Consumption Risk Sharing with Private Information and Limited Enforcement

Online Appendix

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Appendix C : Computing the Value Function

To find the optimal contract, we use value function iteration with finite-element interpolation. Interpolation in this case is somewhat complicated by the fact that the state space $\mathcal{V}$ is not rectangular. Thus we cannot define a grid as a Cartesian product between two fixed one-dimensional grids. What we do instead is to define a grid for v_1, and for each point on that grid we define a grid for v_2. The total number of points is 17250, and the range is more than wide enough in each dimension to cover the ergodic set as discovered in simulations.

The finite-element interpolation method works as follows. First we define the elements as triangles whose vertices are the gridpoints and where the union of these triangles coincides with the convex hull of the set of gridpoints. Clearly such a triangulation is not unique. The particular triangulation that we use is that defined by Delaunay (1934). It is illustrated for a case of four gridpoints in Figure 1. We do not claim that this particular triangulation is optimal in any sense, though optimality can be established in special cases; see, for instance Chen and Xu (2013).

Second, we define, once and for all given the gridpoints and the triangulation, coefficients that we will need to linearly interpolate within each triangle. To see what these coefficients are, let's move one step ahead and suppose we have an arbitrary point (x, y) in the domain and we want to find the interpolating value z given that we know what triangle (x, y) is in. Call the vertices of this triangle (x_0, y_0), (x_1, y_1) and (x_2, y_2) and suppose the function values at these vertices are z_0, z_1 and z_2. Evidently there exist scalars t and u such that

$$(x, y) = (x_0, y_0) + t \cdot (x_1 - x_0, y_1 - y_0) + u \cdot (x_2 - x_1, y_2 - y_1).$$

Figure 2 illustrates this very simple idea. Once we have found the values of t and u, our interpolating value is of course

$$z = z_0 + t \cdot (z_1 - z_0) + u \cdot (z_2 - z_1).$$

We need, then, an efficient way of finding t and u as a function of x and y. It turns out that, for each triangle, the vector $[t \quad u]$ is a fixed affine function of the vector $[x \quad y]$. Thus what we need to do, once and for all given the gridpoints and the triangulation, is to compute, for each triangle, the constant term and the gradient of this affine function. Finding this constant term and this gradient requires us to invert a 2×2 matrix, but the important thing is that this is only done *once* for each triangle. Having computed these coefficients once and for all, finding the interpolating value at an arbitrary point is just a matter of (1) finding the right triangle and (2) evaluating an affine function. This can of course be done very quickly, and this is important because it is done many thousands of times during the value function iteration.

To compute moments of the stationary distribution, we simply use the optimal allocation rule to simulate 100000 observations. To find the general equilibrium, we choose the interest rate R that ensures that mean consumption is equal to available resources in a steady state, as described in Section **??**.

The underlying assumption here is that the insurance providers own the capital. Moving to live in autarky means forgoing any access to this capital. The specific parameter values used are $\delta = .08$ and $\theta = 1/3$.

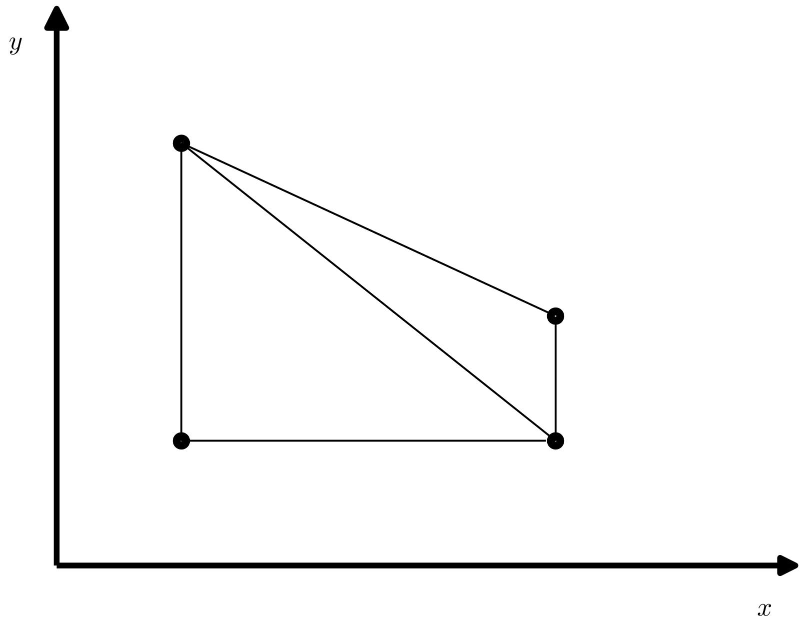

Figure 1: A Delaunay triangulation

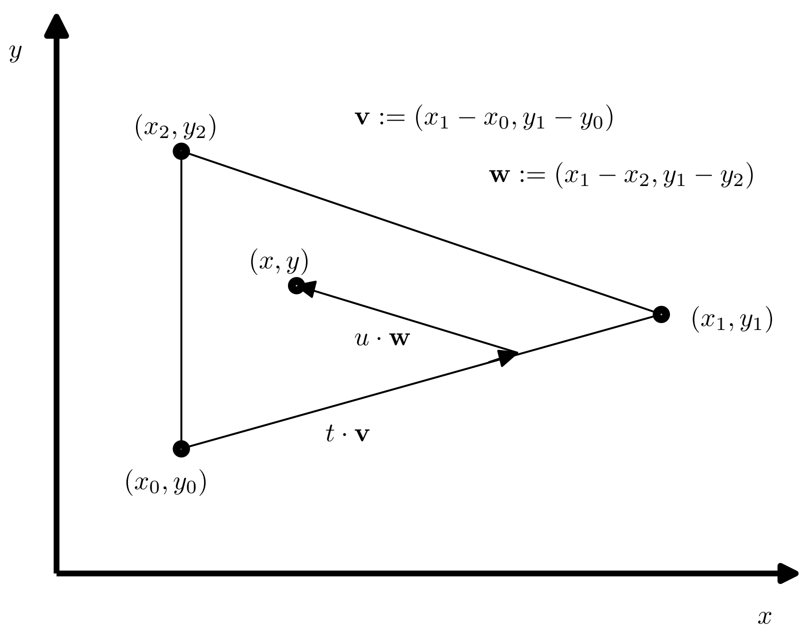

Figure 2: Linear interpolation on a triangle

References

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