left(1+\alpha\beta\right)} = \frac{\beta}{1+\alpha\beta}. \tag{22}$$

Equation (22) shows that an interior solution (with $\lambda = 0$) requires total oil use to fall at the rate $\frac{\beta}{1+\alpha\beta} < 1$ between the two periods.[49] In this case, the standard Hotelling formula applies. Note, however, that an interior solution is only possible when $o_{g,0} + R_0$ is large relative to $o_{g,1}$. If, instead, $o_{g,1}$ is large relative to $o_{g,0} + R_0$, the planner would like to borrow from the future endowment. This is not allowed, so the solution is to set $o_{s,0} = R_0$, which implies $\lambda > 0$ and that the Hotelling formula does not hold.

Also, in the special case when ground oil is constant between periods (which is close to the assumption in this paper), is it possible for total oil consumption to fall between periods if R_0 is large relative to the endowments. Otherwise, the multiplier on the constraint that stored oil cannot be negative will again be binding and the growth rate for the oil price will be strictly lower than the interest rate.

In a setting with an infinite horizon, the Hotelling formula can hold for a finite number of periods if R_0 is large relative to the endowments. However, it is straightforward to verify that without uncertainty, the stock of stored oil will be monotonically depleted over time. As R becomes smaller and smaller, the stock of stored oil will at some point reach a threshold level after which total oil use will fall at rate larger than β. From this point on, the oil price will grow at a rate that is strictly below the interest rate.

Note, finally, that it is not necessary for o_g to be interpreted as oil for the Hotelling formula to break down. This is also true in a setting where oil is given by o_s, whereas o_g is a substitute to oil such as nuclear power. The existence of an exogenous amount of oil, or an exogenous substitute, thus leads to a breakdown of the classical Hotelling relation.

[49] This rate equals β with an infinite horizon.

A.4 Estimating the factor-specific technology shocks

Consistent with the findings in Hassler, Krusell and Olovsson (2017), the technology trends seem to be subject to directed technical change: when A is growing relatively fast, A_o grows relatively slowly and vice versa. As explained in Section 1.3, I therefore estimate the relevant parameters for three sub-periods with distinct average growth rates, 1970-1987, 1987-1993, and 1993-2011, and set the parameters ρ_z, ρ_{z_o}, σ_z and σ_{z_o} to their weighted averages of the estimates over the periods. The three-period average is motivated by the fact that a is similar in concept to the standard TFP shock. This shock should therefore ideally have properties that are not too different from the standard shock, which is the case with three periods. Completely ignoring changing trend growth results in volatilities for a and a_o that respectively are two and more than four times as large as output. The results from this estimation are presented in Table 5.

Table 5: Parameter estimates

	ρ_z	ρ_{z_o}	σ_z	σ_{z_o}	$corr\,(z, z_o)$
1970-1987	0.6631	0.7943	0.0220	0.0545	0.3426
1987-1993	0.0847	-0.1137	0.0066	0.0050	0.0911
1993-2011	0.8109	0.4266	0.0119	0.0181	-0.2094
Weighted average	0.6355	0.5110	0.0157	0.0325	0.1251

Estimated parameters for different time periods, with the weighted averages.

A.5 Numerical approximation of the equilibrium

The decision rules are approximated by cubic splines over a six-dimensional grid. Specifically, the grid is continuous in k and R but, given the curse of dimensionality, the variables a and a_o are discretized and only allowed to take three values each.[50] Similarly, the variables η and ξ are only allowed to take two values each (oil supply is either high or low). The shocks a and a_o are discretized with the algorithm described in Tauchen and Hussey (1991). The model is simulated for 20,000 periods, but the first 1,000 periods are thrown away. Statistics are then computed from the remaining periods.

The numerical approximation of the model is evaluated by computing the Euler equation errors in simulations of the model. Specifically, I

[50]The model has also been solved with four states for the technology shocks and the results are very similar to the results with three states.

evaluate the Euler equation normalized by the left hand side of the Euler equation to get errors in percentage terms. The average error is then of the magnitude 10^{-6}.

A.6 Sensitivity analysis

A.6.1 A higher elasticity of substitution

Table 6 presents results for different values of the elasticity of substitution between capital/labor and energy ε.

Table 6: Business cycle properties for different elasticities

	$std\,(y)$	$std\,(o)$	$std\,(l)$	$std\,(p_o)$	$std\,(R)$
$\varepsilon = 0.5$	1.55	1.76	0.86	4.28	1.55
$\varepsilon = 0.1$	1.55	2.14	0.85	10.45	1.47
	$corr\,(y,l)$	$corr\,(y,o)$	$corr\,(y,\Delta R)$	$corr\,(y,p_o)$	$AR\,(1)$ for p_o
$\varepsilon = 0.5$	0.99	0.26	-0.15	0.51	0.67
$\varepsilon = 0.1$	0.92	0.49	-0.27	0.29	0.79

All variables are in logarithms and have been detrended with the HP filter using a smoothing parameter of 6.25 except in the last column in the bottom part of the table that reports the autocorrelation coefficient for the log-level of the oil price. ΔR is the net change in the stock of stored oil.

A.6.2 No news shocks

This section removes the news shocks from the analysis. Actual oil supply is as volatile as in the benchmark model, but the social planner does not recieve any signals about future supply levels. The purpose is to evaluate the potential importance of news for the business cycle and the oil price. The business cycle statistics are presented in Table 7, and the impulse response functions are presented in Figure 7.

The business cycle statistics are almost identical to those with news shocks. Hence, news about future oil supply does not seem important for understanding the unconditional volatilities of inputs and output. The main effect of ignoring news is to substantially increase the importance of the oil-supply shocks for the oil price, as is shown in Figure 7. In particular, this shock then has a dramatically larger influence on the oil price, but the effects on output and labor supply are also somewhat larger.

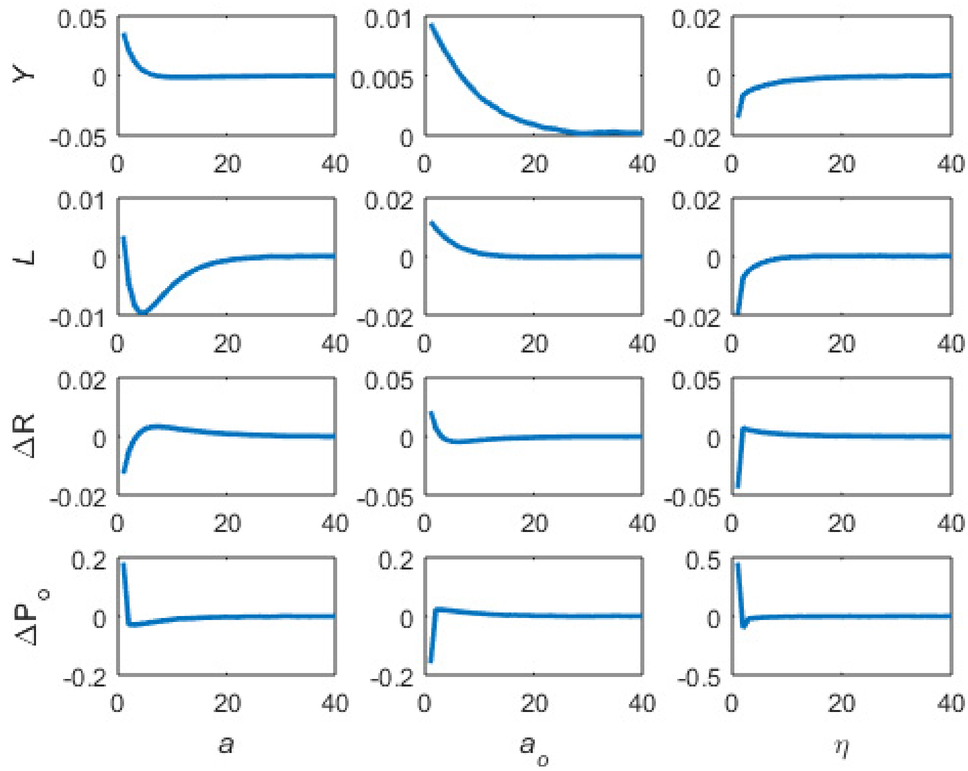

Figure 7: Impulse response functions in the absence of news shocks.

Table 7: Business cycle statistics in the model without news shocks

	$std\,(y)$	$std\,(o)$	$std\,(l)$	$std\,(p_o)$	$std\,(R)$
Model	1.56	2.29	1.18	17.72	1.46
	$corr\,(y,l)$	$corr\,(y,o)$	$corr\,(y,\Delta R)$	$corr\,(y,p_o)$	$AR\,(1)$ for p_o
Model	0.79	0.46	-0.18	0.16	0.84

The upper half of the table provides standard deviations and the lower half presents correlations. ΔR is the net change in the stock of stored oil. All variables are in logarithms and have been detrended with the HP filter using a smoothing parameter of 6.25 except in the last column in the bottom part of the table that reports the autocorrelation coefficient for the log-level of the oil price.

A.6.3 Elastic ground oil

In this section, I analyze to what extent the assumption that ground oil is inelastic matters for the results. It is now, instead, assumed that ground oil in the good state can be increased at a cost. Supply in the bad state is still inelastic, however. Specifically, the amount of ground oil can now be increased by setting the variable $\varkappa$ to a value larger than zero:

$$o_g = \begin{cases} \varphi e^{\eta_l}\left(1+\varkappa\right) & \text{if } \eta = \eta_l \\ \varphi e^{\eta_h} & \text{if } \eta = \eta_h. \end{cases} \tag{23}$$

The cost for increasing ground oil is increasing and convex:

$$\Pi\left(\varkappa\right) = \frac{\omega_1}{\overline{\varkappa}-\varkappa} - \omega_2,$$

where $\overline{\varkappa}$ is the capacity constraint.

The first-order condition with respect to κ becomes

$$\varkappa = \overline{\varkappa} - \sqrt{\omega_1}\left(yF_o\varphi e^{\eta_l}\right)^{-1/2}.$$

The parameters ω_1 and ω_2 are calibrated so that $\varkappa_{ss} = 0$, and that $\Pi\left(\varkappa_{ss}\right) = 0$. This implies that if it was possible to increase the production of ground oil at the steady state, the social planner would not choose to do that. The cost of extraction is then also zero at the steady state. The parameter $\overline{\varkappa}$, finally, denotes the capacity constraint. It is set so ground oil can be increased by ten percent at the most. This number is based on estimates in Kilian (2008a), which argues that the capacity utilization rate for world production of oil is typically running at 90-95 percent.

$$\omega_1 = \overline{\varkappa}^2 y_{ss} \frac{\gamma \varphi^{\frac{\sigma-1}{\sigma}}}{(1-\gamma)\left(k_{ss}^{\alpha} h_{ss}^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + \gamma \varphi^{\frac{\sigma-1}{\sigma}}}, \quad \text{and} \quad \omega_2 = \frac{\omega_1}{\overline{\varkappa}}.$$

The results are presented in Table 8.

Table 8: Business cycle properties with elastic oil supply

	$std\,(y)$	$std\,(o)$	$std\,(l)$	$std\,(p_o)$	$std\,(R)$
Inelastic oil	1.53	2.25	1.19	19.21	1.45
Elastic oil	1.54	2.37	1.01	17.47	1.58
	$corr\,(y,l)$	$corr\,(y,o)$	$corr\,(y,\Delta R)$	$corr\,(y,p_o)$	$AR\,(1)$ for p_o
Inelastic oil	0.77	0.45	-0.19	0.12	0.83
Elastic oil	0.83	0.49	-0.13	0.14	0.76

All variables are in logarithms and have been detrended with the HP filter. The upper half of the table provides standard deviations, and the lower half presents correlations. ΔR is the net change in the stock of stored oil.