

Web Appendix of

“The Evolution of Awareness and Belief Ambiguity in the Process of High School Track Choice”

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A Additional Tables and Figures

Table A1: IDENTITY OF RESPONDENTS

Respondent Identity	Wave 1 Sample (all)		Wave 1 Sample (matched with Wave 3) ^a	
	Children	Parents ^b	Children	Parents ^b
Child (%)	649 (100%)	N.A.	410 (100%)	N.A.
Both parents (%)	N.A.	288 (47.84%)	N.A.	171 (48.44%)
Mother only (%)	N.A.	262 (43.52%)	N.A.	159 (45.04%)
Father only (%)	N.A.	47 (7.81%)	N.A.	23 (6.52%)
Other person (%)	N.A.	5 (0.83%)	N.A.	0 (0%)
N (%)	649 (100%)	602 (100%)	410 (100%)	353 (100%)

[^a]: Based on subjects who responded to wave 3. Subjects who did not respond to wave 3 but responded to wave 2 were also included. [^b]: Parents sample in wave 3 is conditional on families where the same parent(s) responded across waves.

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Table A2: CHILD'S BACKGROUND CHARACTERISTICS

	Children Sample in Wave 1 (all)	Children Sample in Wave 1 (matched with Wave 3) ^a
	(N=649)	(N=410)
Child's gender		
% male	46.53	43.17
% female	53.47	56.83
N (100%)	649	410
% item non-response/missing	0	0
Child's place of birth^b		
% Italy	86.36	88.02
% other country	13.64	11.98
N (100%)	645	409
% item non-response/missing	0.62	0.24
Child's age^c		
mean	13.0929	13.0732
std. dev.	0.4249	0.4072
min	12	12
median	13	13
max	15	15
N (100%)	646	410
% item non-response/missing	0.46	0
Child's age vs. school grade^d		
% regular (born in 1998)	83.9	85.12
% ahead (born after 1998)	3.87	4.15
% behind (born before 1998)	12.23	10.73
N (100%)	646	410
% item non-response/missing	0.46	0
Child's GPA^e		
mean	7.6541	7.7405
std. dev.	0.9663	0.9719
min	6	6
median	7.6	7.7
max	9.8	9.8
N (100%)	567	369
% item non-response/missing	12.63	10
Parent/s' child lives with^f		
% both parents	87.84	88.48
% one parent	11.66	10.99
% none	0.51	0.52
N (100%)	592	382
% item non-response/missing	4.05	4.02
Number of older siblings^g		
mean	0.6248	0.5594
std. dev.	0.7636	0.6966
min	0	0
median	0	0
max	3	3
N (100%)	581	379
% item non-response/missing	10.48	7.56

[^a]: Based on subjects who responded to wave 3. Subjects who did not respond to wave 3 but responded to wave 2 were also included.

[^b]: Constructed using multiple reports by child and parent/s.

[^c]: Constructed from year of birth, using multiple reports child and parent/s.

[^d]: Constructed from year of birth and current grade.

[^e]: Constructed by averaging grades in 9 main subjects.

[^f]: Constructed from co-residing question, using multiple reports by child and parent/s.

[^g]: Constructed by censoring up to 3 older siblings, using multiple reports by child and parent/s.

Table A3: CHILD'S BACKGROUND CHARACTERISTICS (CONTINUED)

	Children Sample in Wave 1 (all) (N=649)	Children Sample in Wave 1 (matched with Wave 3) ^a (N=410)
Mother's country of birth^b		
% Italy	87.79	82.7
% other country	19.21	17.3
N (100%)	609	393
% item non-response/missing	4.25	3.2
Father's place of birth^b		
% Italy	81.16	83.03
% other country	18.84	16.97
N (100%)	584	383
% item non-response/missing	2.99	1.79
Language prevalently spoken at home^c (conditional on one or multiple members being foreign-born)		
% Italian	47.2	56.79
% other language	52.8	43.21
N (100%)	125	81
% item non-response/missing	19.87	14.74
Mother's highest schooling degree^d		
% elementary or less	2.37	1.85
% junior high school degree	20.14	18.78
% HS diploma (includes 3-years vocational degrees)	50.08	52.12
% college degree or higher (includes 3-years degrees)	27.41	27.25
N (100%)	591	378
% item non-response/missing	7.08	6.9
Father's highest schooling degree^d		
% elementary or less	1.94	1.62
% junior high school degree	21.3	22.16
% HS diploma (includes 3-years vocational degrees)	50.35	50.81
% college degree or higher (includes 3-years degrees)	26.41	25.41
N (100%)	568	370
% item non-response/missing	5.65	5.13
Mother's working status^e		
% works full-time	39.43	41.04
% works part-time	37.58	36.36
% does not work	22.90	22.60
N (100%)	596	385
% item non-response/missing	6.29	5.17
Father's working status^e		
% works full-time	92.06	91.84
% works part-time	4.32	4.21
% does not work	3.63	3.95
N (100%)	579	380
% item non-response/missing	3.82	2.56
Mother's occupation^f		
% stay-home mom	24.28	23.76
N (100%)	593	383
% item non-response/missing	3.93	2.46
Father's occupation^f		
% blue collar	28.75	24.54
N (100%)	574	379
% item non-response/missing	4.65	2.82

[^a]: Based on subjects who responded to wave 3. Subjects who did not respond to wave 3 but responded to wave 2 were also included.

[^b]: Conditional on having one. Constructed from country of birth, using multiple reports by child and par/s.

[^c]: Asked of child only.

[^d]: Conditional on having one. Constructed from original question on educational attainment, using multiple reports by child and parent/s.

[^e]: Conditional on having one. Constructed using multiple reports by child and parent/s.

[^f]: Conditional on having one. Constructed from question on occupation, using multiple reports. Selected categories only.

Table A4: RESPONDING PARENTS' BACKGROUND CHARACTERISTICS

	Children Sample in Wave 1 (all) ^a	Children Sample in Wave 1 (matched with Wave 3) ^{a,b}
	(N=649)	(N=410)
Responding mother's age		
mean	44.4614	44.6712
std. dev.	4.8075	4.4908
min	30	32
median	44	45
max	63	63
N (100%)	518	295
Responding father's age		
mean	47.6950	48.8984
std. dev.	5.9205	6.3393
min	28	31
median	47	48
max	73	73
N (100%)	318	128
Responding mother's place of birth		
% Italy	83.24	85.81
% other country	16.76	14.19
N (100%)	525	296
Responding father's place of birth		
% Italy	81.73	85.94
% other country	18.27	14.06
N (100%)	323	128
Responding mother's highest schooling degree		
% elementary or less	1.55	1.04
% junior high school degree	18.64	19.72
% HS diploma (includes 3-years vocational degrees)	50.68	50.52
% college degree or higher (includes 3-years degrees)	29.13	28.72
N (100%)	515	289
Responding father's highest schooling degree		
% elementary or less	1.89	0.79
% junior high school degree	21.45	24.41
% HS diploma (includes 3-years vocational degrees)	50.79	44.88
% college degree or higher (includes 3-years degrees)	25.87	29.92
N (100%)	317	127
Responding mother's working status		
% works full-time	39.65	40.96
% works part-time	37.72	36.18
% does not work	22.63	22.87
N (100%)	517	298
Responding father's working status		
% works full-time	92.26	90
% works part-time	4.02	5.38
% does not work	3.72	4.62
N (100%)	323	130

[^a]: These statistics were constructed by matching responding parents' identity and parents' background characteristics.

[^b]: Based on subjects who responded to wave 3. Subjects who did not respond to wave 3 but responded to wave 2 were also included.

Table A5: PREDICTORS OF THE NUMBER OF CURRICULA OF WHICH THE CHILD IS AWARE AT PRE-ENROLLMENT (WAVE 3)

Estimation Coefficients of a Poisson Regression of the Number of Curricula of which the Child is Aware at Pre-Enrollment (Wave 3)

Predictors	N of 'Know' or 'Heard of'	N of 'Know'	N of 'Know' or 'Heard of'	N of 'Know'	N of 'Know' or 'Heard of'	N of 'Know'
	All Curricula		General		Technical & Vocational	
female student	0.00149 (0.03651)	0.10043** (0.04523)	0.00877 (0.04874)	0.13856** (0.05715)	-0.00789 (0.05513)	0.03039 (0.07419)
foreign-born student	-0.05516 (0.07000)	-0.14402 (0.08828)	-0.04337 (0.09295)	-0.15044 (0.11231)	-0.07044 (0.10642)	-0.13368 (0.14295)
lives with both parents	0.02708 (0.06253)	0.05694 (0.07799)	0.04163 (0.08368)	0.04329 (0.09655)	0.00829 (0.09412)	0.08119 (0.13235)
college+ mother	-0.03861 (0.05797)	0.05390 (0.07220)	-0.00208 (0.07758)	0.16594* (0.09118)	-0.08521 (0.08727)	-0.14779 (0.12016)
high school mother	-0.03145 (0.04911)	0.03008 (0.06106)	-0.00855 (0.06608)	0.07583 (0.07924)	-0.06000 (0.07342)	-0.03140 (0.09610)
stay-at-home mother	0.00528 (0.04353)	0.09627* (0.05290)	0.00438 (0.05819)	0.05806 (0.06748)	0.00641 (0.06560)	0.15500* (0.08539)
blue-collar father	-0.01726 (0.04523)	-0.00166 (0.05546)	-0.02964 (0.06063)	-0.02918 (0.07139)	-0.00164 (0.06793)	0.04052 (0.08831)
n of older siblings	0.01599 (0.02522)	-0.02587 (0.03141)	0.00691 (0.03374)	-0.01786 (0.03944)	0.02760 (0.03798)	-0.03925 (0.05193)
7th-grade GPA	-0.00025 (0.02154)	-0.05114* (0.02659)	0.00084 (0.02874)	0.01780 (0.03340)	-0.00165 (0.03253)	-0.17180*** (0.04424)
constant	2.35354*** (0.17597)	2.19062*** (0.21699)	1.73670*** (0.23515)	1.14312** (0.27462)	1.57927*** (0.26533)	2.19532*** (0.35617)
χ^2	2.06	13.57	0.91	15.64	2.43	32.07
Prob > χ^2	0.9906	0.1385	0.9996	0.0747	0.9826	0.0002
Pseudo R^2	0.0016	0.0083	0.0008	0.0124	0.0022	0.0249
Sample Size	304	304	304	304	304	304

Significance levels: *** significant at 1%; ** significant at 5%; * significant at 10%.

Predictors: female student: dummy=1 if student is female; foreign-born student: dummy=1 if student is foreign-born; lives with both parents: dummy=1 if student lives with both parents; college+ mother: dummy=1 if student's mother has a college or higher degree; high school mother: dummy=1 if the highest degree of student's mother is a high school diploma; stay-at-home mother: dummy=1 if student has a stay-at-home mother; blue-collar father: dummy=1 if student's father works in a blue-collar occupation; n of older siblings: number of student's older siblings (between 0 and 3); 7th-grade GPA: student's GPA in the 7th grade end-of-year report (between 6 and 10).

Table A6: SELECTED EXPECTATIONS QUESTIONS ACROSS FORMATS, RESPONDENTS, AND WAVES

Type	Future Outcome	Format of Expectation Question	Respondent	Wave Asked
Stated choice/preference	Would choose for: himself(/herself/child) today	Percent chance (point only)	Child; mother and father (or resp-parent)	W1 (only top 3 alternatives), W2, W3, W4 (all ranked alt.)
Event following choice	Child enjoys curriculum content	Percent chance (point & range)	Child; responding parent(s)	W1, W2, W3
Event following choice	Child graduates in time with passing (or higher) grades in all subjects	Percent chance (point & range)	Child; responding parent(s)	W1, W2, W3
Event following choice	Child's high school training permits a flexible college-work choice (college & /or work, college only, work only)	Percent chance (point, with no idea option)	Child; responding parent(s)	W1, W2, W3
Event following choice	Child's high school training permits a flexible college major choice (hum. or soc. sciences; math & science or engineering; law or econ.)	Percent chance (point, with no idea option)	Child; responding parent(s)	W1, W2, W3
Event following choice	Child's college-work choice (college only; work only; college & work)	Percent chance (point, with no idea option)	Child; responding parent(s)	W3
Event following choice	Child's job after graduating from high school & no college	Expected job (job name, with no idea option)	Child; responding parent(s)	W3
Event following choice	Child's job after graduating from high school & college	Expected job (job name, with no idea option)	Child; responding parent(s)	W3
Family process	Parents would allow child's choice, without child explaining his/her choice	Percent chance (point, with no idea option)	Child; responding parent(s)	W1, W2, W3
Family process	Parents would allow child's choice, provided child explains his/her choice	Percent chance (point, with no idea option)	Child; responding parent(s)	W1, W2, W3
Family process	Curriculum mother would choose for the child today	Percent chance (point, with no idea option)	Child	W1, W2, W3
Family process	Curriculum father would choose for the child today	Percent chance (point, with no idea option)	Child	(top 3 ranked by mother) W1, W2, W3
Family process	Curriculum child would choose for himself/herself today	Percent chance (point, with no idea option)	Child; responding parent(s)	(top 3 ranked by father) W1, W2, W3
Family process	Curriculum other parent would choose for the child today	Percent chance (point, with no idea option)	Responding parent(s)	(top 3 ranked by child) W1, W2, W3

Table A7: PREDICTORS OF THE NUMBER OF CURRICULA ABOUT WHICH THE CHILD HAS AN AMBIGUOUS SUBJECTIVE LIKELIHOOD OF GRADUATING IN TIME AT PRE-ENROLLMENT (WAVE 3)

Estimation Coefficients of a Poisson Regression of the Number of Curricula about which the Child has an Ambiguous Subjective Likelihood of Graduating on Time at Pre-Enrollment (Wave 3)

Predictors	N of 'Unsure' or 'No Idea'	N of 'No Idea'	N of 'Unsure' or 'No Idea'	N of 'No Idea'	N of 'Unsure' or 'No Idea'	N of 'No Idea'
	All Curricula		General		Technical & Vocational	
female student	0.21185** (0.08497)	0.12449 (0.09354)	0.29581** (0.13782)	0.14386 (0.15928)	0.16081 (0.10799)	0.10995 (0.11574)
foreign-born student	0.31043** (0.14342)	0.30628* (0.16573)	0.47367** (0.20796)	0.42468* (0.25516)	0.15681 (0.19965)	0.20477 (0.21927)
lives with both parents	-0.54472*** (0.11818)	-0.68811*** (0.12845)	-0.55622*** (0.18626)	-0.80558*** (0.21174)	-0.52786*** (0.15304)	-0.61704*** (0.16186)
college+ mother	-0.34171** (0.13720)	-0.60448*** (0.14570)	-0.42426* (0.22943)	-0.89188*** (0.26072)	-0.29317 (0.17136)	-0.46212*** (0.17749)
high school mother	-0.17989 (0.11573)	-0.44635*** (0.12179)	-0.10992 (0.18769)	-0.47005** (0.20100)	-0.22165 (0.14708)	-0.41651*** (0.15353)
stay-at-home mother	-0.12021 (0.10382)	-0.07554 (0.11516)	-0.31069* (0.17191)	-0.19968 (0.19812)	-0.00743 (0.13043)	-0.00941 (0.14138)
blue-collar father	-0.16276 (0.10480)	-0.33446*** (0.12016)	-0.18018 (0.16493)	-0.32649* (0.19459)	-0.14810 (0.13596)	-0.34413** (0.15371)
n of older siblings	0.03679 (0.05754)	-0.08857 (0.06687)	0.11033 (0.09026)	-0.07729 (0.11203)	-0.01703 (0.07495)	-0.09941 (0.08339)
7th-grade GPA	-0.03604 (0.04919)	-0.04788 (0.05449)	-0.21569*** (0.08052)	-0.34563*** (0.09596)	0.07438 (0.06259)	0.10336 (0.06711)
constant	1.70326*** (0.39697)	2.07660*** (0.43684)	2.04617*** (0.63577)	3.44953*** (0.73744)	0.39623 (0.51172)	0.36567 (0.54913)
χ^2	38.33	48.51	40.15	47.74	17.42	28.31
Prob > χ^2	0.0000	0.0000	0.0000	0.0000	0.0426	0.0008
Pseudo R^2	0.0225	0.0312	0.0447	0.0632	0.0154	0.0264
Sample Size	262	262	262	262	262	262

Significance levels: *** significant at 1%; ** significant at 5%; * significant at 10%.

Predictors: female student: dummy=1 if student is female; foreign-born student: dummy=1 if student is foreign-born; lives with both parents: dummy=1 if student lives with both parents; college+ mother: dummy=1 if student's mother has a college or higher degree; high school mother: dummy=1 if the highest degree of student's mother is a high school diploma; stay-at-home mother: dummy=1 if student has a stay-at-home mother; blue-collar father: dummy=1 if student's father works in a blue-collar occupation; n of older siblings: number of student's older siblings (between 0 and 3); 7th-grade GPA: student's GPA in the 7th grade end-of-year report (between 6 and 10).

Table A8: DISTRIBUTION OF OBSERVED PRE-ENROLLMENT CHOICES

Curriculum	All (% students)	Female (% students)	Foreign-born (% students)	GPA > median (% students)
Gen, Humanities	6.42	9.13	3.13	10.86
Gen, Languages	10.70	14.90	3.13	17.14
Gen, Math & Science	24.06	21.15	15.63	38.29
Gen, Art & Music	6.68	6.73	9.38	5.71
Gen, Soc Sciences	9.63	13.94	12.50	5.71
Tech, Economic Sector	10.43	12.50	6.25	5.14
Tech, Technology Sector	16.04	8.17	12.50	11.43
Voc, Services	7.22	7.21	9.38	4.00
Voc, Industry & Crafts	4.01	1.44	21.88	0.57
Voc, Prof Training	4.81	4.81	6.25	1.14
Sample size	374	208	32	175

Pre-enrollment choices were measured in waves 3 and 4.

Table A9: PREDICTORS OF PRE-ENROLLMENT CHOICES: BACKGROUND CHARACTERISTICS ONLY

Multinomial Logit Regression of Pre-enrollment Choice on Background Characteristics											
Predictors	Gen Hum	Gen Lang	Gen Math	Gen Art&Music	Gen Soc Sci	Tech Econ Sect	Tech Tech Sect	Voc Serv	Voc Ind	Voc Prof Train	LR ^a test
female student	1.0457 (0.6626)	2.1555*** (0.5803)		1.8910*** (0.6993)	2.3769*** (0.6759)	2.0144*** (0.7791)	-0.3034 (0.5122)	1.1063 (0.7114)	-1.7132 (1.4923)	1.1642 (0.9489)	***
foreign-born student	0.4427 (1.3036)	-0.9633 (1.2122)		0.6226 (1.0627)	-0.7863 (1.2447)	-17.1930 (17.166)	0.4071 (1.0305)	-0.1123 (1.3285)	4.2054*** (1.6156)	2.6070* (1.4437)	
lives with both parents	-0.1874 (0.9205)	1.5582 (1.1278)		-0.1752 (0.9908)	0.5775 (0.9984)	0.4999 (1.2499)	0.0718 (0.7781)	14.4703 (13.617)	-2.2429 (1.7068)	14.4693 (18.063)	
college+ mother	0.3693 (1.2003)	-0.0130 (0.8446)		-1.0224 (1.1354)	-0.0204 (0.9832)	-3.1359** (1.2547)	-2.3254*** (0.8838)	-0.8390 (1.2581)	-3.3218 (2.1817)	-15.4965 (17.425)	**
high school mother	0.4124 (1.1972)	-0.4264 (0.8364)	Base	-0.1872 (0.9512)	-0.5009 (0.9043)	-1.6771** (0.8209)	-0.8022 (0.7341)	-0.3677 (0.9903)	-1.2396 (1.6308)	-1.8999 (1.1567)	
stay-home mother	0.4712 (0.7657)	0.9017 (0.5996)	choice	0.6941 (0.7650)	1.0173 (0.7015)	1.1198 (0.7685)	0.3264 (0.5809)	0.8857 (0.7642)	1.6385 (1.2337)	0.7439 (0.9489)	
blue-collar father	-0.5859 (1.2402)	-0.3969 (0.8848)		1.8988*** (0.7273)	1.6600** (0.7140)	0.2579 (0.8851)	0.3603 (0.6417)	2.3096*** (0.7863)	0.22679 (1.3722)	2.0313** (1.0196)	**
n of older siblings	-0.1899 (0.4209)	-0.2765 (0.3518)		-0.1125 (0.4488)	0.0554 (0.3975)	-0.2184 (0.5201)	0.1947 (0.3217)	0.5754 (0.4221)	0.22367 (1.7759)	0.0692 (0.5763)	
7th-grade GPA	0.7629* (0.4589)	-0.7064** (0.3231)		-1.5318*** (0.4235)	-1.7347*** (0.3946)	-1.3186*** (0.4502)	-1.5121*** (0.3341)	-1.7988*** (0.4969)	-2.3239** (1.0026)	-3.5594*** (0.9301)	***
constant	-8.8661** (4.1704)	2.2266 (2.8459)		9.4189*** (3.2690)	10.0951*** (3.0281)	8.6114** (3.6401)	12.1075*** (2.6428)	-3.4144 (3.622)	17.374** (7.0653)	9.8497 (8.0825)	
Sample Size	239										
Log likelihood	-369.296										
LR χ^2	233.01										
Prob > χ^2	0.0000										
Pseudo R ²	0.2398										

Significance levels: *** significant at 1%; ** significant at 5%; * significant at 10%.

Predictors: female student: dummy=1 if student is female; foreign-born student: dummy=1 if student is foreign-born; lives with both parents: dummy=1 if student lives with both parents; college+ mother: dummy=1 if student's mother has a college or higher degree; high school mother: dummy=1 if the highest degree of student's mother is a high school diploma; stay-home mother: dummy=1 if student has a stay-home mother; blue-collar father: dummy=1 if student's father works in a blue-collar occupation; n of older siblings: number of student's older siblings (between 0 and 3); 7th-grade GPA: student's GPA in the 7th grade end-of-year report (between 6 and 10).

Sample size: Pre-enrollment choices were measured in waves 3 and 4.

Table A10: χ^2 STATISTICS FOR TESTING EQUALITY OF ROWS IN THE AWARENESS TRANSITION MATRICES (SAMPLE OF CHILDREN WHO RESPONDED TO EACH PAIR OF WAVES)

	Wave 1 vs. Wave 2	Wave 2 vs. Wave 3	Wave 1 vs. Wave 3
Starting state	Unconditional vs. Chosen		
Know	***	***	***
Heard of	*	**	***
Never heard	**	*	**
Starting state	Ranked First vs. Ranked Bottom (Ranked 4th or Below)		
Know	***	***	***
Heard of	***	**	**
Never heard	*	NA	-

***: 99% confidence level, **: 95% confidence level, *: 90% confidence level.

Table A11: χ^2 STATISTICS FOR TESTING EQUALITY OF ROWS IN THE AMBIGUITY TRANSITION MATRICES (SAMPLE OF CHILDREN WHO RESPONDED TO EACH PAIR OF WAVES)

	Wave 1 vs. Wave 2	Wave 2 vs. Wave 3	Wave 1 vs. Wave 3
Starting state	Unconditional vs. Chosen		
Sure	**	**	***
Unsure	-	***	-
No Idea	-	-	-
Starting state	Ranked First vs. Ranked Bottom (Ranked 4th or Below)		
Sure	***	***	***
Unsure	-	-	-
No Idea	*	**	***

***: 99% confidence level, **: 95% confidence level, *: 90% confidence level.

Figure A1: Evolution of Point Beliefs

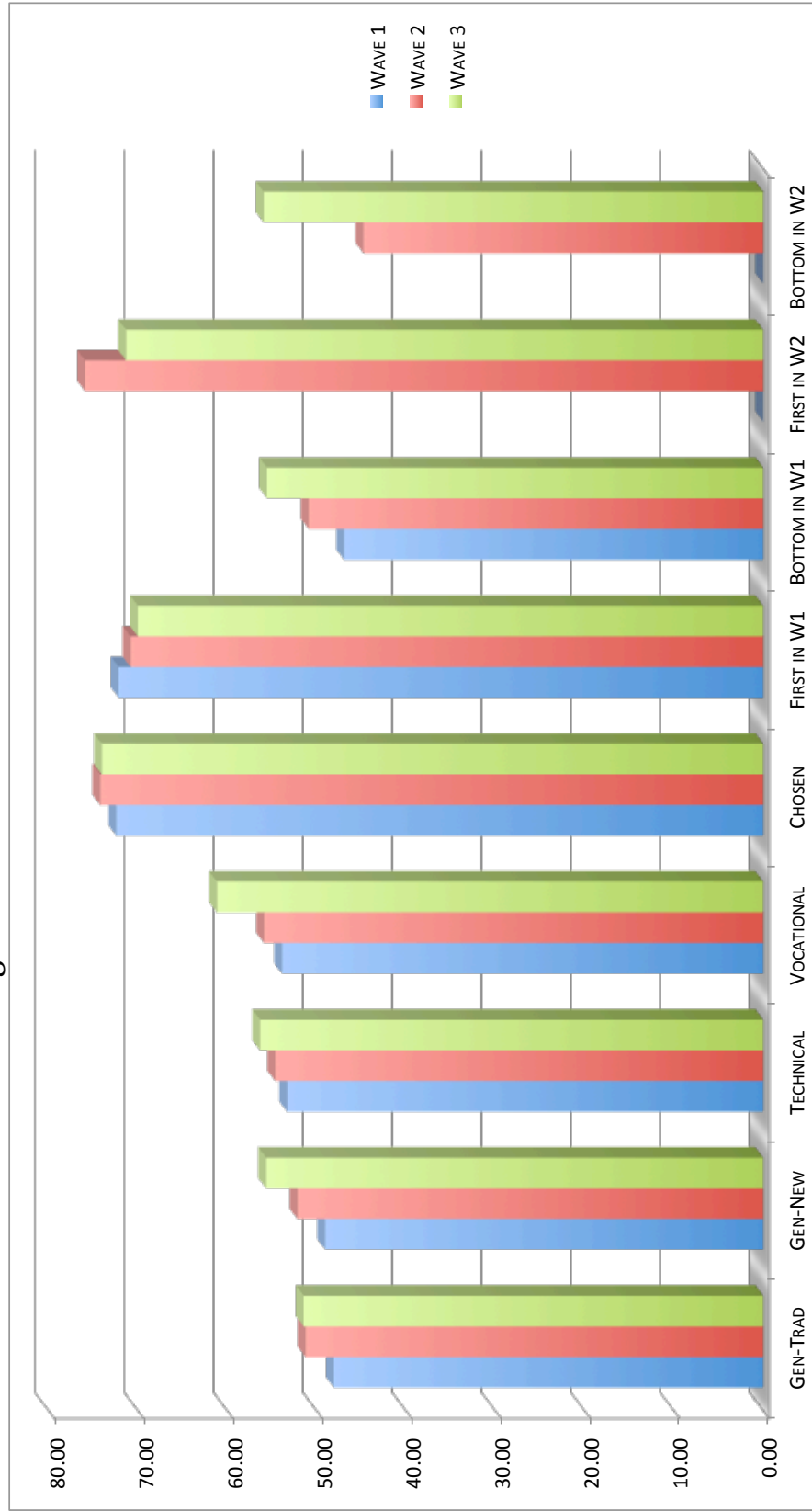


Figure A1: Mean point belief in waves 1-3, conditional on the sets described in the horizontal axis.

Figure A2: Evolution of Subjective Ranges by Curriculum

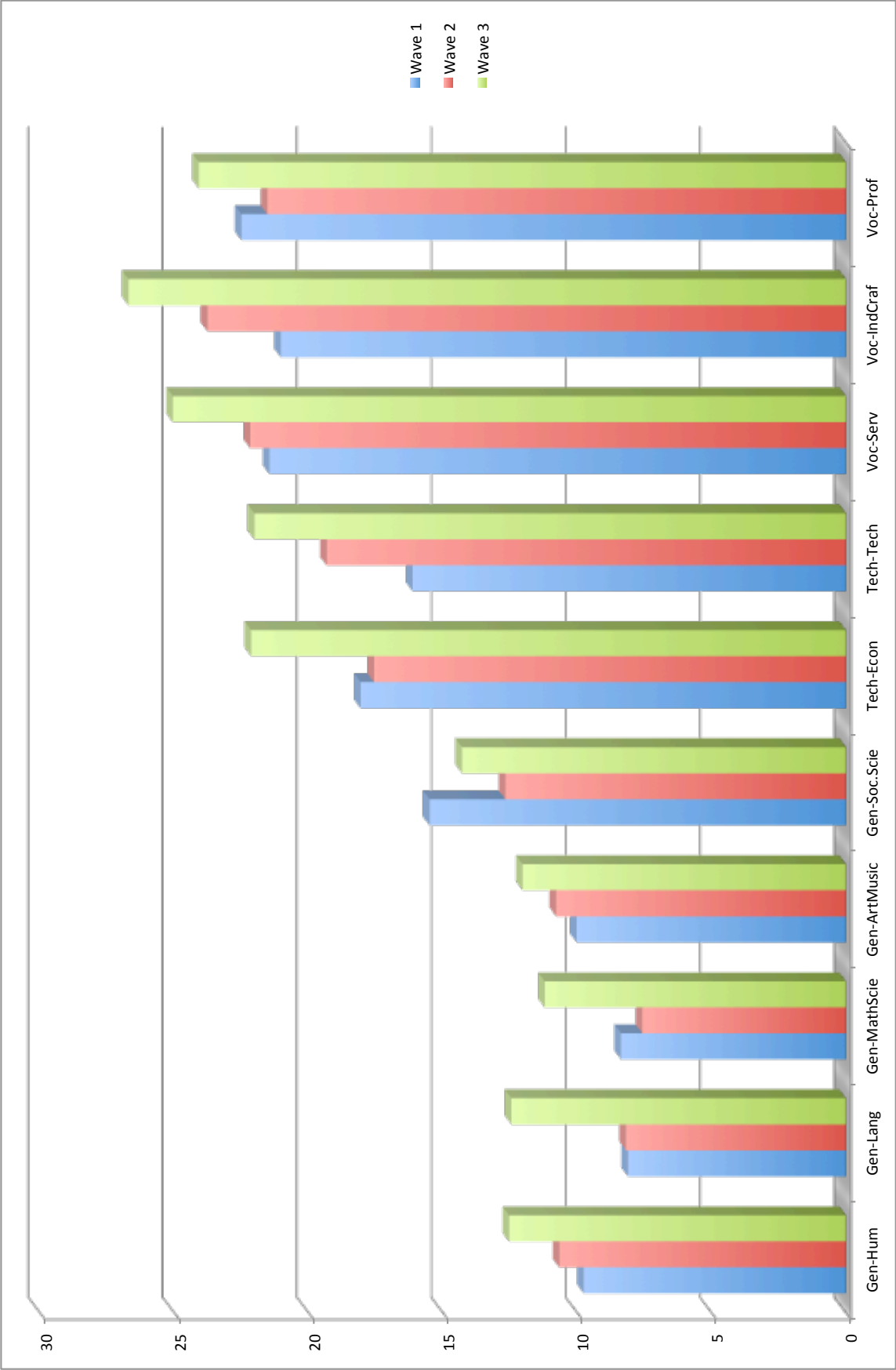


Figure A2: Mean range size in waves 1-3 by curriculum.

Table A12: PREDICTORS OF THE NUMBER OF CURRICULA OF WHICH THE CHILD IS AWARE AT SCHOOL START (WAVE 1)

Estimation Coefficients of a Poisson Regression of the Number of Curricula of which the Child is Aware at School Start (Wave 1)

Predictors	N of 'Know' or 'Heard of'	N of 'Know'	N of 'Know' or 'Heard of'	N of 'Know'	N of 'Know' or 'Heard of'	N of 'Know'
	All Curricula		General		Technical & Vocational	
female student	0.01325 (0.03863)	0.10535** (0.05276)	0.01028 (0.05034)	0.10957* (0.06444)	0.01751 (0.06024)	0.09223 (0.09208)
foreign-born student	-0.04558 (0.07545)	-0.11109 (0.10405)	-0.02633 (0.09759)	-0.16988 (0.13143)	-0.07310 (0.11896)	0.00478 (0.17090)
lives with both parents	0.06250 (0.06604)	0.05096 (0.08900)	0.02483 (0.08459)	-0.04477 (0.10353)	0.11915 (0.10573)	0.28876* (0.17502)
college+ mother	-0.09027 (0.06075)	-0.08589 (0.08249)	-0.02880 (0.07940)	0.12543 (0.10228)	-0.17860* (0.09455)	-0.53047*** (0.14698)
high school mother	-0.07015 (0.05133)	-0.06831 (0.06930)	-0.04352 (0.06779)	0.05395 (0.08929)	-0.10608 (0.07861)	-0.26050** (0.11129)
stay-at-home mother	-0.00543 (0.04594)	0.01612 (0.06214)	-0.01153 (0.06016)	-0.00199 (0.07725)	0.00293 (0.07118)	0.04252 (0.10477)
blue-collar father	0.01185 (0.04691)	0.06087 (0.06295)	0.00700 (0.06147)	-0.00034 (0.07929)	0.01830 (0.07260)	0.16331 (0.10446)
n of older siblings	-0.00326 (0.02691)	-0.02079 (0.03671)	0.01338 (0.03486)	-0.01228 (0.04471)	-0.02750 (0.04236)	-0.03921 (0.06424)
7th-grade GPA	0.01855 (0.02271)	-0.00278 (0.03088)	0.02954 (0.02959)	0.05749 (0.03766)	0.00288 (0.03545)	-0.12735** (0.05445)
constant	2.10210*** (0.18567)	1.60779*** (0.25176)	1.48417*** (0.24180)	0.71945** (0.30852)	1.33444*** (0.28998)	1.40678*** (0.44228)
χ^2	5.06	9.22	1.91	13.51	7.24	42.96
Prob > χ^2	0.8288	0.4176	0.9928	0.1410	0.6126	0.0000
Pseudo R^2	0.0037	0.0061	0.0017	0.0110	0.0063	0.0398
Sample Size	305	305	305	305	305	305

Significance levels: *** significant at 1%; ** significant at 5%; * significant at 10%.

Predictors: female student: dummy=1 if student is female; foreign-born student: dummy=1 if student is foreign-born; lives with both parents: dummy=1 if student lives with both parents; college+ mother: dummy=1 if student's mother has a college or higher degree; high school mother: dummy=1 if the highest degree of student's mother is a high school diploma; stay-at-home mother: dummy=1 if student has a stay-at-home mother; blue-collar father: dummy=1 if student's father works in a blue-collar occupation; n of older siblings: number of student's older siblings (between 0 and 3); 7th-grade GPA: student's GPA in the 7th grade end-of-year report (between 6 and 10).

Table A13: PREDICTORS OF THE NUMBER OF TRANSITIONS IN CHILD’S PERCEIVED AWARENESS BETWEEN SCHOOL START (WAVE 1) AND PRE-ENROLLMENT (WAVE 3)

Estimation Coefficients of a Poisson Regression of the Number of Curricula for which the Child’s Awareness Level between School Start (Wave 1) and Pre-Enrollment (Wave 3):

Predictors	Increases strictly	Decreases from ‘Know’	Increases strictly	Decreases from ‘Know’	Increases strictly	Decreases from ‘Know’
	All Curricula		General		Technical & Vocational	
female student	−0.00767 (0.06489)	0.05085 (0.14265)	0.06030 (0.09592)	0.04756 (0.20325)	−0.06523 (0.08817)	0.05531 (0.20038)
foreign-born student	0.04133 (0.11912)	0.41749* (0.22375)	0.08877 (0.17027)	0.57929* (0.29743)	−0.00060 (0.16680)	0.23126 (0.34165)
lives with both parents	0.04417 (0.11066)	−0.10158 (0.21398)	0.22568 (0.17531)	−0.31843 (0.27806)	−0.09091 (0.14301)	0.16836 (0.33843)
college+ mother	0.29674*** (0.10946)	−0.50877** (0.22141)	0.15781 (0.16136)	−0.48817 (0.32079)	0.40882*** (0.14961)	−0.51970* (0.30649)
high school mother	0.24063** (0.09483)	−0.21418 (0.18004)	0.22128 (0.13624)	−0.10840 (0.26142)	0.25823* (0.13215)	−0.30974 (0.24906)
stay-at-home mother	0.07005 (0.07738)	−0.80092*** (0.20438)	0.05743 (0.11286)	−1.01172*** (0.31667)	0.07962 (0.10630)	−0.62913** (0.26899)
blue-collar father	−0.15015* (0.08267)	−0.03832 (0.16982)	−0.15070 (0.11975)	−0.06671 (0.24104)	−0.15127 (0.11429)	−0.01305 (0.23954)
n of older siblings	0.01556 (0.04473)	0.02927 (0.09880)	−0.01651 (0.06644)	0.01216 (0.14138)	0.04241 (0.06048)	0.04278 (0.13839)
7th-grade GPA	−0.13525*** (0.03844)	−0.12603 (0.08356)	−0.18180*** (0.05685)	−0.16548 (0.11906)	−0.09516* (0.05224)	−0.08865 (0.11742)
constant	2.00651*** (0.31189)	1.05102 (0.66366)	1.45192*** (0.46339)	0.79943 (0.93770)	1.17509*** (0.42292)	−0.13156 (0.94536)
χ^2	24.33	29.89	16.54	24.54	15.65	9.50
Prob > χ^2	0.0038	0.0005	0.0564	0.0035	0.0746	0.3927
Pseudo R^2	0.0171	0.0376	0.0165	0.0488	0.0148	0.0191
Sample Size	301	301	301	301	301	301

Significance levels: *** significant at 1%; ** significant at 5%; * significant at 10%.

Predictors: female student: dummy=1 if student is female; foreign-born student: dummy=1 if student is foreign-born; lives with both parents: dummy=1 if student lives with both parents; college+ mother: dummy=1 if student’s mother has a college or higher degree; high school mother: dummy=1 if the highest degree of student’s mother is a high school diploma; stay-at-home mother: dummy=1 if student has a stay-at-home mother; blue-collar father: dummy=1 if student’s father works in a blue-collar occupation; n of older siblings: number of student’s older siblings (between 0 and 3); 7th-grade GPA: student’s GPA in the 7th grade end-of-year report (between 6 and 10).

Table A14: PREDICTORS OF THE NUMBER OF CURRICULA ABOUT WHICH THE CHILD HAS AN AMBIGUOUS SUBJECTIVE LIKELIHOOD OF GRADUATING IN TIME AT SCHOOL START

Estimation Coefficients of a Poisson Regression of the Number of Curricula about which the Child has an Ambiguous Subjective Likelihood of Graduating on Time at School Start (Wave 1)

Predictors	N of 'Unsure' or 'No Idea'	N of 'No Idea'	N of 'Unsure' or 'No Idea'	N of 'No Idea'	N of 'Unsure' or 'No Idea'	N of 'No Idea'
	All Curricula		General		Technical & Vocational	
female student	0.18506** (0.08818)	0.44430*** (0.12455)	0.05224 (0.12989)	0.12474 (0.22436)	0.29803** (0.12060)	0.57513*** (0.15123)
foreign-born student	0.07068 (0.16301)	0.24856 (0.20097)	0.10444 (0.24261)	0.40870 (0.35747)	0.04160 (0.22016)	0.18672 (0.24361)
lives with both parents	-0.19295 (0.13455)	-0.38817** (0.17102)	-0.09290 (0.20710)	-0.50966 (0.31165)	-0.27277 (0.17720)	-0.34449* (0.20484)
college+ mother	-0.25852* (0.13716)	-0.28417* (0.17110)	-0.36839* (0.20695)	-0.82743** (0.32979)	-0.17320 (0.18345)	-0.07143 (0.20586)
high school mother	-0.07129 (0.11777)	-0.46149*** (0.14971)	-0.05708 (0.17500)	-0.67628*** (0.25746)	-0.08449 (0.15928)	-0.34171* (0.18592)
stay-at-home mother	-0.33794*** (0.11186)	-0.06238 (0.14484)	-0.50902*** (0.17332)	-0.26699 (0.27936)	-0.20454 (0.14685)	0.02583 (0.16936)
blue-collar father	0.08209 (0.10444)	0.11110 (0.13953)	0.02829 (0.15652)	0.24872 (0.24424)	0.12658 (0.14030)	0.04755 (0.17091)
n of older siblings	0.09575 (0.05922)	0.05975 (0.08092)	0.08579 (0.08888)	-0.02467 (0.15525)	0.10413 (0.07939)	0.09369 (0.09461)
7th-grade GPA	0.15004*** (0.05089)	0.02769 (0.06851)	0.13463* (0.07600)	-0.20218 (0.13107)	0.16326** (0.06857)	0.11676 (0.08097)
constant	-0.29330 (0.42064)	0.21102 (0.55318)	-0.91311 (0.62843)	1.30610 (1.01443)	-1.05266* (0.56674)	-1.09685* (0.66479)
χ^2	29.76	34.70	15.70	22.21	19.85	31.52
Prob > χ^2	0.0005	0.0001	0.0733	0.0082	0.0189	0.0002
Pseudo R^2	0.0187	0.0294	0.0178	0.0471	0.0187	0.0352
Sample Size	281	281	281	281	281	281

Significance levels: *** significant at 1%; ** significant at 5%; * significant at 10%.

Predictors: female student: dummy=1 if student is female; foreign-born student: dummy=1 if student is foreign-born; lives with both parents: dummy=1 if student lives with both parents; college+ mother: dummy=1 if student's mother has a college or higher degree; high school mother: dummy=1 if the highest degree of student's mother is a high school diploma; stay-at-home mother: dummy=1 if student has a stay-at-home mother; blue-collar father: dummy=1 if student's father works in a blue-collar occupation; n of older siblings: number of student's older siblings (between 0 and 3); 7th-grade GPA: student's GPA in the 7th grade end-of-year report (between 6 and 10).

Table A15: PREDICTORS OF THE NUMBER OF TRANSITIONS IN CHILD’S PERCEIVED AMBIGUITY BETWEEN SCHOOL START (WAVE 1) AND PRE-ENROLLMENT (WAVE 3)

Estimation Coefficients of a Poisson Regression of the Number of Curricula for which the Child’s Ambiguity Level Between School Start (Wave 1) and Pre-Enrollment (Wave 3):

Predictors	Decreases strictly	Increases from ‘Sure’	Decreases strictly	Increases from ‘Sure’	Decreases strictly	Increases from ‘Sure’
	All Curricula		General		Technical & Vocational	
female student	0.24168* (0.12646)	0.23077* (0.12081)	0.08276 (0.17179)	0.38229* (0.20031)	0.42798** (0.18855)	0.15648 (0.15157)
foreign-born student	−0.06927 (0.24909)	0.42896** (0.20610)	0.06139 (0.33453)	0.70005** (0.29897)	−0.23005 (0.37416)	0.21792 (0.28699)
lives with both parents	−0.00343 (0.20488)	−0.79131*** (0.15693)	0.11876 (0.28734)	−0.90677*** (0.24208)	−0.15194 (0.29301)	−0.71540*** (0.20681)
college+ mother	−0.01513 (0.21880)	−0.05203 (0.20397)	0.08193 (0.31445)	0.07162 (0.36207)	−0.09338 (0.30979)	−0.08054 (0.24714)
high school mother	0.25825 (0.18611)	−0.00508 (0.17836)	0.39463 (0.27624)	0.30882 (0.31752)	0.14421 (0.25415)	−0.14997 (0.21651)
stay-at-home mother	−0.27044* (0.15585)	0.03697 (0.14774)	−0.55709** (0.23542)	−0.42957 (0.27280)	−0.00448 (0.21193)	0.26681 (0.17805)
blue-collar father	0.23829* (0.14506)	−0.28806* (0.15400)	0.03366 (0.21062)	−0.26183 (0.24712)	0.44538** (0.20308)	−0.29186 (0.19743)
n of older siblings	0.08378 (0.08330)	0.04105 (0.08089)	−0.01340 (0.11953)	−0.01535 (0.13263)	0.18087 (0.11650)	0.06750 (0.10205)
7th-grade GPA	0.09723 (0.07292)	−0.09907 (0.06979)	0.09797 (0.10092)	−0.33146*** (0.11777)	0.10463 (0.10601)	0.02904 (0.08722)
constant	−0.96695 (0.61407)	1.54515*** (0.56103)	−1.58481* (0.85698)	2.21524** (0.92270)	−1.85014** (0.88828)	0.09009 (0.71182)
χ^2	15.82	36.02	11.86	37.08	15.30	16.32
Prob > χ^2	0.0707	0.0000	0.2211	0.0000	0.0829	0.0606
Pseudo R^2	0.0165	0.0314	0.0201	0.0664	0.0257	0.0209
Sample Size	247	247	247	247	247	247

Significance levels: *** significant at 1%; ** significant at 5%; * significant at 10%.

Predictors: female student: dummy=1 if student is female; foreign-born student: dummy=1 if student is foreign-born; lives with both parents: dummy=1 if student lives with both parents; college+ mother: dummy=1 if student’s mother has a college or higher degree; high school mother: dummy=1 if the highest degree of student’s mother is a high school diploma; stay-at-home mother: dummy=1 if student has a stay-at-home mother; blue-collar father: dummy=1 if student’s father works in a blue-collar occupation; n of older siblings: number of student’s older siblings (between 0 and 3); 7th-grade GPA: student’s GPA in the 7th grade end-of-year report (between 6 and 10).

B Survey Questions

B.1 Awareness

Awareness was elicited at the beginning of waves 1 through 3 by means of the following question.

What types of high school do you know or have you heard the name of?

Curriculum	Familiarity with the Curriculum
Gen., Art	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Gen., Humanities	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Gen., Languages	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Gen., Music&Choral	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Gen., Math&Science	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Gen., Social Sciences	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Tech., Economic Sector	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Tech., Technology Sector	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Voc., Services	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Voc, Industry&Crafts	☐ I know it ☐ I have heard the name only ☐ I have never heard of it
Voc., Prof. Training	☐ I know it ☐ I have heard the name only ☐ I have never heard of it

B.2 Preference ranking

Stated preferences (preference ranking and choice probabilities) were elicited in waves 1 through 4. In wave 1, students were asked to list up to three curricula which they liked best and to rank them in order of preference. In waves 2-4, students were asked to rank all curricula. In all waves, students were additionally asked to indicate the chances between 0 and 100 that: (i) each of those curricula is the right school for them; (ii) they would choose each of those curricula, if they had to make a choice today. Elicitation of this information was conducted by means of the following set of questions plus the table shown in the next page (which were displayed together on the same page of the survey instrument).

What do you think about the choice today?

Column (I): *Please try and think of ALL factors that you consider important for choosing a curriculum to study in high school. Rank the curricula listed below from 1 = YOUR MOST PREFERRED curriculum to 10 = YOUR LEAST PREFERRED curriculum.*

Column (II): *Between 0 and 100, what are the chances that your most preferred curriculum in column (I) is the “RIGHT SCHOOL” for YOU?*

(Tip: If you feel completely confident that your most preferred curriculum is the right school for you, assign 100 to that curriculum under column (II). If you do not feel completely confident, choose a number smaller than 100 and then split the remainder to 100 among other curricula which MIGHT be the right school for you. Give 0 to all curricula which you think CANNOT be the right school for you.)

Column (III): *Between 0 and 100, what are the chances that YOU WOULD CHOOSE your most preferred curriculum in column (I), if you had to make a choice today?*

(Tip: If you feel completely confident that you would choose your most preferred curriculum, assign 100 to that curriculum under column (III). If you do not feel completely confident, choose a number smaller than 100 and then split the remainder to 100 among other curricula which you MIGHT choose today. Give 0 to all curricula which you are sure you would NOT choose today.)

BE CAREFUL: *Make sure that in each of columns (II) and (III) the numbers sum to 100!*

Curriculum	(I) Your preference order (from 1=BEST to 10=WORST)	(II) Percent chance that this school is right for you (from 0 to 100)		(III) Percent chance that you would choose this school today (from 0 to 100)	
Gen., Art/Music&Choral			+		+
Gen., Humanities			+		+
Gen., Languages			+		+
Gen., Math&Science			+		+
Gen., Social Sciences			+		+
Tech., Economic Sector			+		+
Tech., Technology Sector			+		+
Voc., Services			+		+
Voc, Industry&Crafts			+		+
Voc., Prof. Training			=		=
	Total	100		100	

B.3 Point belief and belief ambiguity

Point beliefs and belief ambiguity about the likelihood of experiencing a regular academic path in each curriculum were elicited in waves 1 through 3 by means of the following question (which refers to the table shown in the next page).

Now please think about your abilities and the school subjects you are good at. Additionally, try and think of how challenging different types of schools might be for you. Between 0 and 100, what are the chances that you would pass each grade on the first try and would graduate in the regular time from each type of school, if you were to enroll in it?

Curriculum	Number of chances (from 0 to 100)	How sure do you feel about your answer?
Gen., Art/Music&Choral	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Gen., Humanities	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Gen., Languages	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Gen., Math&Science	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Gen., Social Sciences	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Tech., Economic Sector	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Tech., Technology Sector	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Voc., Services	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Voc, Industry&Crafts	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances
Voc., Prof. Training	---	☐ I am sure about my answer ☐ I am unsure about my answer minimum chances: maximum chances: ☐ I have no idea about the chances

C Ranges Behavior: Theory

Recall that we assumed that model uncertainty can only be due to the learnable characteristics in Ω_M and that we can associate one model to each element $\omega_M \in \Omega_M$, and hence $M_t = \mathcal{I}_t^M$. It is immediate to see that if the child were to learn over the set of models only (e.g., when the information set $\mathcal{I}^U$ is time-constant), the range measure decreases with time for each a . From equation (3) in Section 4 of the paper, we have $R_t^a(\mathcal{I}_t) = R_t^a(\mathcal{I}^U \times \mathcal{I}_t^M) = \max_{m \in M_t} \pi^{a,m}(\mathcal{I}^U) - \min_{m \in M_t} \pi^{a,m}(\mathcal{I}^U)$, where the max and the min are taken over the set $M_t = \mathcal{I}_t^M$, which ‘shrinks’ over time.

Let us now allow for time-changing $\mathcal{I}^U$ and for ex-ante heterogeneity. Assume that the information $\{\mathcal{I}_t^{U,g}\}_t$, arrives *independently across students*, conditional on the student’s type and the set of models; in particular, after the start of 8th grade there is no new aggregate information. Let us fix a model m . By the law of large numbers and by Bayes’ rule, for each ex-ante homogeneous group $g \in G$, we have $\pi^{a,m,g}(\mathcal{I}_{t-1}^{U,g}) = \int \pi^{a,m,g}(\mathcal{I}_t^{U,g}) dh_t^g(\mathcal{I}_t^{U,g})$, where $h_t^g(\cdot)$ is the distribution over new information in period t for group g , and $\pi^{a,m,g}(\mathcal{I}_{t-1}^{U,g})$ and $\pi^{a,m,g}(\mathcal{I}_t^{U,g})$ are the posteriors reported by the students in group g under model m , in period $t-1$ and t respectively. Since this martingale condition holds for each group, if we let $f(\cdot)$ be the distribution of ex-ante heterogeneous types in the economy and assume that such a distribution is constant over time, we obtain $\hat{\pi}_{t-1}^{a,m} = \int_G \pi_{t-1}^{a,m,g} df(g) = \int_G \int \pi^{a,m,g}(\mathcal{I}_t^{U,g}) dh_t^g(\mathcal{I}_t^{U,g}) df(g) = \hat{\pi}_t^{a,m} \forall a, t, m$. In other terms, given any model, Bayes’ rule and cross-sectional independent signals jointly imply that the cross-sectional average of beliefs $\hat{\pi}_t^{a,m}$ is constant over time.

Since ranges are reported individually, for any two consecutive dates at which the set of models remains constant, $\mathcal{I}_t^{M,g} = \mathcal{I}_{t-1}^{M,g} = M^g$, and any collection of information sets, $\mathcal{I}_t^{U,g} \subset \mathcal{I}_{t-1}^{U,g}$, consistent with assumptions on the learning process, we have

$$\begin{aligned} \hat{R}_t^a &= \int_G \int R_t^{a,g}(\mathcal{I}_t^{U,g} \times \mathcal{I}^{M,g}) dh_t^g(\mathcal{I}_t^{U,g}) df(g) = \int_G \int \left[\max_{m \in M^g} \pi^{a,m,g}(\mathcal{I}_t^{U,g}) - \min_{m \in M^g} \pi^{a,m,g}(\mathcal{I}_t^{U,g}) \right] dh_t^g(\mathcal{I}_t^{U,g}) df(g) \\ &\geq \int_G \left[\max_{m \in M^g} \int \pi^{a,m,g}(\mathcal{I}_t^{U,g}) dh_t^g(\mathcal{I}_t^{U,g}) - \min_{m \in M^g} \int \pi^{a,m,g}(\mathcal{I}_t^{U,g}) dh_t^g(\mathcal{I}_t^{U,g}) \right] df(g) \\ &= \int_G \left[\max_{m \in M^g} \pi^{a,m,g}(\mathcal{I}_{t-1}^{U,g}) - \min_{m \in M^g} \pi^{a,m,g}(\mathcal{I}_{t-1}^{U,g}) \right] df(g) = \int_G R_{t-1}^{a,g} df(g) = \hat{R}_{t-1}^a. \end{aligned} \tag{1}$$

The inequality in the second row follows immediately from the property of the maximum and the minimum and the fact that information arrives independently conditional on the set of models, while the equality at the beginning of the last row uses the martingale property of beliefs. All other equalities are definitional.

This results implies that a decreasing range size is necessarily associated to higher knowledge; whereas an increasing range size has an ambiguous interpretation. To interpret an increase in the cross-sectional average of range size as a ‘deterioration’ of information, we need additional structure, which we provide next.

Proposition C1.

(i) If for some alternative $a \in \hat{A}$ and periods $t, t-1 \geq 0$, we observe a decreasing average range size $\hat{R}_t^a < \hat{R}_{t-1}^a$, it must be that some or all children received some information over the set of models, that is, $M_t^g \subset M_{t-1}^g$ with strict inclusion for at least some g of positive measure.

(ii) Moreover, for a fixed alternative a , if for all t, m, g , and $\mathcal{I}_t^U \subset \mathcal{I}_{t-1}^U$ consistent with the process of arrival of information, we have $\pi^{a,m,g}(\mathcal{I}_t^U) = \pi^{a,m,g}(\mathcal{I}_{t-1}^U) + \varepsilon^{a,g}(\mathcal{I}_t^U)$, then the cross-sectional average range measure $\hat{R}_t^a$ (weakly) decreases over time.

Proof. (i) This is a direct consequence of the fact (shown in (1)) that if $\mathcal{I}^M = M$ is constant, $\hat{R}_t^a$ can only increase.

(ii) In this case with linear updating rules, for any two consecutive dates with fixed $\mathcal{I}_t^{M,g} = M_t^g = M_{t-1}^g$, for each g and $\mathcal{I}_t^{U,g} \subset \mathcal{I}_{t-1}^{U,g}$, we have

$$\int R_t^{a,g}(\mathcal{I}_t^{U,g} \times \mathcal{I}_t^{M,g}) d h_t^g(\mathcal{I}_t^{U,g}) = R_{t-1}^{a,g}(\mathcal{I}_{t-1}^{U,g} \times \mathcal{I}_{t-1}^{M,g}).$$

The monotonicity is implied by the fact that for each g and each $\mathcal{I}_t^{U,g}$ generated by the learning process, we showed above that the individual ranges $R_t^{a,g}(\mathcal{I}_t^{U,g} \times M_t^g) = \max_{m \in M_t^g} \pi^{a,m,g}(\mathcal{I}_t^{U,g}) - \min_{m \in M_t^g} \pi^{a,m,g}(\mathcal{I}_t^{U,g})$ decrease in M_t^g , and learning implies that M_t^g (weakly) decreases with t for each g . **Q.E.D.**