Online Technical Appendix
(not for publication)

April 29, 2018

1 An Analytical Explantation on Mechanism

In this section, we show how the sectoral output are affected by the total export and the relative price adjustment. We first define $\widehat{X_t} = \log(X_t) - \log(X_{ss})$ for all variables. From equation 2.8, we have

$$\begin{aligned}
\widehat{Y^c_{Xt}} &= -\lambda(\widehat{P^c_{Xt}} - \widehat{P_{Xt}}) + \widehat{Y_{Xt}} \\
\widehat{Y^u_{Xt}} &= -\lambda(\widehat{P^u_{Xt}} - \widehat{P_{Xt}}) + \widehat{Y_{Xt}}
\end{aligned}$$

It is very straightfoward that the sectoral output increases with total trade and decreases with their relative prices. From the aggregate index, we have

$$\widehat{P_{Xt}} = \kappa\widehat{P^c_{Xt}} + (1-\kappa)\widehat{P^u_{Xt}}$$

For small open economy, the traded-good price is determined by the world price, $P_{Xt} = P^*_{Xt}$ and the imported goods price is normalized as unit, $P^*_{ft} = 1$. Equation 2.14 shows that, when there is a shock (ε_t) to terms of trade $\frac{P^*_{Xt}}{P^*_{ft}}$, $\widehat{P^c_{Xt}}$ simply follows the stochastic process. $\widehat{P^c_{Xt}} = \rho_e\widehat{P^c_{Xt-1}} + \varepsilon_t$.

In this small open economy, a negative shock to terms of trade is equivalent to the real exchange rate depreciation. In face of a negative shock to terms of trade, the total export $\widehat{Y_{Xt}}$ declines, which increases the trade in both credit constrained sector and credit unconstrained sector. However, there is a substitution effect between two subsectors due to the relative price adjustment, which affect two subsectors in a asymmetric way. Using the aggregate price index, we can have

$$\begin{aligned}
\widehat{P^c_{Xt}} - \widehat{P_{Xt}} &= (1-\kappa)(\widehat{P^c_{Xt}} - \widehat{P^u_{Xt}}) \\
\widehat{P^u_{Xt}} - \widehat{P_{Xt}} &= -\kappa(\widehat{P^c_{Xt}} - \widehat{P^u_{Xt}})
\end{aligned}$$

In equilibrium, we have $\widehat{P^c_{Xt}} = \widehat{MC^c_{Xt}}$ and $\widehat{P^u_{Xt}} = \widehat{MC^u_{Xt}}$. Given production function $Y^j_X = (\frac{K^j_X}{\alpha_X})^{\alpha_X}(\frac{L^j_X}{1-\alpha_X})^{1-\alpha_X}$, where $j = c, u$. we can solve for marginal cost as below,

$$\begin{aligned}
\widehat{MC^c_{Xt}} &= \alpha_X\widehat{R^c_t} + (1-\alpha_X)\widehat{W_t} \\
\widehat{MC^u_{Xt}} &= \alpha_X\widehat{R_t} + (1-\alpha_X)\widehat{W_t}
\end{aligned}$$

where R^c_t and R_t are the rental cost of capital in credit constrained sector and unconstrained sector, respectively, and W_t is the wage for both sectors as labor is perfect mobile across sectors. Due to the presence of credit constraints, R^c_t

differs from R_t . Now, we can show clearly that the relative price is simply determined by the difference between R_t^c and R_t.

$$\widehat{P_{Xt}^c} - \widehat{P_{Xt}^u} = \alpha_X(\widehat{R_t^c} - \widehat{R_t})$$

In face of a negative shock to terms of trade, the traded goods sector will expand, the demand for capital increases as well. In such a case, the credit constrained firm is subject to a relatively tighten constrained. Consequently, the rental cost of capital goes up, in particular, we must

$$\widehat{R_t^c} > \widehat{R_t},$$

which implies that $\widehat{P_{Xt}^c} - \widehat{P_{Xt}^u} > 0$. Therefore, when the real exchange rate depreciates, the total export increases, which benefits both credit constrained and unconstrained sectors. However, the substitution effect will shift the demand from credit constrained sector to unconstrained sector, which reduces the export in credit constrained and increases the export in unconstrained. When λ is sufficiently large, the substitution effect may dominate the aggregate trade effect, which leads the export in credit constrained sector decline. In the opposite case when the real exchange rate appreciates, the total export will decline, but the sign of substitution effect is not as clear as that in the case with exchange rate depreciation.

2 Alternative Credit Constraint

In this note we consider an alternative specification of credit constraint in entrepreneur's optimization problem, where the entrepreneur's debt limit depends on the value of the firm. To solve this problem, we first define the value of the firm as

$$V^f(B_t^c, K_{X,t}^c) = \max_{(K_{X,t+1}^c, B_{t+1}^c, L_{X,t}^c)} d_{xt}^c + \beta(1-\xi)E_t V^f(B_{t+1}^c, K_{X,t+1}^c)$$

where $d_{xt}^c = P_{X,t}^c Y_{X,t}^c - W_t L_{X,t}^c + (1-\delta)P_t K_{X,t}^c + B_{t+1}^c - P_t K_{X,t+1}^c - (1+i_t)B_t^c$ is the firm's dividend or net worth at end of period t. $Y_{X,t}^c$ is the output, $L_{X,t}^c$ and $K_{X,t}^c$ are labor input and capital input. B_{t+1}^c is the debt level. $P_{X,t}^c$ is the price of goods in constrained tradable sector and W_t is the wage. The consumption amount is C_t^e and we assume the utility function is $\ln C_t^e$. Given the firm's value function, the entrepreneur maximizes his utility by solving the following Bellman Equation

$$V(B_t^c, K_{X,t}^c) = \max_{c_t^e, B_{t+1}^c, K_{X,t+1}^c, L_{X,t}^c} \ln C_t^e + \beta(1-\xi)E_t V(B_{t+1}^c, K_{X,t+1}^c) \quad (1)$$

which is subject to the budget constraint and the credit constraint:

$$P_t(C_t^e + K_{X,t+1}^c) + (1+i_t)B_t^c = P_{X,t}^c Y_{X,t}^c - W_t L_{X,t}^c + (1-\delta)P_t K_{X,t}^c + B_{t+1}^c \quad (2)$$

$$B_{t+1}^c \leqslant \phi E_t V^f(B_{t+1}^c, K_{X,t+1}^c) \quad (3)$$

ϕ is the fraction of the firm value that can be used as collateral. where $Y_{X,t}^c = A_{X,t}(K_{X,t}^c)^\alpha (L_{X,t}^c)^{1-\alpha}$.Let's denote λ_{1t} and λ_{2t} as the Lagrangian multipliers for the budget constraint and the credit constraint respectively, the first order conditions respect to $\{c_t^e, B_{t+1}^c, K_{X,t+1}^c, L_{X,t}^c\}$ are given by:

$$\begin{aligned}
\frac{1}{P_t C_t^e} &= \lambda_{1t} \quad (4)\\
\beta(1-\xi)E_t \frac{\partial V(B_{t+1}^c, K_{X,t+1}^c)}{\partial K_{X,t+1}^c} - \lambda_{1t}P_t + \phi\lambda_{2t}E_t \frac{\partial V^f(B_{t+1}^c, K_{X,t+1}^c)}{\partial K_{X,t+1}^c} &= 0\\
\beta(1-\xi)E_t \frac{\partial V(B_{t+1}^c, K_{X,t+1}^c)}{\partial B_{t+1}^c} + \lambda_{1t} + \phi\lambda_{2t}E_t \frac{\partial V^f(B_{t+1}^c, K_{X,t+1}^c)}{\partial B_{t+1}^c} - \lambda_{2t} &= 0\\
P_{X,t}^c \frac{\partial Y_{X,t}^c}{\partial L_{X,t}^c} &= W_t
\end{aligned}$$

From the envelope theorem,

$$\begin{aligned}
\frac{\partial V(B_t^c, K_{X,t}^c)}{\partial K_{X,t}^c} &= \lambda_{1t}\left(R_{Xt}^c + (1-\delta)P_t\right) \quad (5)\\
\frac{\partial V(B_t^c, K_{X,t}^c)}{\partial B_t^c} &= -(1+i_t)\lambda_{1t} \quad (6)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial V^f(B_t^c, K_{X,t}^c)}{\partial K_{X,t}^c} &= R_{Xt}^c + (1-\delta)P_t \quad (7)\\
\frac{\partial V^f(B_t^c, K_{X,t}^c)}{\partial B_t^c} &= -(1+i_t) \quad (8)
\end{aligned}$$

where $R_{t+1}^c = \frac{\lambda-1}{\lambda} P_{X,t+1}^c \frac{\partial Y_{X,t+1}^c}{\partial K_{X,t+1}^c}$ is the return of capital. The FOCs can be simplified as

$$\begin{aligned}
E_t[\beta(1-\xi)\frac{1}{C_{t+1}^e P_{t+1}}\left(R_{Xt+1}^c + (1-\delta)P_{t+1}\right)] + \phi\lambda_{2t}E_t\left(R_{Xt+1}^c + (1-\delta)P_{t+1}\right) &= \frac{1}{C_t^e} \quad (9)\\
E_t\beta(1-\xi)\frac{(1+i_{t+1})}{P_{t+1}C_{t+1}^e}] + \phi\lambda_{2t}E_t(1+i_{t+1}) &= \frac{1}{C_t^e P_t} - \lambda_{2t} \quad (10)
\end{aligned}$$

FOC (10) can get the expression of λ_{2t}

$$\frac{\frac{1}{C_t^e P_t} - \beta(1-\xi)E_t(1+i_{t+1})\frac{1}{P_{t+1}C_{t+1}^e}}{1+\phi E_t(1+i_{t+1})} = \lambda_{2t}$$

By replacing λ_{2t} in (9), we have

$$\begin{aligned} & E_t\left[\beta(1-\xi)\frac{\left(R_{Xt+1}^c + (1-\delta)P_{t+1}\right)}{C_{t+1}^e P_{t+1}}\right] \\ & + \frac{\frac{1}{C_t^e P_t} - \beta(1-\xi)E_t(1+i_{t+1})\frac{1}{P_{t+1}C_{t+1}^e}}{1+\phi E_t(1+i_{t+1})}\phi E_t\left(R_{Xt+1}^c + (1-\delta)P_{t+1}\right) \\ = & \frac{1}{C_t^e} \end{aligned}$$

We simplify the above equation and have

$$\beta(1-\xi)E_t\{\frac{C_t^e P_t}{C_{t+1}^e P_{t+1}}[\frac{P_{t+1}}{P_t}\left(\frac{R_{t+1}^c}{P_{t+1}} + (1-\delta)\right) - \phi(1+i_{t+1})\Theta_t]\} = 1-\phi\Theta_t \quad (11)$$

Where $\Theta_t = \frac{E_t[\frac{P_{t+1}}{P_t}\left(\frac{R_{t+1}^c}{P_{t+1}}+(1-\delta)\right)]}{1+\phi E_t(1+i_{t+1})}$

In the paper, the credit constraint is subject to the value of physical capital, which gives us the following condition:

$$\beta(1-\xi)E_t\{\frac{C_t^e P_t}{C_{t+1}^e P_{t+1}}[\frac{P_{t+1}}{P_t}\left(\frac{R_{t+1}^c}{P_{t+1}} + (1-\delta)\right) - (1+i_{t+1})\phi]\} = 1-\phi \quad (12)$$

2.1 Quantitative exercise

In the calibration, all the parameter are the same as those in the benchmark case except the parameter ϕ in the new credit constraint. We choose $\phi = 0.223$ so as to keep the steady state ratio of $\frac{B^c}{PK_X^c} \approx 0.2$,which is the ratio we set in the benchmark case. All the results are report in Table 1. We can show that our results still hold in the alternative credit constraint. Two credit constraint specifications yield similar quantitative results.

Table 1: Mean of output under high volatility/mean of output under low volatility

		$\frac{\text{mean of output under high volatility } (\sigma=0.5)}{\text{mean of output under low volatility } (\sigma=0.1)}$
$\phi = 0.223$,$\kappa = 0.5$ $tb/y = 0$,$\lambda = 11$	constrained subsector	79.12%
	unconstrained subsector	92.03%
	overall tradable sector	88.34%
$\phi = 0.223$,$\kappa = 0.5$ $\mathbf{tb/y = 2\%}$,$\lambda = 11$	constrained subsector	78.16%
	unconstrained subsector	91.97%
	overall tradable sector	88.02%
$\phi = 0.223$,$\kappa = 0.5$ $\mathbf{tb/y = -2\%}$,$\lambda = 11$	constrained subsector	80.01%
	unconstrained subsector	92.11%
	overall tradable sector	88.64%
$\phi = 0.223$,$\kappa = 0.5$ $tb/y = 0$,$\boldsymbol{\lambda = 6}$	constrained subsector	96.60%
	unconstrained subsector	98.73%
	overall tradable sector	97.90%
$\phi = 0.223$,$\boldsymbol{\kappa = 0.75}$ $tb/y = 0$,$\lambda = 11$	constrained subsector	96.26%
	unconstrained subsector	100.03%
	overall tradable sector	97.95%
$\boldsymbol{\phi = 0.9}$,$\kappa = 0.5$ $tb/y = -2\%$,$\lambda = 11$	constrained subsector	89.10%
	unconstrained subsector	94.90%
	overall tradable sector	92.82%