For Online Publication: Technical Appendix "Reputation and Liquidity Traps"*

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Section A sets up notations. Section B provides a formal proof of the conditions for the revert-to-discretion plan to be credible. Section C describes the details of the Ramsey planner's problem and the solution method, and reports the solution accuracy. Section D provides detailed analyses of Markov-perfect equilibria and demonstrates the existence of an equilibrium in which the government chooses to keep the nominal interest rate at zero in both high and low states. Section E analyzes the implications of repeated crises for the Ramsey outcome. Section F constructs the *revert-to-deflation* plan and examines its credibility.

Section G defines and analyzes the *revert-to-discretion(N)* plan in which the reversion to a regime in which the central bank cannot manipulate future expectations lasts for finite periods N. Section H provides some details on the fully nonlinear version of the model. Section I provides detailed exposition of the mechanism behind the result on how frequency and persistence of the shock affects credibility. Section J provides detailed sensitivity analyses in the reduced-form parameter space, whereas Section K provides the results of sensitivity analyses in the structural-parameter space. Section L provides additional analyses on the revert-to-discretion(N) plan. Section M discusses the discontinuity of the threshold crisis frequency at $p_L = 0$.

A Notations

Throughout the paper, I use the following notations. For any variable x, its state-contingent sequence is denoted by $\boldsymbol{x}$ (bold font). In other words, $\boldsymbol{x} := \{x_k(s^k)\}_{k=1}^{\infty}$.

To help make this section self-contained, I write the consumption Euler equation

*The views expressed in this paper, and all errors and omissions, should be regarded as those of the author, and are not necessarily those of the Federal Reserve Board of Governors or the Federal Reserve System.

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and the Phillips curve here:

$$\chi_c c_t(s^t) = \chi_c E_t c_{t+1}(s^{t+1}) - \left[r_t(s^t) - E_t \pi_{t+1}(s^{t+1})\right] + s_t \tag{1}$$

$$\pi_t(s^t) = \kappa c_t(s^t) + \beta E_t \pi_{t+1}(s^{t+1}) \tag{2}$$

A state-contingent sequence up to time t and *a continuation state-contingent sequence starting at time t* are respectively denoted by $\boldsymbol{x}^t$ and $\boldsymbol{x}_t$. In other words, $\boldsymbol{x}^t := \{x_k(s^k)\}_{k=1}^t$ and $\boldsymbol{x}_t := \{x_k(s^k)\}_{k=t}^\infty$. x^t (non-bold font) is used to denote a particular realization of $\boldsymbol{x}^t$, and should not be confused with $\boldsymbol{x}^t$ (bold font).

For any variable x with a range $\mathbb{X}$, $\boldsymbol{x}(s)$ denotes a state-contingent sequence with $s_1 = s$, which is defined by a sequence of functions mapping a history of states with $s_1 = s$ to $\mathbb{X}$. In other words,

$$\begin{aligned} x_1 &: s \rightarrow \mathbb{X} \\ x_t &: s \times \mathbb{S}^{t-1} \rightarrow \mathbb{X} \end{aligned}$$

$\boldsymbol{x}^t(s)$ denotes *a state-contingent sequence with $s_1 = s$ up to time t*. $\boldsymbol{x}_t(s)$ denotes *a continuation state-contingent sequence starting at time t with $s_t = s$*, which is formally given by the following sequence of functions.

$$\begin{aligned} x_t &: s \rightarrow \mathbb{X} \\ x_{t+k} &: s \times \mathbb{S}^{k-1} \rightarrow \mathbb{X} \end{aligned}$$

$CE(s)$ denotes the set of all competitive outcomes with $s_1 = s$. That is, for each $s \in \mathbb{S}$,

$$\begin{aligned} CE(s) := \{&(\boldsymbol{c}(s), \boldsymbol{\pi}(s), \boldsymbol{r}(s)) \in \mathbb{R}^\infty \times \mathbb{R}^\infty \times \mathbb{R}^\infty_{\geq 0} \\ &|\text{Equations(1) and (2) hold for all } t \geq 1 \text{ and for all } s^t \in \mathbb{S}^t \text{ with } s_1 = s\} \end{aligned}$$

$CE_k(s)$ to denotes the set of continuation competitive outcomes starting at period k with $s_k = s$. That is, for each $s \in \mathbb{S}$,

$$\begin{aligned} CE_k(s) := \{&(\boldsymbol{c}_k(s), \boldsymbol{\pi}_k(s), \boldsymbol{r}_k(s)) \in \mathbb{R}^\infty \times \mathbb{R}^\infty \times \mathbb{R}^\infty_{\geq 0} \\ &|\text{Equations(1) and (2) hold for all } t \geq k \text{ and for all } s^t \in \mathbb{S}^t \text{ with } s_k = s\} \end{aligned}$$

B Proofs

Proposition: The revert-to-discretion plan is credible if and only if $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$ for all $t \geq 1$ and all $s^t \in \mathbb{S}^t$.

Proof of "if"-part: Let σ_g^{ram} and σ_p^{ram} be the government and private sector strategies associated with the revert-to-discretion plan. We need to show that, if $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$ for all $t \geq 1$ and all $s^t \in \mathbb{S}^t$, (i) σ_g^{ram} is admissible, (ii) given σ_g^{ram}, after any history r^t and s^t, the continuation of σ_p^{ram} and σ_g^{ram} induce a $(\boldsymbol{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \boldsymbol{r}_t(s_t)) \in CE_t(s_t)$, and (iii) after any history r^{t-1} and s^t, the sequence $\boldsymbol{r}_t(s_t)$ induced by σ_g^{ram}

maximizes the government's objective over $CE_t^R(s_t)$ given σ_p^{ram}.

Proof of (i): Consider a history r^{t-1} and s^t where $r_j = r_{ram,j}(s^j)$ for all $j \leq t-1$ and all $s^j \in \mathbb{S}^t$. The continuation of σ_g^{ram} will induce $\boldsymbol{r}_t(s_t)$ in which $r_{t+k}(s^{t+k}) = r_{ram,t+k}(s^{t+k})$ for all $k \geq 0$ and all $s^{t+k} \in \mathbb{S}^{t+k}$. Call this sequence $\boldsymbol{r}_{ram,t}$. Take $\boldsymbol{c}_{ram,t}(s_t)$ and $\boldsymbol{\pi}_{ram,t}(s_t)$. Clearly, $(\boldsymbol{c}_{ram,t}(s_t), \boldsymbol{\pi}_{ram,t}(s_t), \boldsymbol{r}_{ram,t}(s_t))$ belongs to $CE_t(s_t)$.

Now consider a history r^{t-1} and s^t in which $r_j \neq r_{ram,j}(s^j)$ for some $j \leq t-1$. The continuation of σ_g^{ram} will induce $\boldsymbol{r}_t$ in which $r_{t+k}(s^{t+k}) = r_{d,t+k}(s^{t+k})$ for all $k \geq 0$ and all $s^{t+k} \in \mathbb{S}^{t+k}$. Call this sequence $\boldsymbol{r}_{d,t}$. Take $\boldsymbol{c}_{d,t}(s_t)$ and $\boldsymbol{\pi}_{d,t}(s_t)$. Clearly, $(\boldsymbol{c}_{d,t}(s_t), \boldsymbol{\pi}_{d,t}(s_t), \boldsymbol{r}_{d,t}(s_t))$ belongs to $CE_t(s_t)$.

These two cases cover all possible histories. Thus, after *any* history r^{t-1} and s^t, the continuation of σ_g^{ram} will induce $\boldsymbol{r}_t(s_t)$ that belongs to $CE_t^R(s_t)$.

Proof of (ii): Consider a history r^t and s^t where $r_j = r_{ram,j}(s^j)$ for all $j \leq t$. The continuation of σ_g^{ram} and σ_p^{ram} induces $(\boldsymbol{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \boldsymbol{r}_t(s_t))$ where $c_{t+k}(s^{t+k}) = c_{ram,t+k}(s^{t+k})$, $\pi_{t+k}(s^{t+k}) = \pi_{ram,t+k}(s^{t+k})$, and $r_{t+k}(s^{t+k}) = r_{ram,t+k}(s^{t+k})$ for all $k \geq 0$ and all $s^{t+k} \in \mathbb{S}^{t+k}$. Call this sequence $(\boldsymbol{c}_{ram,t}(s_t),\ \boldsymbol{\pi}_{ram,t}(s_t),\ \boldsymbol{r}_{ram,t}(s_t))$. Clearly, $(\boldsymbol{c}_{ram,t}(s_t), \boldsymbol{\pi}_{ram,t}(s_t), \boldsymbol{r}_{ram,t}(s_t))$ belongs to $CE_t(s_t)$.

Consider a history r^t and s^t in which $r_j \neq r_{ram,j}(s^j)$ for some $j \leq t$. The continuation of σ_g^{ram} and σ_p^{ram} induces $(\boldsymbol{c}_t, \boldsymbol{\pi}_t, \boldsymbol{r}_t)$ where $c_{t+k}(s^{t+k}) = c_{d,t+k}(s^{t+k})$, $\pi_{t+k}(s^{t+k}) = \pi_{d,t+k}(s^{t+k})$, and $r_{t+k}(s^{t+k}) = r_{d,t+k}(s^{t+k})$ for all $k \geq 1$ and all $s^{t+k} \in \mathbb{S}^{t+k}$. At time t, $c_t(s^t) = c_{br}(r_t, s^t)$ and $\pi_t = \pi_{br,t}(r_t(s^t), s^t)$.

Call this continuation sequence $(\boldsymbol{c}_{dev,t}(s_t), \boldsymbol{\pi}_{dev,t}(s_t), \boldsymbol{r}_{dev,t}(s_t))$. Clearly, the consumption Euler equation and the Phillips curve are satisfied for all $k \geq 1$. If $r_t(s^t) = r_{d,t}(s^t)$, it is clear that the consumption Euler equation and the Phillips curve is satisfied at $k = 0$ as well. At time t, the Phillips curve is satisfied trivially, and you can rearrange the expression for $c_t(s^t)$ to confirm that the consumption Euler equation is also satisfied. Thus, $(\boldsymbol{c}_{dev,t}(s_t), \boldsymbol{\pi}_{dev,t}(s_t), \boldsymbol{r}_{dev,t}(s_t))$ belongs to $CE_t(s_t)$.

These two cases cover all possible histories. Thus, after any history r^t and s^t, the continuation of σ_g^{ram} and σ_p^{ram} will induce $(\boldsymbol{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \boldsymbol{r}_t(s_t))$ that belongs to $CE_t(s_t)$.

Proof of (iii): By one-shot deviation principle, it suffices to show that, after any history r^{t-1} and s^t, there is no profitable deviation today from the government strategy in order to prove (iii).

First, consider a history r^{t-1} in which $r_j \neq r_{ram,j}(s^j)$ for some $j \leq t-1$. In this case, we want to show

$$\begin{aligned} & u(c_{d,t}(s^t), \pi_{d,t}(s^t)) + \beta E_t w_{d,t+1}(s^{t+1}) \\ \geq \quad & u(c_{br}(r_t, s_t), \pi_{br}(r_t, s_t)) + \beta E_t w_{d,t+1}(s^{t+1}) \end{aligned}$$

for all $r_t \in \mathbb{R}_{\geq 0}$. The left-hand side is the value of following the instruction given by the government strategy, and the right hand-side is the value of deviating from it. The continuation value in the case of the deviation is $E_t w_{d,t+1}(s^{t+1})$ because the government and private sector strategies would induce the discretionary outcome from next period on even in the case of the government deviation today. Notice that, by the definition of the discretionary outcome,

$$\begin{aligned} & u(c_{d,t}(s^t), \pi_{d,t}(s^t)) + \beta E_t w_{d,t+1}(s^{t+1}) \\ = & \max_{r_t \in \mathbb{R}_{\geq 0}} u(c_{br}(r_t, s_t), \pi_{br}(r_t, s_t)) + \beta E_t w_{d,t+1}(s^{t+1}) \end{aligned}$$

Thus, the aforementioned inequality holds, and there is no profitable one-shot deviation.

Now, consider a history r^t where $r_j = r_{ram,j}(s^j)$ for all $j \leq t$. We want to show that

$$\begin{aligned} & u(c_{ram,t}(s^t), \pi_{ram,t}(s^t)) + \beta E_t w_{ram,t+1}(s^{t+1}) \\ \geq & u(c_{br}(r_t, s_t), \pi_{br}(r_t, s_t)) + \beta E_t w_{d,t+1}(s^{t+1}) \end{aligned}$$

for all $r_t \in \mathbb{R}_{\geq 0}$. The left-hand side is the continuation value when the government chooses the nominal interest rate consistent with the Ramsey outcome (i.e., $r_t(s^t) = r_{ram,t}(s^t)$), and the right hand side is the possible continuation values in the case of deviation (i.e., $r_t(s^t) \neq r_{ram,t}(s^t)$). Notice that the left-hand side of the inequality is equal to $w_{ram,t}(s^t)$. If $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$ for all $t \geq 1$, then

$$\begin{aligned} & u(c_{ram,t}(s^t), \pi_{ram,t}(s^t)) + \beta E_t w_{ram,t+1}(s^{t+1}) \\ = & w_{ram,t}(s^t) \\ \geq & w_{d,t}(s^t) \\ = & \max_{r_t \in \mathbb{R}} \left[u(c_{br}(r_t, s_t), \pi_{br}(r_t, s_t)) + \beta E_t w_{d,t+1}(s^{t+1}) \right] \\ \geq & u(c_{br}(r_t), \pi_{br}(r_t)) + \beta E_t w_{d,t+1}(s^{t+1}) \end{aligned}$$

for all $r_t \in \mathbb{R}_{\geq 0}$. Thus, there is no profitable one-shot deviation in this case as well.

These two cases cover all possible histories. Thus, there is no profitable one-shot deviation after *any* history, and (iii) holds.

Proof of "only if"-part:

We want to prove that, if a plan is credible, then $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$ for all $t \geq 1$ and all $s^t \in \mathbb{S}^t$. We will do so by proving the contraposition, i.e., by showing that a plan is not credible if $w_{ram,t}(s^t) < w_{d,t}(s^t)$ for some $t \geq 1$ and some $s^t \in \mathbb{S}^t$.

Let t_v and s^{t_v} be such that $w_{ram,t}(s^{t_v}) < w_{d,t}(s^{t_v})$. Then,

$$\begin{aligned} & u(c_{ram,t_v}(s^{t_v}), \pi_{ram,t}(s^{t_v})) + \beta E_{t_v} w_{ram,t_v+1}(s^{t_v+1}) \\ < & u(c_{br}(r_{dev,t}, s_t), \pi_{br}(r_{dev,t}, s_t)) + \beta E_t w_{d,t+1}(s^{t+1}) \end{aligned}$$

for some $r_{dev,t} \in \mathbb{R}$. Then, consider a government strategy that instructs the government to choose $r_{dev,t}$ today and follow the discretionary outcome from tomorrow. The strategy delivers the better value today than the continuation of the revert-to-discretion plan, violating the third condition of credibility.

C Details of the Ramsey Outcome

C.1 Recursive Formulation

Following Adam and Billi (2006), I characterize the Ramsey equilibrium recursively based on the saddle-point functional equation.[1] The saddle-point functional equation corresponding to the infinite-horizon problem of the Ramsey planner is give by

$$\begin{aligned}&W_t(\delta_t, \phi_{1,t-1}, \phi_{2,t-1})\\ =&\min_{\{\gamma_{1,t},\gamma_{2,t}\}}\max_{\{d_t\}}\ h(c_t, \pi_t, i_t, \phi_{1,t-1}, \phi_{2,t-1}, \gamma_{1,t}, \gamma_{2,t}) + \beta \mathrm{E}_t W_{t+1}(\delta_{t+1}, \phi_{1,t}, \phi_{2,t})\end{aligned}$$

where

$$\begin{aligned}&h(c_t, \pi_t, r_t, \phi_{1,t-1}, \phi_{2,t-1}, \gamma_{1,t}, \gamma_{2,t})\\ =&-\frac{1}{2}\left[\pi_t^2 + \lambda c_t^2\right] - \frac{\chi_c}{\beta}\phi_{1,t-1}c_t - \phi_{2,t-1}\pi_t + \gamma_{1,t}\chi_c c_t + \gamma_{2,t}\left[\pi_t - \kappa c_t\right]\end{aligned}$$

and the optimization is subject to the ZLB constraint,

$$r_t \geq 0,$$

and the following law of motions for $\phi_{1,t}$ and $\phi_{2,t}$,

$$\begin{aligned}\phi_{1,t} &= \gamma_{1,t},\\ \phi_{2,t} &= \gamma_{2,t}.\end{aligned}$$

Let $h_t \equiv [s_t, \phi_{1,t-1}, \phi_{2,t-1}]$. Let $\phi_{3,t}$ be the Lagrange multiplier on the ZLB constraint. The Ramsey equilibrium can be characterized by a set of time-invariant policy and value functions, $[c(h_t)$, $\pi(h_t)$, $r(h_t)$, $\phi_1(h_t)$, $\phi_2(h_t)$, $\phi_3(h_t)]$, that solve the saddle-point functional equation above.

The first-order necessary conditions characterizing the solution to the saddle-point functional equation are given by

$$\begin{aligned}0 &= -\lambda c_t + \phi_{1,t}\chi_c - \phi_{1,t-1}\frac{\chi_c}{\beta} - \phi_{2,t}\kappa\\ 0 &= -\pi_t - \frac{1}{\beta}\phi_{1,t-1} + \phi_{2,t} - \phi_{2,t-1}\\ 0 &= \phi_{1,t} + \phi_{3,t}\end{aligned}$$

with $\phi_{3,t}r_t = 0$, $\phi_{3,t} \geq 0$, and $r_t \geq 0$. These conditions, together with the consumption Euler equation and the Phillips curve, characterize the solution to the Ramsey equilibrium.

[1] See Marcet and Marimon (2016) for the theory behind this recursive characterization of the Ramsey planner's problem.

C.2 Solution Method

The computational problem is to find a set of time-invariant policy functions, $\{c(\cdot), \pi(\cdot), r(\cdot), \phi_1(\cdot), \phi_2(\cdot), \phi_3(\cdot)\}$, that solves the system of *functional* equations given by

$$\begin{aligned}
\chi_c c(h_t) &= \chi_c E_t(h_{t+1}) - \big[r(h_t) - E_t\pi(h_{t+1})\big] + s_t \\
\pi(h_t) &= \kappa c(h_t) + \beta E_t \pi(h_{t+1}) \\
0 &= -\lambda c(h_t) + \phi_1(h_t)\chi_c - \phi_1(h_{t-1})\frac{\chi_c}{\beta} - \phi_2(h_t)\kappa \\
0 &= -\pi_t - \frac{1}{\beta}\phi_1(h_{t-1}) + \phi_2(h_t) - \phi_2(h_{t-1}) \\
0 &= \phi_1(h_t) + \phi_3(h_t)
\end{aligned}$$

where $h_t \equiv [s_t, \phi_{1,t-1}, \phi_{2,t-1}]$. Following the idea of Christiano and Fisher (2000), I decompose these policy functions into two parts using an indicator function: One in which the policy rate is allowed to be less than zero, and the other in which the policy rate is assumed to be zero. That is, for any variable Z,

$$Z(\cdot) = I_{\{R(\cdot)\geq 1\}} Z_{unc}(\cdot) + (1 - I_{\{R(\cdot)\geq 1\}}) Z_{zlb}(\cdot).$$

The problem then becomes finding a set of a *pairs* of policy functions, $\Big[\big[c_{unc}(\cdot), c_{zlb}(\cdot)\big], \big[y_{unc}(\cdot), y_{zlb}(\cdot)\big], \big[\pi_{unc}(\cdot), \pi_{zlb}(\cdot)\big], \{\big[\phi_{unc,i}(\cdot), \phi_{zlb,i}(\cdot)\big]\}_{i=1}^{3}\Big]$ that solves the system of functional equations above. This method can achieve a given level of accuracy with a considerable less number of grid points relative to the standard approach.

The time-iteration method starts by specifying a guess for the value and policy functions take on a finite number of grid points. Let $X(\cdot)$ be a vector of policy functions that solves the functional equations above and let $X^{(0)}$ be the initial guess of such policy functions.[2] At the s-th iteration and at each point of the state space, we solve the system of nonlinear equations associated with each problem to find today's consumption, output, inflation, and the policy rate, given that $X^{(s-1)}(\cdot)$ is in place for the next period. In solving the system of nonlinear equations, I use Gaussian quadrature to evaluate the expectation terms in the consumption Euler equation and the Phillips curve, and the value of future variables not on the grid points are evaluated with linear interpolation. The system is solved numerically by using a nonlinear equation solver, dneqnf, provided by the IMSL Fortran Numerical Library. For both models, I use 10 grid points for the Gaussian quadrature. If the updated policy functions are sufficiently close to the guessed policy functions, then the algorithm ends. Otherwise, using the updated policy functions just obtained as the guess for the next period's policy functions, I iterate on this process until the difference between the guessed and updated policy functions is sufficiently small ($\|vec(X^s(\delta) - X^{s-1}(\delta))\|_\infty <$ 1E-10 is used as the convergence criteria).

[2] For all models and all variables, I use flat functions at the deterministic steady-state values as the initial guess.

C.3 Solution Accuracy

In this section, I report the accuracy of our numerical solutions for the models with and without commitment. Following Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015), we evaluate these residuals functions along a simulated equilibrium path. The length of the simulation is 100,000. We focus on two key residual equations of the model—one associated with the consumption Euler equation, the other associated with the sticky-price equilibrium condition—given by,

$$\mathrm{R}_{1,t} = \left| c_t - \mathrm{E}_t c_t - \frac{1}{\chi_c}(\mathrm{E}_t \pi_{t+1} - r_t + \delta_t) \right|,$$
$$\mathrm{R}_{2,t} = |\pi_t - \kappa c_t - \beta \mathrm{E}_t \pi_{t+1}|.$$

The residual $\mathrm{R}_{1,t}$ measures the difference between the chosen consumption today and today's consumption consistent with the optimization behavior of the household, as a percentage of the chosen consumption. The residual $\mathrm{R}_{2,t}$ measures how much the chosen inflation rate differs from the inflation rate consistent with the optimization behavior of firms.

Table C.1: Solution Accuracy

$\xi = \lceil \log_{10}(\mathrm{R}_{k,t}) \rceil$	$\mathrm{E}[\xi]$	95th-percentile of ξ
$k=1$: Euler equation error	-4.8	-3.1
$k=2$: Sticky-price equation error	-6.5	-5.3

Table C.1 shows the average and the 95th percentile of the residuals for the two equations for the models with and without commitment. Reflecting the large number of grid points used to solve the model without commitment, the approximation errors are very small. The average errors on the consumption Euler equation is 0.002 percent ($= 100 * 10^{-4.8}$) with the 95th percentile being 0.08 percent ($= 100 * 10^{-3.1}$). The average errors on the sticky price equation is 0.01 basis points ($= 400 * 100 * 10^{-6.5}$) with the 95th percentile being 0.2 basis points ($= 400 * 100 * 10^{-5.3}$).

D Analyses of Markov-Perfect Equilibria

In this section, I analyze the existence and multiplicity of Markov-Perfect equilibria—time-invariant solutions for the discretionary government's problem described in the main text. In addition to the equilibrium in which the ZLB binds only in the low state (Type-I equilibrium), there are three other possible equilibria: one in which the ZLB binds in both states (Type-II equilibrium), one in which the ZLB binds in neither states (Type-III equilibrium), and the other in which the ZLB binds only in the high state (Type-IV equilibrium). This section aims to understand how frequency and persistent of the shock affect the existence of these four types of Markov-Perfect equilibria.

The first order necessary conditions for the discretionary government's problem are given by

$$
\begin{aligned}
\chi_c c(H) &= \chi_c\big[(1-p_H)c(H)+p_H c(L)\big] + \big[(1-p_H)\pi(H)+p_H\pi(L)\big] - r(H) + H\\
\pi(H) &= \kappa c(H) + \beta\big[(1-p_H)\pi(H)+p_H\pi(L)\big]\\
0 &= \lambda c(H) - \phi_3(H) + \kappa\pi(H)\\
\chi_c c(L) &= \chi_c\big[(1-p_L)c(H)+p_L c(L)\big] + \big[(1-p_L)\pi(H)+p_L\pi(L)\big] - r(L) + L\\
\pi(L) &= \kappa c(L) + \beta\big[(1-p_L)\pi(H)+p_L\pi(L)\big]\\
0 &= \lambda c(L) - \phi_3(L) + \kappa\pi(L)
\end{aligned}
$$

together with one of the following sets of conditions regarding the nominal interest rate and the Lagrange multiplier on the ZLB constraint, $\phi_3(\cdot)$.

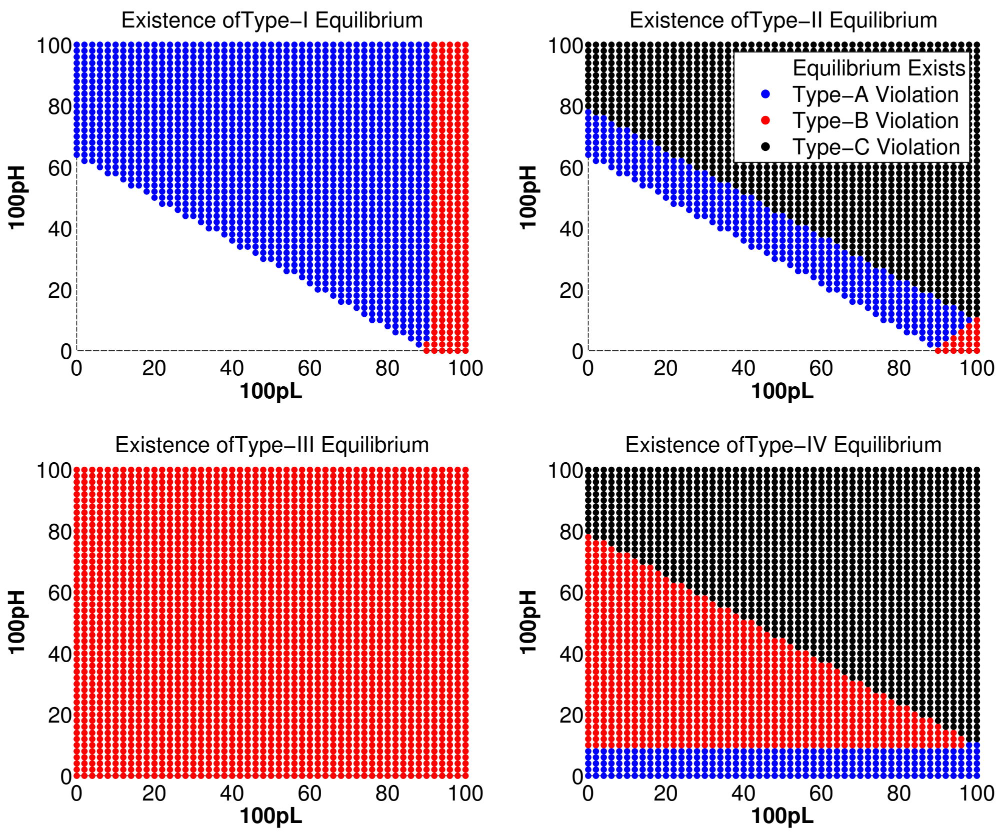
$$
\begin{aligned}
&r(H)\geq 0, r(L)=0, \phi_3(H)=0, \text{ and } \phi_3(L)\leq 0 &&\text{(for type-I Equilibrium)}\\
&r(H)= 0, r(L)=0, \phi_3(H)\leq 0, \text{ and } \phi_3(L)\leq 0 &&\text{(for type-II Equilibrium)}\\
&r(H)\geq 0, r(L)\geq 0, \phi_3(H)=0, \text{ and } \phi_3(L)= 0 &&\text{(for type-III Equilibrium)}\\
&r(H)= 0, r(L)\geq 0, \phi_3(H)\leq 0, \text{ and } \phi_3(L)= 0 &&\text{(for type-IV Equilibrium)}
\end{aligned}
$$

To check the existence of an equilibrium in which the ZLB binds only in the low state (Type-I), you need to solve the system of linear equations above by assuming (i) $\phi_3(H)=0$ and (ii) $r(L)=0$ and then check whether or not (i) $r(H)\geq 0$ and (ii) $\phi_3(L)\leq 0$. If either one or both of these two inequalities are violated, this means that there is no equilibrium in which the ZLB binds only in the low state.

The existence of other three equilibria can be checked in a similar way. For the equilibrium in which the ZLB binds in the both states (Type-II), you first solve the system of equations above by assuming (i) $r(H)=0$ and (ii) $r(L)=0$ and then check that (i) $\phi_3(H)\leq 0$ and (ii) $\phi_3(L)\leq 0$ to verify its existence. If either one or both of these two inequalities are violated, then this means that there is no equilibrium in which the ZLB binds in both states. For the equilibrium in which the ZLB does not bind in both states (Type-III), you solve the system of linear equations above by assuming (i) $\phi_3(H)=0$ and (ii) $\phi_3(L)=0$ and then check that (i) $r(H)\geq 0$ and (ii) $r(L)\geq 0$. Violation of either one or both of these two inequalities would mean that there is no equilibrium in which the ZLB does not bind in both states. Finally, for the equilibrium in which the ZLB binds only in the high state (Type-IV), you solve the system of equations above by assuming (i) $r(H)=0$ and (ii) $\phi_3(L)=0$ and then check that (i) $\phi_3(H)\leq 0$ and (ii) $r(L)\geq 0$. Violation of either one or both of these two inequalities would mean that there is no equilibrium in which the ZLB binds only in the high state. In each of four alternative Markov-Perfect equilibria, I say the Type-A violation occurs if the first of the two inequalities alone is violated, Type-B violation if the second of the two inequalities alone is violated, and Type-C violation if both inequalities are violated.

Figure D.1 shows the existence of four possible equilibria for different combinations of p_L and p_H. In each panel, white areas show the combinations of frequency and persistence under which the equilibrium of a particular type exists. Colored dots indicate that either one or both of the relevant inequality constraints are violated and thus that the equilibrium does not exist. Different colors indicate different reasons for why the equilibrium does not exist. Red, blue and black dots respectively indicate Type-A, Type-B, and Type-C violations.

Figure D.1: Existence of Four Markov-Perfect Equilibria



According to the top two panels, Type-I and Type-II Markov-Perfect equilibria exist when frequency and persistence of the shock are sufficiently low. According to the bottom two panels, Type-III and Type-IV Markov-Perfect equilibria do not exist regardless of frequency and persistence. In what follows, I will take a closer look at each possible equilibrium to understand why an equilibrium of a particular type does and does not exist.[3]

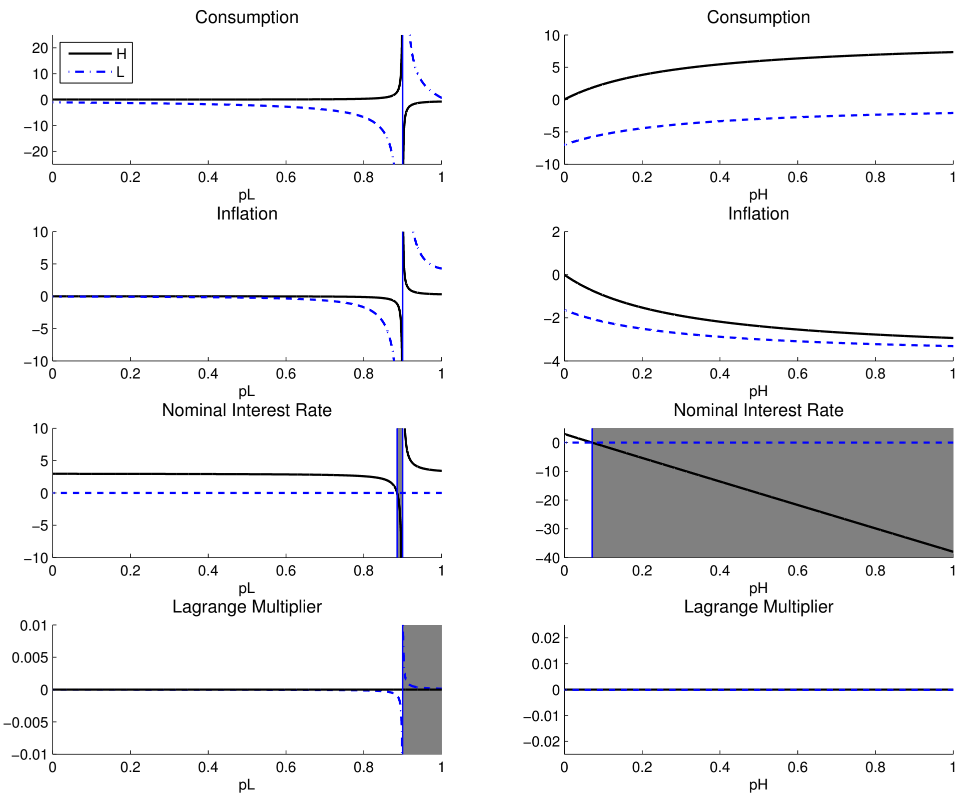
Type-I: The ZLB binds only in the low state

Top-left panel in figure D.1 shows that the equilibrium in which the ZLB binds only in the low state does not exist if either the frequency or the persistence of the shock is sufficiently high. To understand why, figure D.2 plots how the solution to the

[3]See Nakata and Schmidt (2014) for analytical results.

system of linear equations above depends on p_L and p_H. The left panels shows how consumption, inflation, nominal interest rate, and the Lagrange multiplier that solves the linear system vary with p_H, holding p_L constant at 0.5. The right panels shows how they vary with p_L holding p_H constant at 0.01.

Figure D.2: Allocations in Type-I Markov-Perfect Equilibrium



Note: Left panels show how allocations and the Lagrange multiplier vary with p_L holding $p_H = 0.01$ while right panels show how they vary with p_H holding $p_L = 0.5$. In all panels, solid black and dash blue lines are for high and low states respectively. Shaded areas in the panels for the nominal interest rate show the parameter region where the nominal interest rate is below zero, and the shaded area in the Lagrange multiplier panels show the region where it is positive.

As described in the main text, the household and firms have incentives to reduce their consumption and prices even before the contractionary shock hits the economy since the anticipation of future shocks increases the expected real interest rate and reduces the marginal costs they face in the high state. The government lowers the nominal interest rate to offset these effects. In equilibrium, consumption is positive, inflation is negative, and the nominal interest rate is below the deterministic steady-state in the high state. A higher frequency of shocks (p_H) means this anticipation effect is stronger. The right panels in figure D.2 indeed show that high-state consumption increases and high-state inflation and nominal interest rate decrease with p_H. When the frequency is sufficiently high, the nominal interest rate in the high state is negative, violating the ZLB constraint. Therefore, Type-I equilibrium does not exist with sufficiently large p_H.

The equilibrium also does not exist when persistence is sufficiently high. A more persistent shock means that the household and firms expect to be in the low state longer on average, which increases the expected real interest rate and decreases the expected marginal costs in the low state. Thus, the household and firms reduce consumption and prices in the low state by more in the economy with a higher p_L, as depicted in figure D.2. However, there is a cut-off value p_L above which low-state consumption and inflation that solve the first order necessary conditions turn positive. With more persistence, low-state consumption and inflation are influenced more by the future low-state consumption and inflation, and positive low-state consumption and inflation can be self-fulfilling if persistence is sufficiently large. However, for such high persistence, the Lagrange multiplier on the ZLB constraint becomes positive as the government has the incentive to raise the nominal interest rate to lower consumption and inflation, violating one of the inequality constraints that needs to be satisfied for the equilibrium to exist.

The result described in the previous paragraph is related to those in Eggertsson (2011), Christiano, Eichenbaum, and Rebelo (2011), and Carlstrom, Fuerst, and Paustian (2014). Namely, they all show that, in the economy in which the policy rate is determined by a standard Taylor rule and in which the high state is an absorbing state (that is, $p_H = 0$), consumption and inflation at the low state switch their sign and become positive when the crisis persistence becomes sufficiently high.[4] In their models with a Taylor rule, an equilibrium still exists with a very high crisis persistence in which the policy rate is above the ELB in the low state. However, in an economy in which the central bank is optimizing, positive consumption and inflation in the low state are not consistent with the optimizing behavior of the central bank, and thus the equilibrium does not exist, as shown in this section. Note that this feature of the model is present even in a fully nonlinear model. Christiano and Eichenbaum (2012) show that, in a fully nonlinear Rotemberg model, there are two equilibria consistent with the policy rate being at the ZLB in the low state, but with the policy rate above the ZLB in the high state, provided that the low state is not too persistent. They report that these equilibria cease to exist when the persistence of the low state is sufficiently high.[5]

Type-II: The ZLB binds in both states

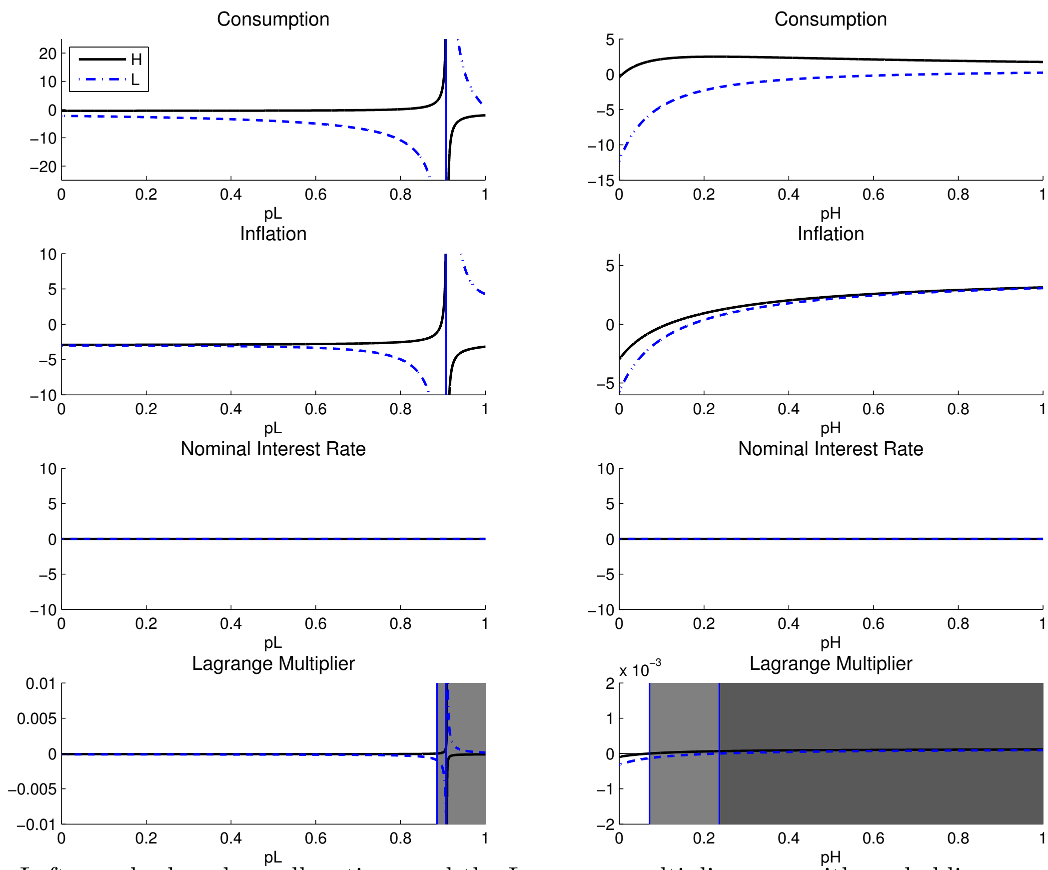
According to the top-right panel in figure D.1, the equilibrium in which the ZLB binds in both states exists when the frequency and persistence are sufficiently low. Figure D.3 shows that inflation and output are below their steady-states in both states. Declines in inflation and output in the low state are much larger in this equilibrium than those in the Type-I Markov-Perfect equilibrium. Similarly to the Type-I Equilibrium, an increase in the shock persistence leads to larger declines in consumption and output in the low state, and there is a cut-off value of p_L above which the low-state consumption and inflation that solve the system of linear equations turn positive. According to the

[4]See Proposition 3 in Eggertsson (2011), page 91 in Christiano, Eichenbaum, and Rebelo (2011), and page 1297 in Carlstrom, Fuerst, and Paustian (2014)

[5]See page Section 3.4 in Christiano and Eichenbaum (2012).

right panels, consumption and inflation in both states increase with the frequency in this equilibrium. Nakata and Schmidt (2014) analytically show that the region of equilibrium existence for this case is identical to the existence region of the first Markov-Perfect equilibrium.

Figure D.3: Allocations in Type-II Markov-Perfect Equilibrium



Note: Left panels show how allocations and the Lagrange multiplier vary with p_L holding $p_H = 0.01$ while right panels show how they vary with p_H holding $p_L = 0.5$. In all panels, solid black and dash blue lines are for high and low states respectively. Light gray areas in the panels for the Lagrange multiplier show the parameter region where one of low-state and high-state Lagrange multipliers is positive, while dark gray areas show the parameter region where the Lagrange multipliers are positive in both states.

The existence of such an equilibrium with this Type-II Markov-Perfect equilibrium with permanently binding ZLB is at first surprising. If the government is optimizing, why can the government keep the economy out of a permanent liquidity trap? The key ingredient to understanding the existence of this equilibrium is the lack of commitment by the government, i.e. the government takes future policy functions as given. If the household and firms expect low consumption and inflation in the low state tomorrow, they would like to choose low consumption and inflation even in the high state today. The government would like to reduce the nominal interest rate in order to prevent the declines in inflation and consumption, but it cannot do so if the ZLB constraint is binding. Thus, below-trend consumption and deflation can be self-fulfilling in the high state.

The existence of a steady state in which the ZLB constraint binds in the absence of any fundamental shocks is well-known since Benhabib, Schmitt-Grohe, and Uribe (2002). However, the equilibrium properties of an economy in which fundamental shocks generate fluctuations around the ZLB steady state are less known than those of an economy in which fundamental shocks generates fluctuations around the standard steady state. The closest to my result that the conditions under which the Type-I equilibrium exists are identical to those under which the Type-II equilibrium exists is a result in Armenter (2016) who shows that, in the same economy considered in this paper, but with a more general shock process, if a Markov equilibrium exists, then there is at least one additional Markov equilibrium.

Type-III and Type-IV: The ZLB does not bind in the low state

According to the bottom two panels in figure D.1, the equilibrium in which the ZLB does not bind in both states (Type-III) and the equilibrium in which the ZLB binds only in the high state (Type-IV) do not exist for any combinations of frequency and persistence. In the hypothetical Type-III equilibrium in which the low-state nominal interest rate is unconstrained, the government would like to lower the nominal interest rate below zero in the low state in order to stabilize consumption and output, which violates the inequality constraint that the nominal interest rate has to be positive (Type-B violations). In the hypothetical Type-IV equilibrium in which the low-state nominal interest rate is unconstrained, the reasons for non-existence are not always the same and depend on the frequency and persistence of the shock.

To summarize, given a pair of (p_H, p_L), we either have two equilibria—one in which the ZLB binds only in the low state and the other in which the ZLB bind in both states—or do not have any equilibria. In the main text, I construct the *revert-to-discretion* plan—a plan in which the deviation from the Ramsey outcome would be punished by the first of these two Markov-Perfect equilibria—and analyze the conditions under which this plan can make the Ramsey outcome time-consistent. In the next section, I will construct a plan in which deviation from the Ramsey outcome is punished by the second Markov perfect equilibrium and study the conditions under which such a plan can make the Ramsey outcome time-consistent.

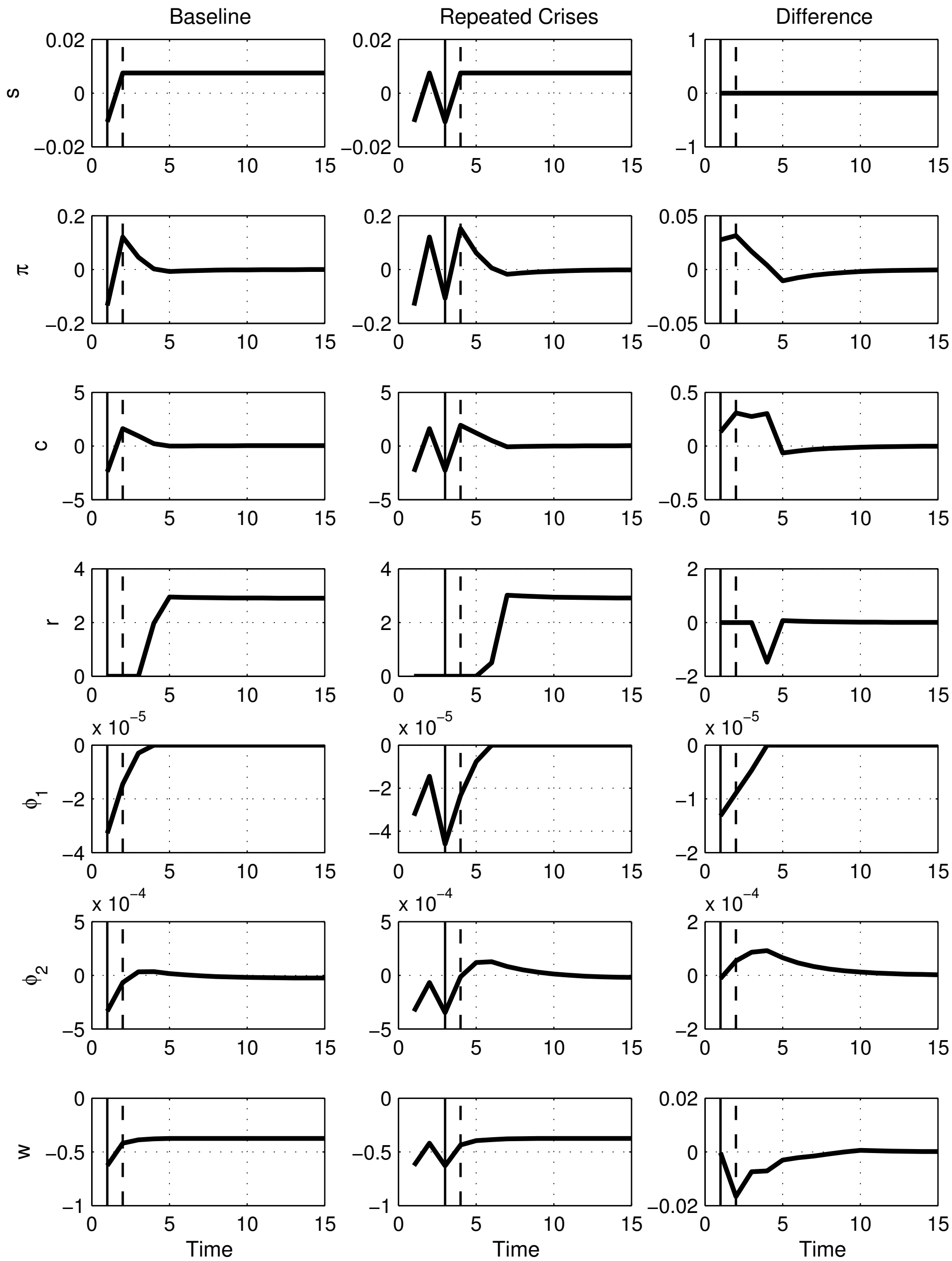
E Implications of Repeated Crises for the Ramsey Outcome

In this section, I explore the implications of repeated crises for the Ramsey outcome under three scenarios that differ in (i) the duration of the crisis (k) and (ii) how many periods the economy is in a normal state before the second crisis shock hits the economy (m).

E.1 Scenario 1 ($k = 1$ and $m = 1$):

In figure E.1, the left panels show the dynamics of the economy when the crisis shock hits the economy at time 1 and disappears afterward ($s_1 = H$ and $s_t = L$ for all $t > 1$). In the middle panels, the crisis shock hits the economy at time 1, disappears at time 2, but another crisis shock hits the economy again at time 3, which disappears next period ($s_1 = H$, $s_2 = L$, $s_3 = H$, and $s_t = L$ for all $t > 3$). The right panels show the difference between the left panels from time 1 and the right panels from time 3—when the second crisis shock hits the economy.

Figure E.1: Repeated Crises: Scenario 1



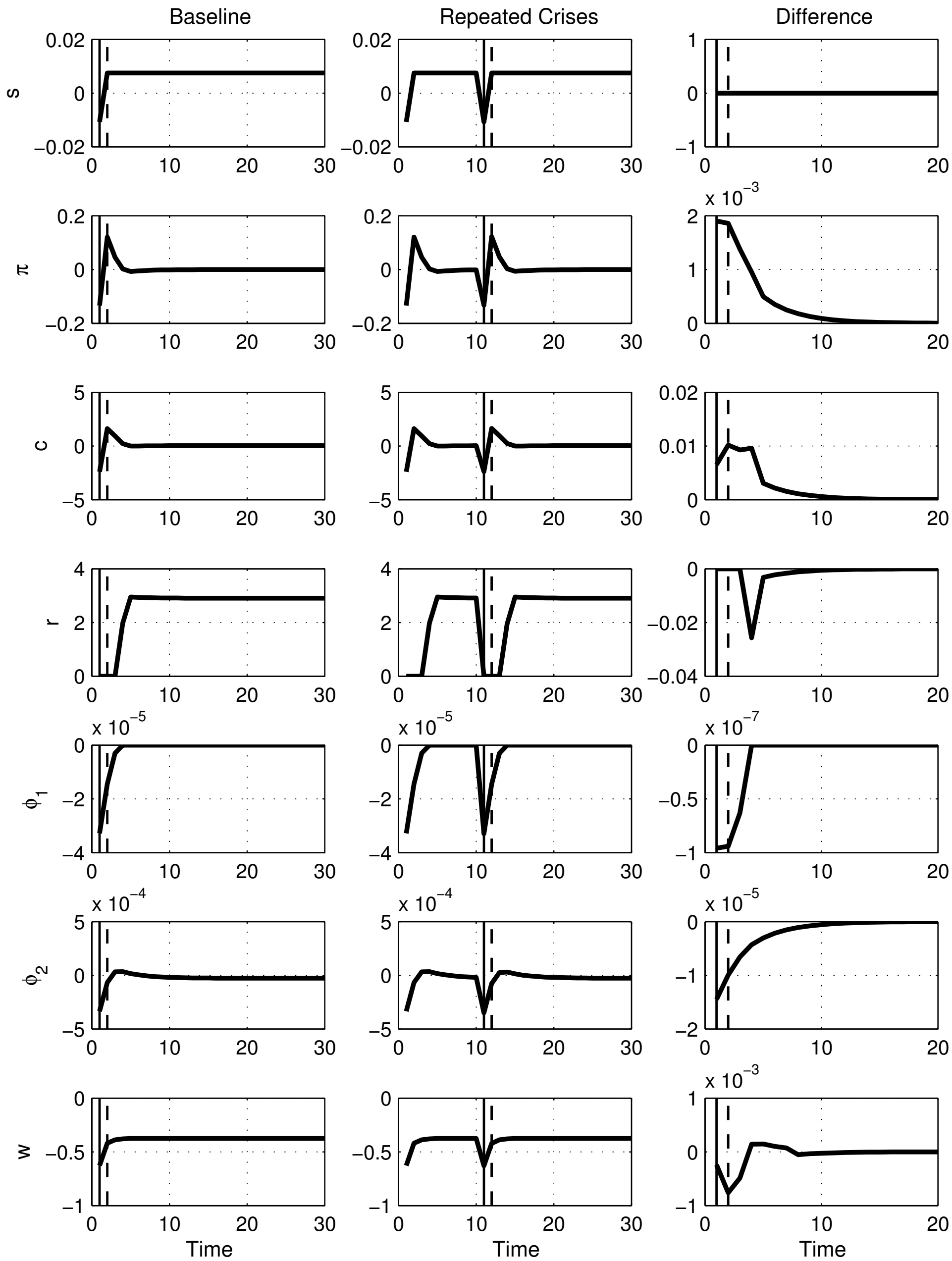
Note: The third column shows the differences between the dynamics of the economy from time 1 in the baseline case and those in the repeated-crises case from time 3—when the second crisis hits the economy.

According to the figure, the crisis is slightly less severe when it is preceded by another crisis than when it occurs at time 1. A less severe crisis is due to a larger overshooting of consumption and inflation in the aftermath of the crisis, which is in turn associated with more accommodative monetary policy stance. Mechanically, these differences arise because of the differences in the initial lagged Lagrange multipliers. When the crisis occurs at time 1, the initial lagged Lagrange multipliers are zero, as the economy start at time 1. However, when the crisis occurs at time 3 in the middle panel, the lagged Lagrange multipliers ($\phi_{1,2}$ and $\phi_{2,2}$) are nonzero, reflecting the dynamics of the Lagrange multipliers associated with the first crisis.

E.2 Scenario 2 ($k = 1$ and $m = 9$):

In figure E.2, the left panels show the dynamics of the economy when the crisis shock hits the economy at time 1 and disappears afterward ($s_1 = H$ and $s_t = L$ for all t¿1). In the middle panels, the crisis shock hits the economy at time 1, disappears at time 2, but another crisis shock hits the economy again at time 11, which disappears next period ($s_1 = H$, $s_2 = \; = s_{10} = L$, $s_{11} = H$, and $s_t = L$ for all $t > 11$). The right panels show the difference between the left panels from time 1 and the right panels from time 11—when the second crisis shock hits the economy.

Figure E.2: Repeated Crises: Scenario 2



Note: The third column shows the differences between the dynamics of the economy from time 1 in the baseline case and those in the repeated-crises case from time 11—when the second crisis hits the economy.

As in the first scenario, the crisis is less severe when it is preceded by another crisis than when it occurs at time 1. However, the differences are negligible in the second scenario, reflecting the fact that the Lagrange multipliers have returned to their stochastic steady states by the time the second crisis shock hits.[6]

[6]At time 10, a period before the second crisis, the Lagrange multiplier on the consumption Euler equation is zero because the policy rate is strictly positive. However, the Lagrange multiplier on

E.3 Scenario 3 ($k = 10$ and $m = 1$):

The previous two scenarios consider situations in which the crisis shock lasts only for one period. Now, I will examine the implication of repeated crises in which the crisis shock lasts for multiple periods.

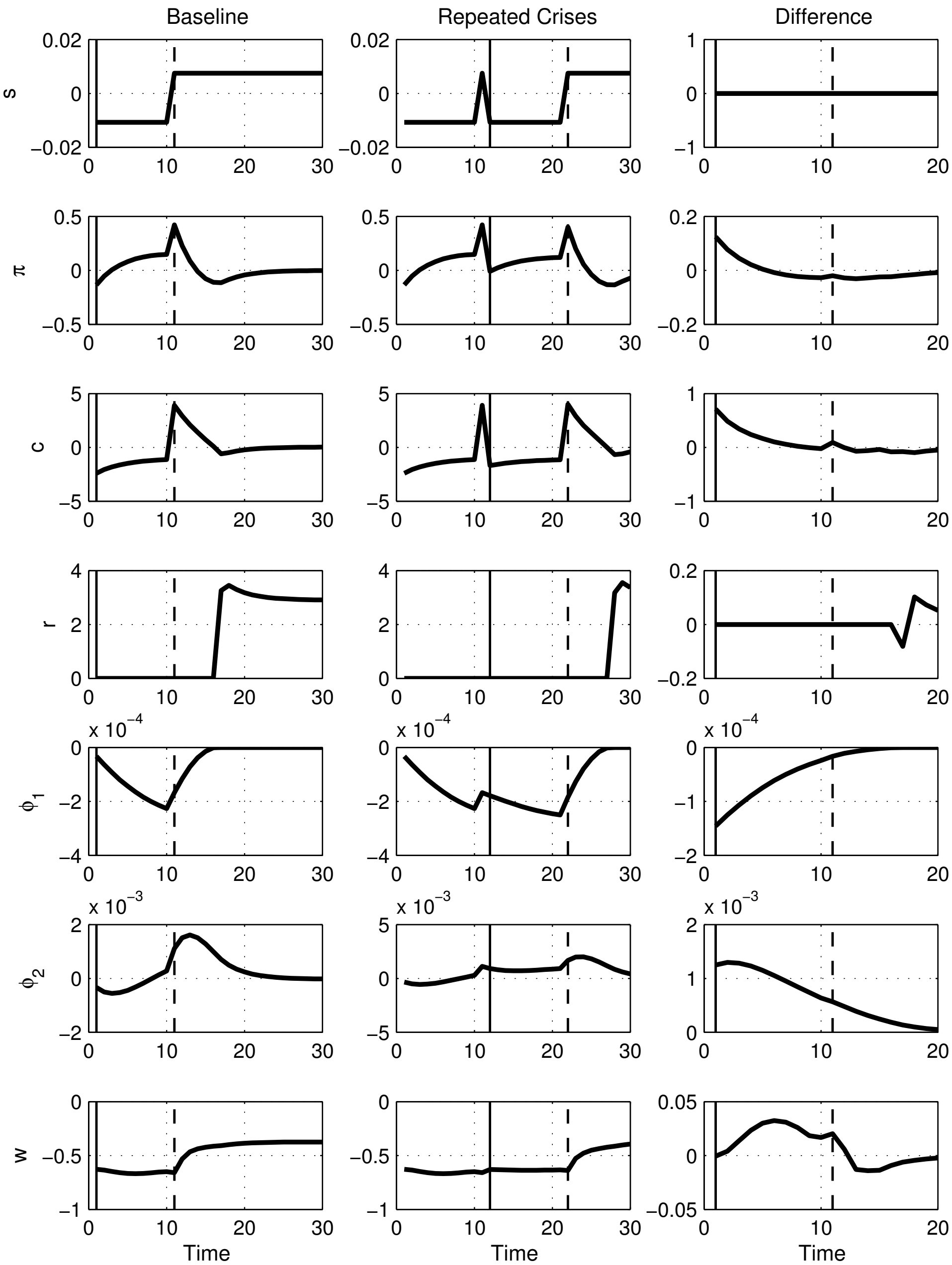
In figure E.3, the left panels show the dynamics of the economy when the crisis shock hits the economy at time 1, lasts for 10 periods, and disappears at time 11 ($s_1 = ... = s_{10} = H$ and $s_t = L$ for all $t > 11$). In the middle panels, the crisis shock hits the economy at time 1, lasts for 10 periods, disappears at time 11, but another crisis shock hits the economy again at time 11, which again lasts for 10 periods ($s_1 = ... = s_{10} = H$, $s_{11} = L$, $s_{12} = ... = s_{21} = H$, and $s_t = L$ for all $t > 22$). The right panels show the difference between the left panels from time 1 and the right panels from time 12—when the second crisis shock hits the economy.

When the crisis shock lasts for long periods, the Lagrange multipliers drift away from zero, as shown in the fifth and sixth panels in the left column. When the crisis shock disappears, the Lagrange multipliers start reverting back to their stochastic steady states. The longer the crisis has lasted, the longer it takes for the Lagrange multipliers to return to their stochastic steady states. If the second crisis shock hits soon after the first crisis shock disappears, then the absolute values of the Lagrange multipliers are still large. As a result, the extent to which the second crisis is less severe than the first crisis is more pronounced when the preceding crisis has lasted for longer periods.

However, as the second crisis shock continues, the initial differences in the Lagrange multipliers diminish, making the differences between the first and second crises smaller. By the time the second crisis shock disappears, the differences between the Lagrange multipliers are so small that the magnitudes of consumption and inflation overshooting are similar across the aftermaths of the first and second crises, in contrast to what we saw in the first scenario.

the Phillips curve is slightly negative, reflecting the fact that inflation is slightly negative due to the possibility of future crises. The deflationary bias is much smaller under the Ramsey outcome than under the discretionary outcome, but is still present.

Figure E.3: Repeated Crises: Scenario 3



Note: The third column shows the differences between the dynamics of the economy from time 1 in the baseline case and those in the repeated-crises case from time 12—when the second crisis hits the economy.

F The revert-to-deflation plan and its credibility

In this section, I define the revert-to-deflation plan and examine their credibility.

First, let $(c_{def}(\cdot),\ \pi_{def}(\cdot),\ r_{def}(\cdot))$ be the set of policy functions that solves the problem of the discretionary government in which the ZLB binds in both states. *The* *revert-to-deflation plan*, $(\sigma_g^{rtdef}, \sigma_p^{rtdef})$, consists of the following government strategy

- $\sigma_{g,1}^{rtdef}(s_1) = r_{ram,1}(s_1)$ for any $s_1 \in \mathbb{S}$
- $\sigma_{g,t}^{rtdef}(r^{t-1}, s^t) = r_{ram,t}(s^t)$ if $r_j = r_{ram,j}(s^j)$ for all $j \leq t-1$
- $\sigma_{g,t}^{rtdef}(r^{t-1}, s^t) = r_{def}(s_t)$ otherwise.

and the following private-sector strategy

- $\sigma_{p,t}^{rtdef}(r^t, s^t) = (c_{ram,t}(s^t), \pi_{ram,t}(s^t))$ if $r_j = r_{ram,j}(s^j)$ for all $j \leq t$
- $\sigma_{p,t}^{rtdef}(r^t, s^t) = (c_{br,def}(r_t, s_t), \pi_{br,def}(r_t, s_t))$ otherwise.

where

$$c_{br,def}(s_t, r_t) = E_t c_{def}(s_{t+1}) - \frac{1}{\chi_c}\Big[\big[r_t - E_t \pi_{def}(s_{t+1})\big] - s_t\Big]$$
$$\pi_{br,def}(s_t, r_t) = \kappa c_{br,def}(s_t, r_t) + \beta E_t \pi_{def}(s_{t+1})$$

The government strategy instructs the government to choose the nominal interest rate consistent with the deflationary outcome, but chooses the interest rate consistent with the deflationary outcome if it has deviated from the Ramsey outcome at some point in the past. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the Ramsey outcome as long as the government has never deviated from the Ramsey outcome. If the government has ever deviated from the nominal interest rate consistent with the Ramsey outcome, the private sector strategy instructs the household and firms to choose consumption and inflation today based on the belief that the government in the future will choose the nominal interest rate consistent with the deflationary outcome.

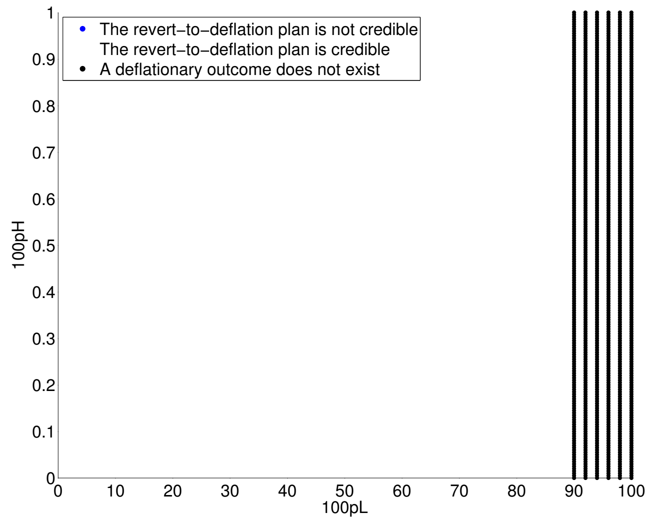
By construction, the revert-to-deflation plan induces the Ramsey outcome, and the implied value sequence is identical to the Ramsey value sequence. It is straightforward to show that the revert-to-deflation plan is credible if and only if $w_{ram,t}(s^t) \geq w_{def,t}(s^t)$ for all $t \geq 1$ and all $s^t \in \mathbb{S}^t$. The proof proceeds in the same way as the proof for the statement regarding the credibility of the revert-to-discretion plan.

Figure F.1 shows how the credibility of the revert-to-deflation plan depends on the frequency and severity of the shocks. Blank areas indicate the combinations of (p_H, p_L) for which the revert-to-deflation plan is credible. Blue dots indicate the combinations of (p_H, p_L) for which the revert-to-deflation plan is not credible. Black dots indicate the combinations of (p_H, p_L) for which the revert-to-deflation plan is not defined because the discretionary outcome does not exist.

According to the figure, the revert-to-deflation plan is credible for any pairs of frequency and persistence, provided that the pair of crisis frequency and persistence is consistent with the existence of the deflationary outcome.[7] To understand why, notice

[7]As described in online appendix D, the deflationary Markov-Perfect equilibrium (Type II-equilibrium) does not exist, and thus the deflationary outcome cannot be defined when p_H and p_L are sufficiently high

Figure F.1: Credibility of the revert-to-deflation plan



that the deflationary outcome is associated with deflation and consumption declines even in the high-state. After the shock disappears, if the government were to renege the promise of zero nominal interest rate and raises the nominal interest rate, the private sector agents adjust their expectations and believe the economy will be followed by deflationary outcome in the future. By reneging on the Ramsey promise after the shock disappears, the government would see deflation and below-trend consumption in the period of reneging, instead of above-trend consumption and high inflation. Inflation and consumption in the high state of the deflationary outcome are so low that there is no short-run gain from reneging on the promise. Thus, the revert-to-deflation plan is credible for any combinations of frequency and persistence.[8]

G The revert-to-discretion(N) plan

One may feel that the private sector's punishment strategy of reverting to the discretionary outcome forever after the government's deviation may be too harsh. In reality, the household and firms do not live forever. The head of the central bank changes at some frequencies. Even if the same central banker is in charge for an extended period

[8] It is useful to note that the "best" deviation in the aftermath of a crisis is not well defined. Suppose that, right after the crisis shock disappears, the government wants to deviate from the promise of keeping the policy rate at zero and raise the policy rate to $\epsilon > 0$, which would send the economy into the deflationary outcome today and future. The government can achieve a higher welfare by deviating by less, that is, by increasing the policy rate by $0 < \hat{\epsilon} < \epsilon$. If the government reduces the deviation to zero, then that means there is no deviation (as a result, the government has no incentive to deviate). However, the fact that the best deviation is not well defined does not mean that the revert-to-deflation plan is not well defined. Even though the best deviation is not well defined in some states, the revert-to-deflation plan is well defined because it specifies what happens to the economy after any history of the government action and shocks.

of time, central bank doctrines can change over the course of his/her tenure.[9] Based on these considerations, I explore plans in which the punishment regime lasts for a finite period of time and the economy reverts back to the Ramsey outcome afterward. After introducing a few concepts, I formally define the revert-to-discretion(N) plan in which the punishment lasts for a finite N periods and discuss a proposition that is useful in analyzing its credibility. The results are discussed in the paper.

G.1 Setup

Definition of the resurrected Ramsey outcome: For any positive integer j, *the resurrected Ramsey outcome starting at time* j is defined as, and denoted by, $\{c^j_{ram,t}(s^t),\ \pi^j_{ram,t}(s^t),\ r^j_{ram,t}(s^t)\}_{t=j}^{\infty}$ such that $c^j_{ram,t}(s^t) := c_{ram,t-j+1}(s^t_j)$, $\pi^j_{ram,t}(s^t) := r_{ram,t-j+1}(s^t_j)$, and $r^j_{ram,t}(s^t) := r_{ram,t-j+1}(s^t_j)$ where $t \geq j$ and s^t_j denotes a recent history of s^t starting at time j (i.e., $s^t_j := \{s_j, s_{j+1}, ..., s_t\}$).

This outcome specifies the outcome the Ramsey planner would choose if the economy hypothetically starts at time j. This will be the outcome the economy reverts to after the punishment period ends at time $j-1$. *The resurrected Ramsey value starting at time* j is defined as the value sequence associated with the resurrected Ramsey outcome starting at time j.

Government's problem in the temporary punishment periods

Consider the following problem of the discretionary government in the final period of the temporary punishment regime. Taking as given the fact that the resurrected Ramsey outcome prevails starting from tomorrow, say T, the government chooses today's consumption, inflation, and the nominal interest rate in order to maximize the value at $T-1$.

$$w_{T-1}(s_{T-1}) = \max_{\{c_{T-1}\in\mathbb{R},\pi_{T-1}\in\mathbb{R},r_{T-1}\in\mathbb{R}_{\geq 0}\}} u(c_{T-1},\pi_{T-1}) + \beta E_{T-1} w^T_{ram,T}(s^T_T)$$

subject to

$$\begin{aligned}
\chi_c c_{T-1} &= \chi_c E_{T-1} c^T_{ram,T}(s^T_T) - \left[r_{T-1} - E_{T-1}\pi^T_{ram,T}(s^T_T)\right] + s_{T-1} \\
\pi_{T-1} &= \kappa c_{T-1} + \beta E_{T-1}\pi^T_{ram,T}(s^T_T)
\end{aligned}$$

Let us denote the solution to this problem by $\{c_{d,1}(\cdot), \pi_{d,1}(\cdot), r_{d,1}(\cdot), w_{d,1}(\cdot)\}$. The discretionary government in the prior period takes them as given and maximizes the value at time T-2. As such, the allocations and value during the temporary punishment regime are recursively defined as follows. For any $j \geq 2$, the discretionary government's problem is given by

[9] For example, consider the gradual move toward transparency during the tenure of Alan Greenspan.

$$w_{T-j}(s_{T-j}) = \max_{\{c_{T-j}\in\mathbb{R},\pi_{T-j}\in\mathbb{R},r_{T-j}\in\mathbb{R}_{\geq 0}\}} u(c_{T-j},\pi_{T-j}) + \beta E_{T-j} w_{d,j+1}(s_{T-j+1}^{T-j+1})$$

subject to

$$\begin{aligned}
\chi_c c_{T-j} &= \chi_c E_{T-j} c_{d,T-j+1}(s_{T-j+1}) - \left[r_{T-j} - E_{T-j+1}\pi_{d,T-j+1}(s_{T-j+1})\right] + s_{T-j} \\
\pi_{T-j} &= \kappa c_{T-j} + \beta E_{T-j}\pi_{d,T-j+1}(s_{T-j+1})
\end{aligned}$$

Let $\{c_{d,j}(\cdot), \pi_{d,j}(\cdot), r_{d,j}(\cdot), w_{d,j}(\cdot)\}$ be the solution to these problems at time T-j. $\{c_{d,j}(\cdot),\ \pi_{d,j}(\cdot),\ r_{d,j}(\cdot),\ w_{d,j}(\cdot)\}_{j=1}^{T}$ will be the allocations that would prevail during the temporary punishment regime of length N.

G.2 Definition of the revert-to-discretion(N) plan

For any positive integer N, *the revert-to-discretion(N)* plan, $(\sigma_g^{rtd(N)}, \sigma_p^{rtd(N)})$, consists of the following government and private sector strategies.

G.2.1 The government strategy

For $t = 1$, set $\sigma_{g,1}^{rtd(N)}(s_1) = r_{ram,1}(s_1)$ for any $s_1 \in \mathbb{S}$.

For $t \geq 2$, determine $\sigma_{g,t}^{rtd(N)}(s^t)$ as follows.
Case 1: If $r_k = r_{ram,k}(s^k)$ for all $1 \leq k \leq t-1$, then

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{ram,t}(s^t)$

Case 2: If $r_k \neq r_{ram,k}(s^k)$ for some $1 \leq k \leq t-1$, let $k^{(1)}$ be the period of the first deviation from the Ramsey policy. In other words, if $r_1 \neq r_{ram,1}(s_1)$,

$$k^{(1)} = 1 \tag{3}$$

Otherwise,

$$k^{(1)} = \min\{k > 1 | r_k \neq r_{ram,k}(s^k) \text{ and } r_{k-1} = r_{ram,k-1}(s^{k-1})\} \tag{4}$$

Case 2.A [when $k^{(1)} = t-1$]: Set $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{d,N}(s^t)$
Case 2.B [when $t - N \leq k^{(1)} \leq t-2$]: Let $j := t - k^{(1)}$.

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{d,N+1-j}(s_t)$ if $r_{t-h+1} = r_{d,N+1-h}(s_{t-h+1})$ for all $1 \leq h \leq j-1$
- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_d(s_t)$ otherwise.

Case 2.C [when $k^{(1)} \leq t - N - 1$]: If $r_{k^{(1)}+j} \neq r_{d,N-j+1}(s_{k^{(1)}+j})$ for some $1 \leq j \leq N$,

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_d(s_t)$

If $r_{k^{(1)}+j} = r_{d,N-j+1}(s_{k^{(1)}+j})$ for all $1 \le j \le N$, then set $m = 1$ and follow the steps below.

Recursive steps to follow if you are following the resurrected Ramsey outcome

Case 1: If $r_k = r_{ram,k}^{k^{(m)}+N+1}(s^k)$ for all $k^{(m)} + N + 1 \le k \le t-1$, then

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{ram,k}^{k^{(m)}+N+1}(s^t)$

Case 2: If $r_k \neq r_{ram,k}^{k^{(m)}+N+1}(s^k)$ for some $k^{(m)} + N + 1 \le k \le t-1$, let $k^{(m+1)}$ be the period of the first deviation from the resurrected Ramsey policy starting at $k^{(m)}+N+1$. In other words, if $r_{k^{(m)}+N+1} \neq r_{ram,k^{(m)}+N+1}^{k^{(m)}+N+1}(s^{k^{(m)}+N+1})$,

$$k^{(m+1)} = k^{(m)} + N + 1 \tag{5}$$

Otherwise,

$$k^{(m+1)} = \min\{k > k^{(m)} + N + 1 | r_k \neq r_{ram,k}^{k^{(m)}+N+1}(s^k) \text{ and } r_{k-1} = r_{ram,k-1}^{k^{(m)}+N+1}(s^{k-1})\} \tag{6}$$

Case 2.A [when $k^{(m+1)} = t-1$]: Set $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{d,N}(s_t)$
Case 2.B [when $t - N \le k^{(m+1)} \le t-2$]: Let $j := t - k^{(m+1)}$.

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{d,N+1-j}(s_t)$ if $r_{t-h+1} = r_{d,N+1-h}(s_{t-h+1})$ for all $k^{(m+1)} + 1 \le h \le k^{(m+1)} + j - 1$
- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_d(s_t)$ otherwise.

Case 2.C [when $k^{(m+1)} \le t - N - 1$]: If $r_{k^{(m+1)}+j} \neq r_{d,N-j+1}(s^{k^{(m+1)}+j})$ for some $k^{(m+1)} + 1 \le j \le k^{(m+1)} + N$,

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_d(s_t)$

If $r_{k^{(m+1)}+j} = r_{d,N-j+1}(s_{k^{(m+1)}+j})$ for all $1 \le j \le N$, then set $m = m+1$ and go back to the beginning of the recursive step.

At each time t, this government strategy instructs the government to choose the nominal interest rate consistent with the Ramsey outcome as long as the past government has not deviated from the Ramsey prescription. If the past government deviated from either the Ramsey prescription or the resurrected Ramsey outcome at some $k \le t - N - 1$, but chose the nominal interest rate consistent with temporary punishment regime afterward, then the strategy instructs the government to choose the nominal interest rate consistent with the resurrected Ramsey outcome. If the government deviated from either the Ramsey or resurrected Ramsey policies in the recent past $t - N \le k \le t-1$, then the strategy instructs the government to choose the nominal interest rate consistent with the temporary punishment regime. If the government has ever deviated from the temporary punishment regime during some punishment periods, then the strategy prescribes the government to choose the nominal interest rate consistent with the discretionary outcome.

G.2.2 The private sector strategy

For any positive integer t, determine $\sigma^{rtd(N)}_{p,t}(r^t, s^t)$ as follows.
Case 1: If $r_k = r_{ram,k}(s^k)$ for all $1 \leq k \leq t$, then

- $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c_{ram,t}(s^t), \pi_{ram,t}(s^t))$

Case 2: If $r_k \neq r_{ram,k}(s^k)$ for some $1 \leq k \leq t$, let $k^{(1)}$ be the period of the first deviation from the Ramsey policy. That is, $k^{(1)}$ is given by equations (3) and (4).
Case 2.A [when $k^{(1)} = t$]: Set $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c_{d,N}(s_t), \pi_{d,N}(s_t))$
Case 2.B [when $t - N - 1 \leq k^{(1)} \leq t - 1$]: Let $j := t - k^{(1)}$.

- $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c_{d,N+1-j}(s_t), \pi_{d,N+1-j}(s_t))$ if $r_{t-h+1} = r_{d,N+1-h}(s_{t-h+1})$ for all $1 \leq h \leq j - 1$
- $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$ otherwise.

Case 2.C [when $k^{(1)} \leq t - N - 2$]: If $r_{k^{(1)}+j} \neq r_{d,N-j+1}(s_{k^{(1)}+j})$ for some $1 \leq j \leq N$,

- $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$

If $r_{k^{(1)}+j} = r_{d,N-j+1}(s_{k^{(1)}+j})$ for all $1 \leq j \leq N$, then set $m = 1$ and follow the steps below.

Recursive steps to follow if you are following the resurrected Ramsey outcome

Case 1: If $r_k = r^{k^{(m)}+N+1}_{ram,k}(s^k)$ for all $k^{(m)} + N + 1 \leq k \leq t$, then

- $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c^{k^{(m)}+N+1}_{ram,k}(s^t), \pi^{k^{(m)}+N+1}_{ram,k}(s^t))$

Case 2: If $r_k \neq r^{k^{(m)}+N+1}_{ram,k}(s^k)$ for some $k^{(m)} + N + 1 \leq k \leq t$, let $k^{(m+1)}$ be the period of the first deviation from the resurrected Ramsey policy starting at $k^{(m)}+N+1$. That is, $k^{(m+1)}$ is given by equations (5) and (6).
Case 2.A [when $k^{(m+1)} = t - 1$]: Set $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = r_{d,N}(s_t)$
Case 2.B [when $t - N \leq k^{(m+1)} \leq t - 2$]: Let $j := t - k^{(m+1)}$.

- $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c_{d,N+1-j}(s_t), \pi_{d,N+1-j}(s_t))$ if $r_{t-h+1} = r_{d,N+1-h}(s_{t-h+1})$ for all $k^{(m+1)} + 1 \leq h \leq k^{(m+1)} + j - 1$
- $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$ otherwise.

Case 2.C [When $k^{(m+1)} \leq t - N - 1$]: If $r_{k^{(m+1)}+j} \neq r_{d,N-j+1}(s_{k^{(m+1)}+j})$ for some $k^{(m+1)} + 1 \leq j \leq k^{(m+1)} + N$,

- $\sigma^{rtd(N)}_{p,t}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$

If $r_{k^{(g+1)}+j} = r_{d,N-j+1}(s_{k^{(m+1)}+j})$ for all $1 \leq j \leq N$, then set $m = m+1$ and go back to the beginning of the recursive step.

At each time t, this private sector strategy instructs the household and firms to choose consumption and inflation consistent with the Ramsey outcome as long as the past government has not deviated from the Ramsey prescription. If the government deviated from either the Ramsey prescription or the resurrected Ramsey prescription at some $k \leq t - N$, but chose the nominal interest rate consistent with temporary punishment regime afterward, then the strategy instructs the household and firms to choose consumption and inflation consistent with the resurrected Ramsey outcome. If the government deviated from either the Ramsey or resurrected Ramsey policies in the recent past $t - N - 1 \leq k \leq t$, then the strategy instructs the household and firms to choose consumption and inflation consistent with the temporary punishment regime. If the government has ever deviated from the temporary punishment regime during some punishment periods, then the strategy prescribes the household and firms to choose consumption and inflation consistent with the discretionary outcome.

It is straightforward to show that the revert-to-discretion(N) plan is credible if and only if (i) $w_{ram,t}(s^t) \geq w_{d,N}(s_t)$ for all t and $s^t \in \mathbb{S}^t$ and (ii) $w_{d,j}(s_t) \geq w_d(s_t)$ for all $1 \leq j \leq N$. The first condition makes sure that the government does not have incentives to deviate from either the Ramsey or resurrected Ramsey outcomes. The second condition guarantees that the government does not have incentives to deviate from the strategy's prescription in the temporary punishment regime.

H Details of the Nonlinear Model

In this section, I describe some details of the fully nonlinear model considered in the main text.

The model is a standard New Keynesian model with imperfect-competition in the product market and nominal rigidities in the form of quadratic price-adjustment costs (so-called Rotemberg pricing). The model is standard and its description can be found in Boneva, Braun, and Waki (2016) and Nakata (2017), among many others.

The only exogenous shock of the model is the shock to the discount rate in the household's utility function and is denoted by δ_t. δ_t evolves according to a two-state Markov process. Transition probabilities are given by

$$\text{Prob}(\delta_{t+1} = \delta_{crisis} | \delta_t = \delta_{normal}) = p_N \tag{7}$$

$$\text{Prob}(\delta_{t+1} = \delta_{crisis} | \delta_t = \delta_{crisis}) = p_C \tag{8}$$

where $\delta_{crisis} > 1/\beta$ and $\delta_{normal} = 1$. p_N is the probability of moving to the crisis state next period when today's state is normal, whereas p_C is the probability of staying in the crisis state next period when today's state is the crisis state. Let $\mathbb{D} := \{\delta_{normal}, \delta_{crisis}\}$The competitive outcome of the fully nonlinear economy is given by a state-contingent sequence of consumption, output, inflation, and the nominal interest rate, $\{C_t(\delta^t),\, N_t(\delta^t),\, Y_t(\delta^t),\, \Pi_t(\delta^t),\, R_t(\delta^t)\}_{t=1}^{\infty}$ such that, for all $t \geq 1$ and $\delta^t \in \mathbb{S}^t$,

(i) $C_t(\delta^t) \in \mathbb{R}$, $N_t(\delta^t) \in \mathbb{R}$, $Y_t(\delta^t) \in \mathbb{R}$, $\Pi_t(\delta^t) \in \mathbb{R}$, $R_t(\delta^t) \in \mathbb{R}_{\geq 0}$ where $\mathbb{R}$ denote a set of real numbers and $\mathbb{R}_{\geq 0}$ denote a set of non-negative real number and (ii)

$$
\begin{aligned}
& C_t(\delta^t)^{-\chi_c} = \beta\delta_t R_t(\delta^t) E_t C_{t+1}(\delta^{t+1})^{-\chi_c}\Pi_{t+1}(\delta^{t+1})^{-1} && (9) \\
& \frac{N_t(\delta^t)}{C_t(\delta^t)^{\chi_c}}\left[\varphi(\Pi_t(\delta^t)-1)\Pi_t(\delta^t)-(1-\theta)-\theta N_t(\delta^t)^{\chi_n}C_t(\delta^t)^{\chi_c}\right] \\
& \qquad\qquad = \beta\delta_t E_t \frac{N_{t+1}(\delta^{t+1})}{C_{t+1}(\delta^{t+1})^{\chi_c}}\varphi(\Pi_{t+1}(\delta^{t+1})-1)\Pi_{t+1}(\delta^{t+1}) && (10) \\
& Y_t(\delta^t) = C_t(\delta^t) + \frac{\varphi}{2}\left[\Pi_t(\delta^t)-1\right]^2 Y_t(\delta^t) && (11) \\
& Y_t(\delta^t) = N_t(\delta^t) && (12)
\end{aligned}
$$

The government's objective function at period t is given by the expected discounted sum of future utility flow of the household

$$
W_t(\delta^t) := E_t \sum_{j=1}^{\infty} \beta^{j-1}\left[\prod_{k=0}^{j-1}\delta_k\right]U(C_{t+j}, N_{t+j}) \tag{13}
$$

where the utility flow at each period is given by the following function.

$$
U(C,N) := \frac{C^{1-\chi_c}}{1-\chi_c} - \frac{N^{1+\chi_n}}{1+\chi_n} \tag{14}
$$

The problem of the discretionary central bank is to choose allocations, prices, and the policy rate today to maximize the welfare, taking as given the future value and policy and functions as given:

$$
W_t(\delta_t) = \max_{C_t, N_t, Y_t, \Pi_t, R_t}\left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1-\chi_n}}{1-\chi_n}\right] + \beta\delta_t \mathrm{E}_t V_{t+1}(\delta_{t+1}) \tag{15}
$$

where the optimization is subject to the equations characterizing the competitive outcome given by equations (9-12). Let $\{W_d(.), C_d(.), N_d(.), Y_d(.), \Pi_d(.), R_d(.)\}$ be the set of time-invariant value and policy functions that solve the Bellman equation above in which the ZLB only binds in the low state. The discretionary outcome is defined as, and denoted by, the state-contingent sequence of consumption, labor supply, output, inflation, and the policy rate, $\{C_{d,t}(\delta^t), N_{d,t}(\delta^t), Y_{d,t}(\delta^t), \Pi_{d,t}(\delta^t), R_{d,t}(\delta^t)\}_{t=1}^{\infty}$ such that $C_{d,t}(\delta^t) := C_d(s_t)$, $N_{d,t}(\delta^t) := N_d(s_t)$, $Y_{d,t}(\delta^t) := Y_d(s_t)$, $\Pi_{d,t}(\delta^t) := \Pi_d(s_t)$, and $R_{d,t}(\delta^t) := R_d(s_t)$ and *the discretionary value sequence* is defined as, and denoted by, $\{W_{d,t}(\delta^t)\}_{t=1}^{\infty}$ such that $W_{d,t}(\delta^t) := W_d(s_t)$.

For each $\delta_1 \in \mathbb{D}$, the problem of the Ramsey planner is to choose a state-contingent sequence of the model's variables at time 1 in order to maximize the welfare of the household:

$$
\max_{\{\boldsymbol{C}(\delta_1), \boldsymbol{N}(\delta_1), \boldsymbol{Y}(\delta_1), \boldsymbol{\Pi}(\delta_1), \boldsymbol{R}(\delta_1)\}\in CE(\delta_1)} \mathrm{E}_1 \sum_{t=1}^{\infty}\beta^{t-1}\prod_{k=0}^{t-1}\delta_k\left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1-\chi_n}}{1-\chi_n}\right] \tag{16}
$$

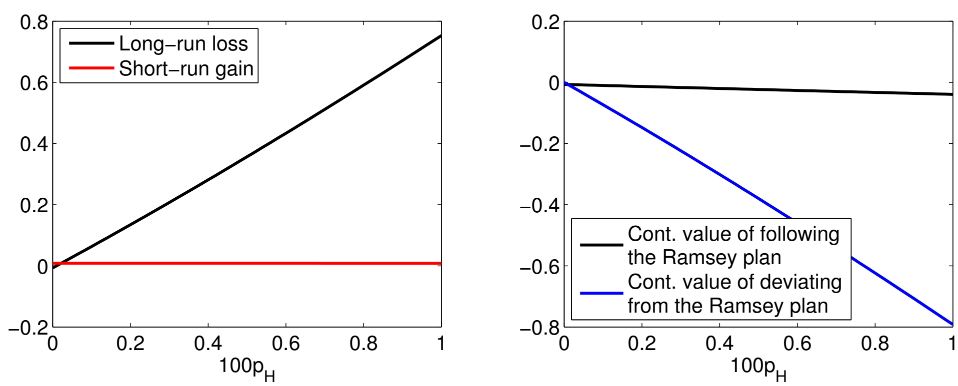
subject to the equations characterizing the competitive outcome. *The Ramsey outcome* is defined as the state-contingent sequence of consumption, labor supply, output, inflation, and the nominal interest rate that solves the problem above. In other words, the Ramsey outcome is a competitive outcome with the highest time-one value. The Ramsey outcome is denoted by $\{C_{ram,t}(\delta^t),\ N_{ram,t}(\delta^t),\ Y_{ram,t}(\delta^t),\ \Pi_{ram,t}(\delta^t),\ R_{ram,t}(\delta^t)\}_{t=1}^{\infty}$. The value sequence associated with the Ramsey outcome is denoted by $\{W_{ram,t}(\delta^t)\}_{t=1}^{\infty}$, and will be referred to as *the Ramsey value sequence*.

A plan, credibility of a plan, and the revert-to-discretion plan in the fully nonlinear economy are defined analogously to their counterparts in the semi-loglinear economy. As in the semi-loglinear model, I follow Adam and Billi (2006) and characterize the Ramsey equilibrium recursively, using the lagged Lagrange multipliers as pseudo state variables.

I More on the Main Results

In this section, I elaborate on the mechanism behind the two results highlighted in section 4. The first result is that there is a threshold crisis frequency above which the revert-to-discretion plan is credible and below which it is not credible. The second result is that there exists a threshold crisis persistence above which the revert-to-discretion plan is credible and below which it is not credible.

Figure I.1: The short-run gain and the long-run loss of deviating from the Ramsey policy (with alternative shock frequencies)



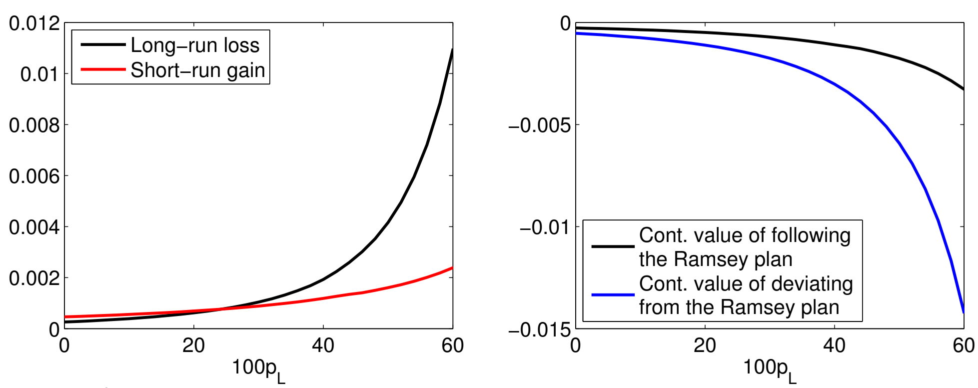
Note: On the left panel, *the long-run loss* shows the loss in the continuation value if the government deviates from the Ramsey policy at t=5, given by $\beta E_5\left[w_{ram,6}(s^6) - w_{d,6}(s^6)\right])$, and *the short-run gain* shows the gain in today's utility flow if the government deviates from the Ramsey policy at t=5, given by $\left[u(c_{d,5}(s^5), \pi_{d,5}(s^5)) - u(c_{ram,5}(s^5), \pi_{ram,5}(s^5))\right]$, where $s_t = L$ for $1 \leq t \leq 4$ and $s_5 = H$. On the right panel, the continuation values of following and deviating from the Ramsey plan at t=5 are respectively given by $\beta E_5 w_{ram,6}(s^6)$ and $\beta E_5 w_{d,6}(s^6)$ where $s_t = L$ for $1 \leq t \leq 4$ and $s_5 = H$.

Frequency: Red and black lines in the left panel of figure I.1 show how the short-run temptation and the long-run loss of deviating from the Ramsey path at time 5—right after the crisis shock disappears in the scenario depicted in figure 4 in the main body of the paper—vary with the crisis frequency (p_H), respectively. Since the magnitude of the overshooting of consumption and inflation does not materially depend on the crisis frequency, the short-run temptation to renege on the Ramsey promise does not materially depends on the crisis frequency, as captured by the flat red line. However, the long-run loss of reneging on the Ramsey promise depends importantly on the crisis frequency. In particular, the long-run loss increases with the crisis frequency.

The long-run loss of deviating from the Ramsey policy and thus of following the discretionary path thereafter increases with the crisis frequency because an increase in the crisis frequency reduces the continuation value associated with the discretionary outcome by more than it reduces the continuation value associated with the Ramsey outcome. While the crisis is associated with mild declines in consumption and inflation under the Ramsey outcome, it is associated with large declines in consumption and inflation under the discretionary outcome. As a result, the same increase in the crisis frequency reduces the discretionary value by more than it reduces the Ramsey value, as shown by the right panel of figure I.1. Accordingly, the long-run loss of deviating from the Ramsey path, which is the difference between the continuation Ramsey and discretionary values, increases with the crisis frequency.

Persistence: Red and black lines in the left panel of figure I.2 show how the short-run temptation and the long-run loss of deviating from the Ramsey path at time 5—right after the crisis shock disappears in the scenario depicted in figure 5 in the main body of the paper—vary with the crisis persistence (p_L), respectively. On the one hand, since the magnitude of the overshooting of consumption and inflation is large when the shock is more persistent, the short-run temptation to renege on the Ramsey promise is larger when the shock is more persistent, as captured by the red line. On the other hand, the long-run loss of reneging on the Ramsey promise is higher when the shock is more persistent the crisis persistence. In particular, the long-run loss increases with the crisis persistence.

Figure I.2: The short-run gain and the long-run loss of deviating from the Ramsey policy (with alternative shock persistence)



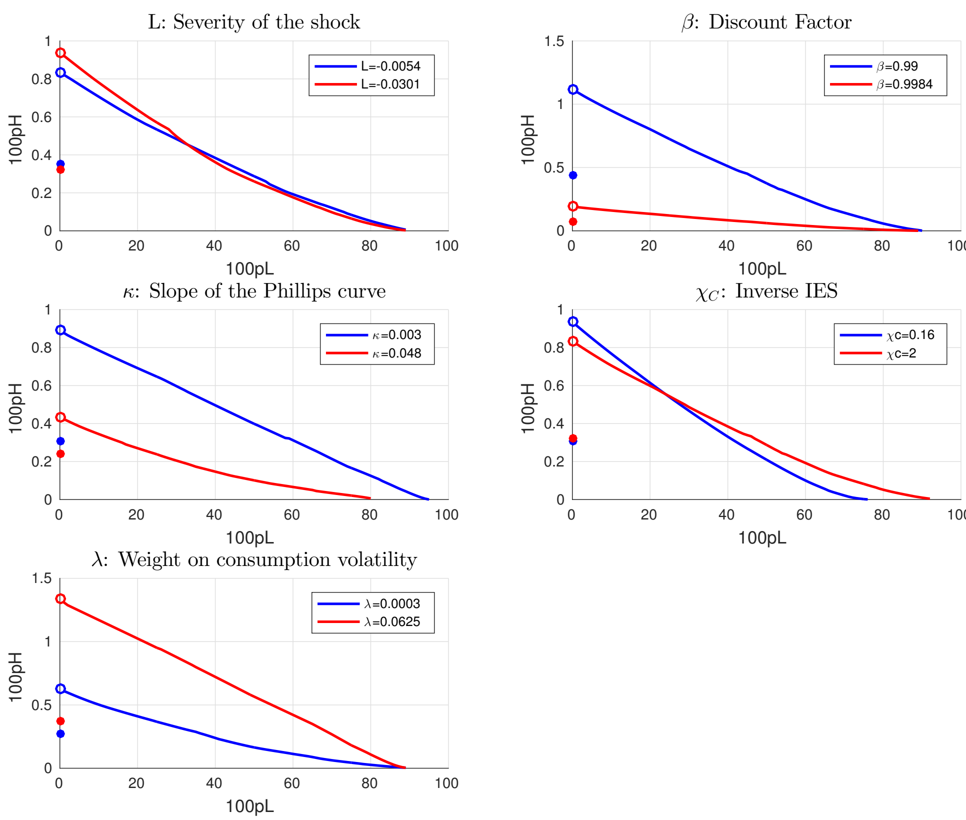
See the note on figure I.1

The long-run loss of deviating from the Ramsey policy and thus of following the discretionary path thereafter increases with the crisis persistence because an increase in the crisis persistence reduces the continuation value associated with the discretionary outcome by more than it reduces the continuation value associated with the Ramsey outcome. Under both the discretionary and Ramsey outcome, an increase in the crisis persistence makes the crisis more severe. However, the Ramsey planner mitigates the adverse effect of a more persistent shock by promising to overheat the economy by more in the aftermath of the crisis. As a result, the same increase in the crisis frequency reduces the discretionary value by more than it reduces the Ramsey value, as shown by the right panel of figure I.2. Accordingly, the long-run loss of deviating from the Ramsey path, which is the difference between the continuation Ramsey and discretionary values, increases with the crisis persistence.

J Sensitivity Analysis—Reduced-Form Parameter Space

In this section, I examine how each reduced-form parameter of the model affects the set of (p_H, p_L) under which the revert-to-discretion plan is credible. In the following section, I examine how each structural parameter affects the credible region.

Figure J.1: Credibility of the revert-to-discretion plan: Sensitivity Analysis



Note: In all charts, colored lines (and dots for the case with $p_L = 0$) show the threshold frequency above which the revert-to-discretion plan is credible.

J.1 Severity of the shock (L)

The top-left panel of figure J.1 shows the credible region under two alternative values for the shock size. In one scenario in which $L = -0.0054$, the (absolute) size of the shock implies 5 percent decline in the crisis output under the discretionary outcome, smaller than the 7 percent decline in the baseline parameterization. In the other scenario in which in which $L = -0.0301$, the size of the shock implies 25 percent decline in the crisis output under the discretionary outcome, larger than in the baseline parameterization.

According to the panel, the effect of the shock size on credibility of the revert-to-discretion plan is ambiguous when p_L is high, the threshold crisis frequency above which the revert-to-discretion plan is credible is higher when the absolute size of the shock is larger. When p_L is low, the threshold crisis frequency is higher when the absolute size of the shock is smaller.

To understand this result, the left and right panels in figure J.2 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economies with small and large shocks (that is, small and large $|L|$), respectively. The contractionary shock is assumed to take a low value for the first eight periods and to return to the high value afterward. In this figure, $(p_H, p_L) = (0.04/100, 0.8)$ and other parameters are set to their baseline values, except for the parameter being investigated.

Comparison of the IRFs for consumption and inflation across two economies shows that a more severe shock leads to larger declines in consumption and inflation under both discretionary and Ramsey outcomes. However, the Ramsey planner can promise a higher inflation, a larger consumption boom, and a longer period of zero nominal interest rates to mitigate the declines in consumption and inflation during the period of contractionary shocks. Thus, a marginal increase in the shock severity leads to larger marginal declines in consumption and inflation in the low state in the discretionary outcome than in the Ramsey outcome, implying large marginal declines in both the high-state and low-state values. This means that the long-run loss from reneging on the Ramsey promise and reverting back to the discretionary outcome is larger in the economy with more severe shocks.

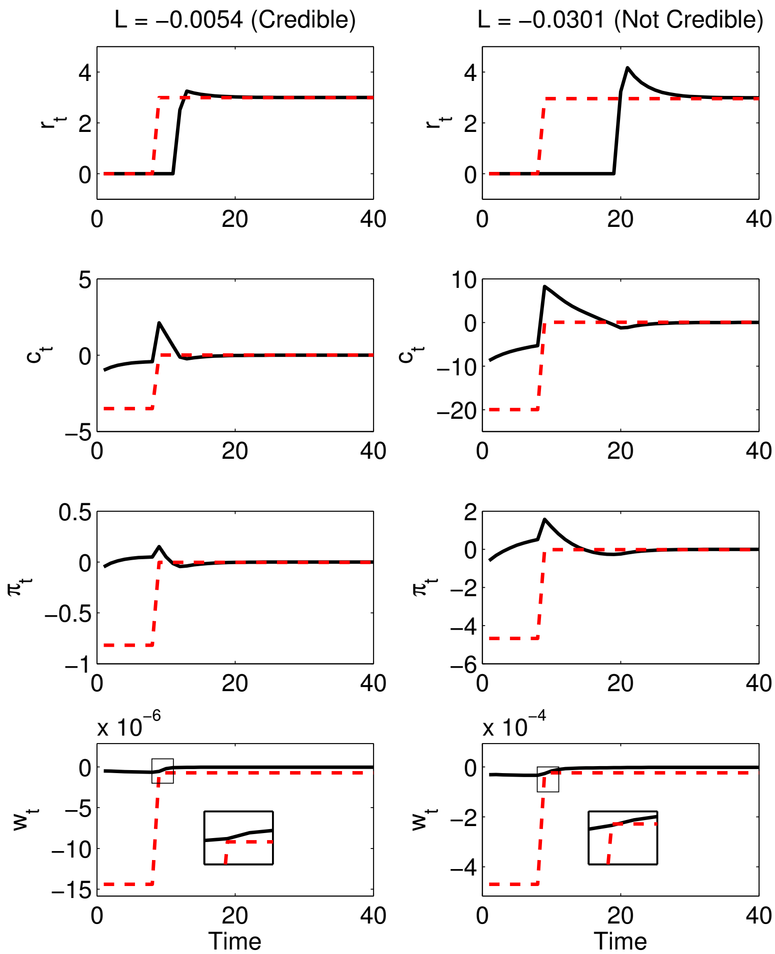
On the other hand, as the Ramsey promise entails a higher inflation and larger consumption boom, the short-run gain from reneging on the promise is also larger with a larger shock. As such, the overall effects are mixed. For the specific choice of parameter values used in figure J.2, the Ramsey outcome is credible if the shock size is sufficiently small.

J.2 Discount rate (β)

The top-right panel of figure J.1 shows how the threshold crisis frequency depends on the crisis persistence for two alternative values of β. A low value of 0.99 is a common value in the literature, used for example in Eggertsson and Woodford (2003). A high value of 0.9984 is the estimate from Smets and Wouters (2007). According to the panel, the threshold crisis frequency is lower when β is higher for any p_L.

To understand this pattern, the left and right panels in figure J.2 show the impulse

Figure J.2: The discretionary/Ramsey outcomes and values with alternative sizes of the shock

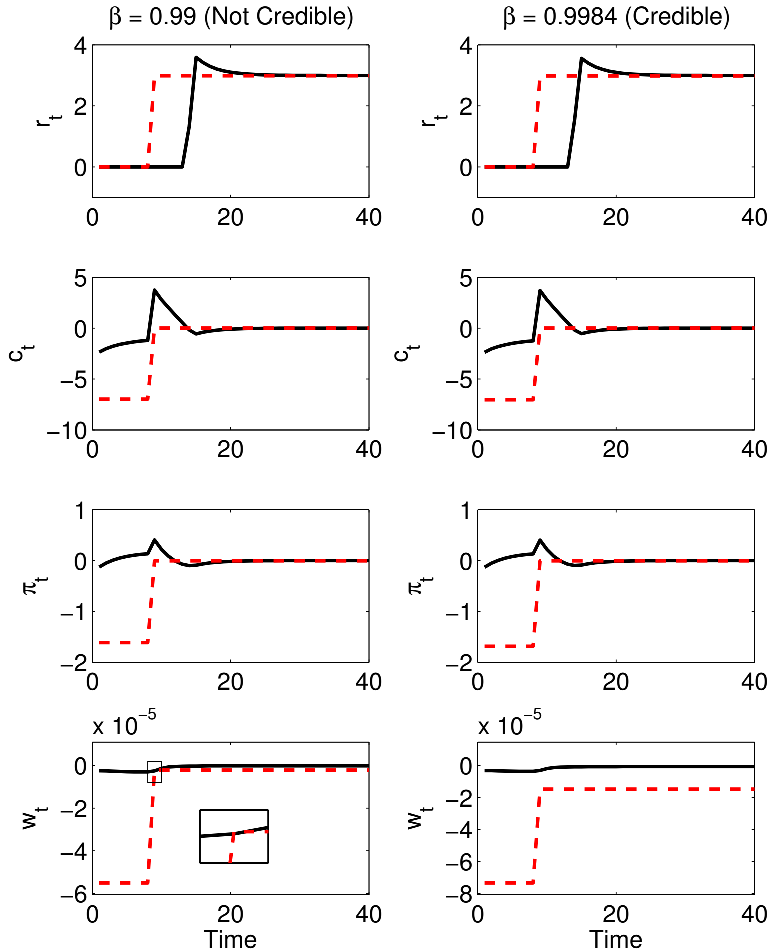


Note: *The long-run loss* shows the loss in the continuation value if the government deviates from the Ramsey policy at $t = 9$, given by $\beta E_9\left[w_{ram,10}(s^{10}) - w_{d,10}(s^{10})\right]$), and *the short-run gain* shows the gain in today's utility flow if the government deviates from the Ramsey policy at $t = 9$, given by $\left[u(c_{d,9}(s^9), \pi_{d,9}(s^9)) - u(c_{ram,9}(s^9), \pi_{ram,9}(s^9))\right]$, where $s_t = L$ for $1 \leq t \leq 8$ and $s_9 = H$.

response functions for the discretionary/Ramsey outcomes and values in the economy with small and large discount factors, respectively. The contractionary shock is assumed to take a low value for the first eight periods and return to the high value afterward. In this figure, $(p_H, p_L) = (0.04/100, 0.8)$ and other parameters are set to their baseline values, except for the parameter being investigated.

When β is large, inflation today depends more on future marginal costs. As a result, a promise of future inflation is more effective in mitigating the declines in inflation in the low state and the Ramsey planner promises higher inflation in the economy with higher β. However, this effect is quantitatively negligible, as the comparison of solid black lines in the right and left column in figure J.3 shows. As a result, the short-run gain from reneging on the Ramsey promise and thus stabilizing consumption and inflation are essentially insensitive to the discount rate.

Figure J.3: The discretionary/Ramsey outcomes and values with alternative discount rates



Note: *The long-run loss* shows the loss in the continuation value if the government deviates from the Ramsey policy at $t = 5$, given by $\beta E_5\big[w_{ram,6}(s^6) - w_{d,6}(s^6)\big]$), and *the short-run gain* shows the gain in today's utility flow if the government deviates from the Ramsey policy at $t = 5$, given by $\big[u(c_{d,5}(s^5), \pi_{d,5}(s^5)) - u(c_{ram,5}(s^5), \pi_{ram,5}(s^5))\big]$, where $s_t = L$ for $1 \leq t \leq 4$ and $s_5 = H$.

On the other hand, with a higher β, the same difference between the discretionary and Ramsey continuation values translates into a larger difference between discounted continuation values. As a result, a high discount factor implies a larger long-run loss of reneging on the promise. Accordingly, the threshold p_H above which the revert-to-discretion plan is credible is lower in the economy with a larger β. This result is consistent with previous literature on credible plans which has shown that a sufficiently large β can make the Ramsey policy credible in various contexts (see, for example, Chari and Kehoe (1990), Phelan and Stacchetti (2001), and Kurozumi (2008).)

J.3 Slope of the Phillips Curve (κ)

The middle-left panel of figure J.1 shows the credible regions for two alternative values of the slope of the Phillips curve. The low (high) value of 0.003 (0.048) is consistent with the Calvo probability of 90 percent (65 percent), as opposed to 81 percent in the baseline parameterization. 90 percent is the value used on studies such as Erceg and Lindé (2014). 65 percent is the value in line with the estimate of this parameter from pre-crisis studies, such as Smets and Wouters (2007). According to the panel, the threshold crisis frequency is lower when the value of κ is higher.[10]

To understand this pattern, the left panels in figure J.4 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economy with a small κ. The right panels show the same objects in the economy a large κ. The contractionary shock is assumed to take a low value for the first four periods and return to the high value afterward. In this figure, $(p_H, p_L) = (0.1/100, 0.6)$ and other parameters are set to their baseline values, except for the parameter being investigated.

Comparison of left and right panels tells us that the declines in consumption and inflation in the low state are exacerbated under both discretionary and Ramsey outcomes under more flexible prices (i.e. higher slope of the Phillips Curve). When the prices are more flexible, the same shock leads to larger deflation in the low state, which in turn amplifies the decline in low-state consumption by increasing the expected real interest rate. However, the Ramsey planner mitigates those declines by promising a higher inflation, consumption booms, and a longer period of the ZLB. Thus, a marginal increase in the slope parameter leads to larger marginal declines in consumption and inflation, and thus values, in the discretionary outcome than in the Ramsey outcome. Accordingly, the long-run loss from reverting back to the discretionary plan is larger in the economy with more flexible prices.

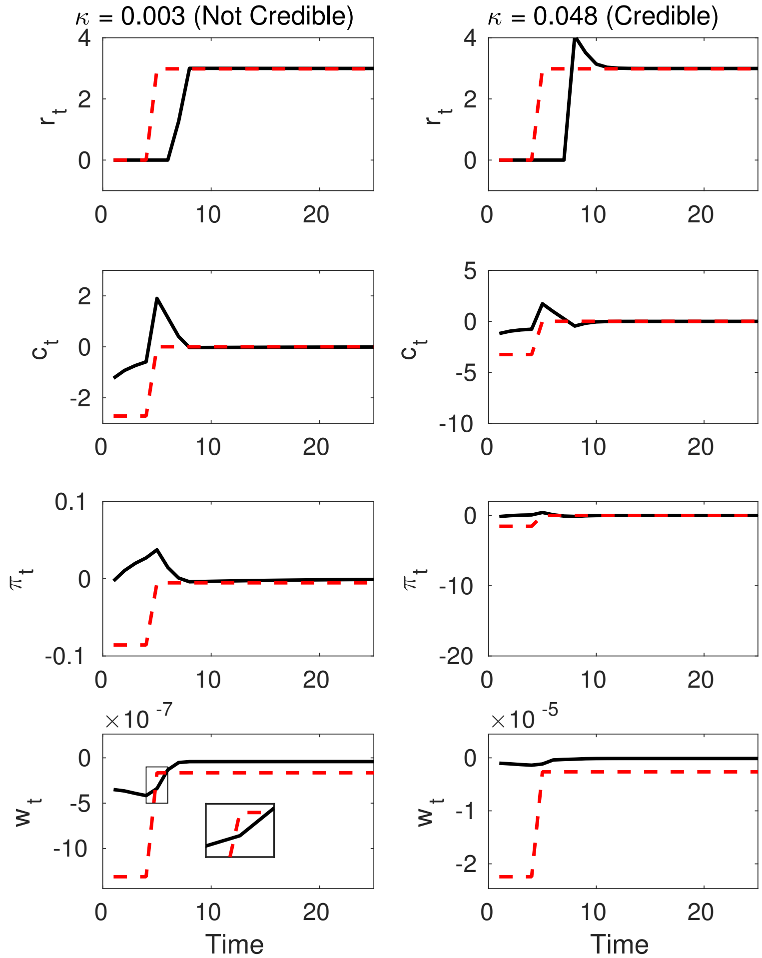
On the other hand, the Ramsey promise of higher inflation and larger consumption booms means that short-run gain from reneging on the promise once the shock disappears is higher under a more flexible price environment. Quantitatively, for the calibration considered in this paper, the second effect dominates the first effect. The threshold value of p_H above which the revert-to-discretion plan is credible is lower for any given p_L.

J.4 Inverse IES (χ_c)

The middle-left panel of figure J.1 shows the credible regions for two alternative values of the slope of the Phillips curve. A low value of 1/6.25 is the value used in Adam and Billi (2006). A higher value of 2 is used in Christiano, Eichenbaum, and Rebelo (2011). According to the panel, χ_c has ambiguous effect on credibility of the revert-to-discretion plan. When p_L is low, the threshold crisis frequency is higher when χ_c is lower. When p_L is high, the threshold crisis frequency is higher when χ_c is higher.

[10] Note that, in this exercise, I change the value of κ, holding the value of λ constant. An alternative way to conduct a sensitivity analysis is to change the value of λ together with κ, reflecting the fact that, when the objective function is derived as the second-order approximation, $\lambda = \kappa/\theta$ where θ is the degree of substitutability across intermediate goods in the original fully nonlinear model.

Figure J.4: The discretionary/Ramsey outcomes and values with alternative slopes of the Phillips curve

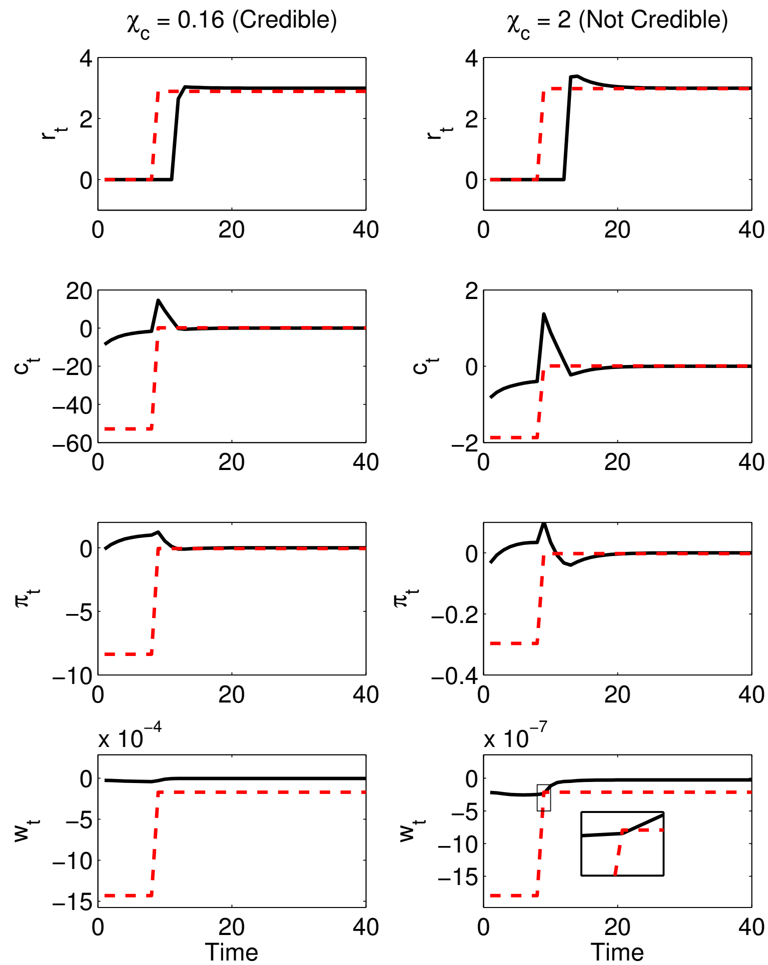


Note: See the note in figure J.3

To understand this result, the left panels in figure J.5 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economy with a small shock. The right panels show the same objects in the economy with a large shock. The contractionary shock is assumed to take a low value for the first eight periods and return to the high value afterward. In this figure, $(p_H, p_L) = (0.1/100, 0.7)$ and other parameters are set to their baseline values, except for the parameter being investigated.

Comparison of the IRFs across left and right panels shows that a higher χ_c implies larger declines in consumption and inflation in the low state under both discretionary and Ramsey outcomes. When the inverse IES is high, the household's consumption decision is more sensitive to the fluctuations in s_t. Since firms' pricing today depends on consumption today, inflation today is more sensitive to the fluctuations in s_t with a higher χ_c. While the Ramsey planner can mitigate declines in low-state consumption and inflation by future promises, the discretionary government has no tool to mitigate them. As a result, a marginal increase in the inverse IES leads to larger marginal

Figure J.5: The discretionary/Ramsey outcomes and values with alternative risk aversion



Note: See the note in figure J.2

declines in low-state consumption and inflation under the discretionary outcome than in the Ramsey outcome. Since lower low-state consumption and inflation reduce values in both states, the long-run loss from reneging on the promise is large with a smaller χ_c.

On the other hand, higher promised consumption and inflation with a larger χ_c mean that short-run gain from reneging on the promise is larger when χ_c is larger. Thus, the overall effect of χ_c on credibility of the revert-to-discretion plan is mixed. Similarly with the severity of shocks, while the threshold frequency is higher when the inverse IES is larger in the economy with highly persistent shocks, the threshold frequency is lower when the inverse IES is larger in the economy in which the shock persistence is low.

J.5 Weight on consumption volatility (λ)

The bottom-left panel of figure J.1 shows how the credible region depends on the weight on consumption volatility (λ). A low value of λ is the value that would be consistent with the lower value of κ if we had used the microfounded value of λ (that is, $\lambda = \frac{\kappa}{\theta}$ where θ is the degree of imperfect substitution across intermediate goods). A higher value of $\lambda = 1/16$ is a value that would give equal weights to the consumption volatility and the (annualized) inflation volatility. In practical applications at central banks, this weight is often used in analyzing optimal policy.[11] According to the panel, the threshold frequency is lower when the value of λ is lower, regardless of the crisis persistence.[12] Note that, as in the case for κ, this sensitivity analysis on λ is conducted in the reduced-form parameter space. That is, I keep the value of κ constant as I change the value of λ.

To understand this result, the left panels in figure J.6 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economy with a small weight on consumption volatility in the government's objective function. The right panels show the same objects in the economy with a large weight. The contractionary shock is assumed to take a low value for the first four periods and return to the high value afterward. In this figure, $(p_H, p_L) = (0.04/100, 0.8)$ and other parameters are set to their baseline values, except for the parameter being investigated.

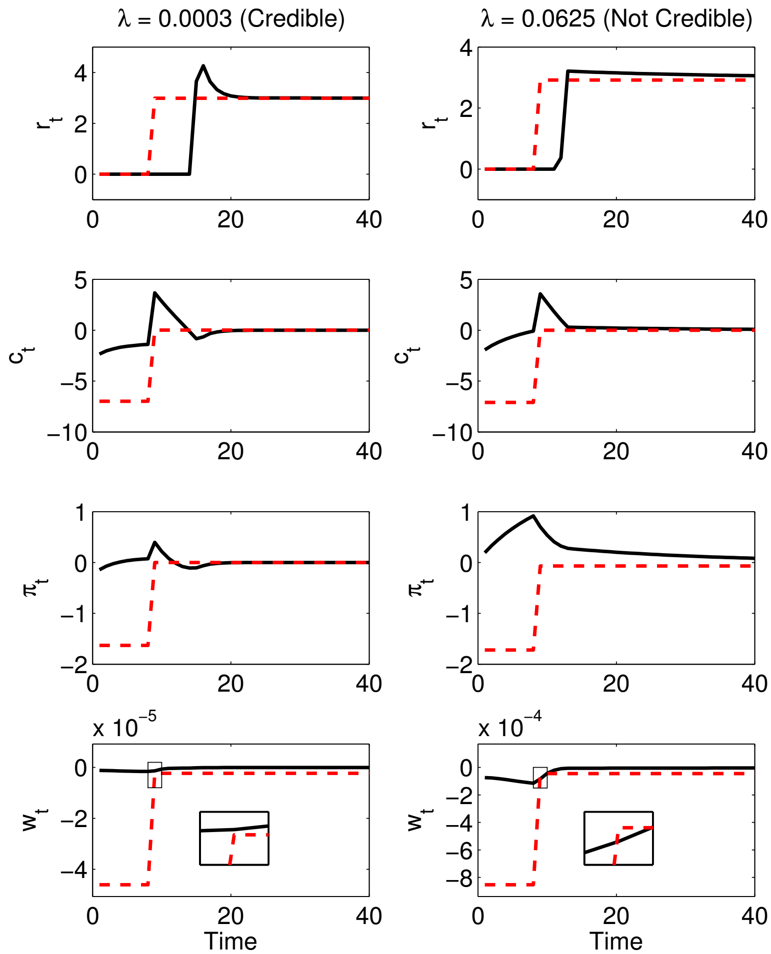
A larger λ means that the government cares more about consumption volatility relative to inflation volatility. Under the discretionary outcome, a greater concern for consumption volatility exacerbates the deflation bias in the high state, in turn amplifying deflation and consumption drops in the low state. The Ramsey planner can mitigate this effect by promising a higher, and more prolonged, inflation and consumption booms in the future, and marginal increases in the weight on consumption volatility reduces the low-state consumption and inflation, and also the values in both states, by more under the discretionary outcome than under the Ramsey outcome. Thus, the long-run loss of reneging on the promise, and therefore accepting the continuation value associated with the discretionary outcome, is higher.

On the other hand, promises of higher inflation and consumption hikes mean that the short-run gain of deviating from the promise is larger. Quantitatively, the second effect dominates the first effect unless the persistence of the shock is very high. For most values of p_L, the threshold frequency above which the revert-to-discretion plan is credible is higher when the central bank places a greater weight on consumption volatility in its objective function.

[11] See, for example, Ajello, Laubach, López-Salido, and Nakata (2016).

[12] Kurozumi (2008) and Sunakawa (2015) similarly find that the credible region decreases with λ in the model with stabilization bias in the sense that the threshold β above which the Ramsey policy is credible increases with λ.

Figure J.6: The discretionary/Ramsey outcomes and values with alternative weights on consumption stabilization



Note: See the note in figure J.3

K Sensitivity Analysis—Structural Parameter Space

If the semi-loglinear model is seen as the approximation to the Calvo model, the reduced-form parameters of the semi-loglinear model are related to the structural parameters of the Calvo model as follows:

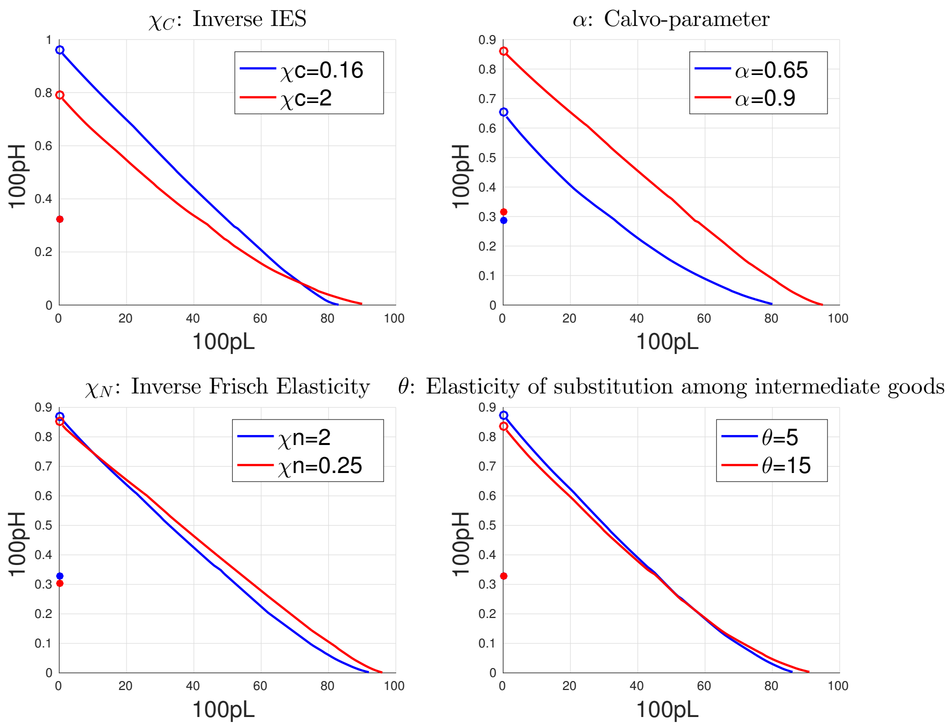
$$\kappa = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\frac{\chi_c+\chi_n}{1+\chi_n\theta}, \tag{17}$$

$$\lambda = \frac{\kappa}{\theta}, \tag{18}$$

where β is the discount factor, χ_c is the inverse intertemporal elasticity of substitution for consumption, χ_n is the inverse intertemporal labor supply elasticity, θ is the elasticity of substitution among intermediate goods, and α is the probability that firms cannot reoptimize their prices. Note that β and χ_c show up directly in the semi-loglinear model. With $\beta = 0.9925$ and $\chi_c = 1$, the values of the structural parameters consistent with the baseline values of κ and λ are $(\chi_n, \theta, \alpha) = (1, 0.5, 10, 0.81)$.

Figure K.1 shows how the structural parameters of the Calvo model affect the set of (p_H, p_L) under which the revert-to-discretion plan is credible.

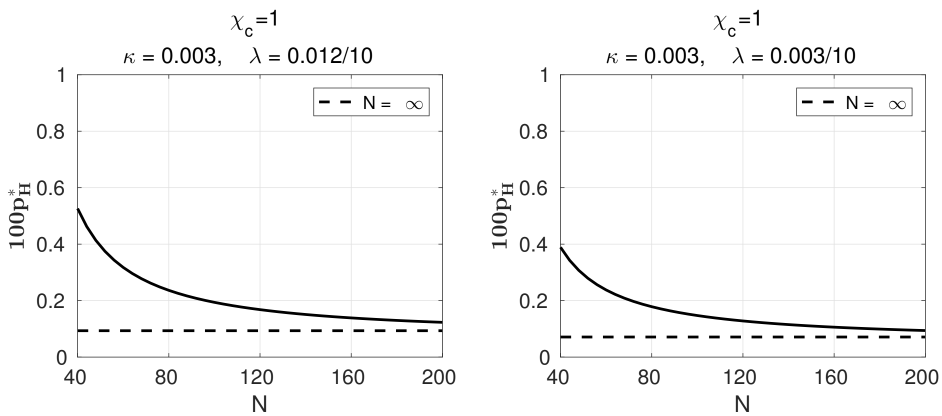
Figure K.1: Sensitivity Analysis
Structural-Parameter Space



One way to understand how a change in a structural parameter affects the credibility of the revert-to-discretion plan is by understanding (i) how it affects the reduced-form parameters and (ii) how the associated changes in the reduced-form parameters affect the credibility of the revert-to-discretion plan. Suppose that we want to understand how an increase in the Calvo parameter (α) affects the threshold crisis frequency. A higher α is associated with a lower κ and a lower λ. As shown in section 5.2 and online appendix J, a lower κ is associated with a higher threshold crisis frequency, while a lower λ is associated with a lower threshold crisis frequency. As a result, the threshold crisis frequency under $\alpha = 0.9$—shown in the right panel of figure K.2—is lower than that associated with the reduced-form reduction in κ consistent with $\alpha = 0.9$—shown in section 5.2 and reproduced in the left panel of figure K.2. Overall, the threshold crisis frequency is a bit higher under $\alpha = 0.9$ than under the baseline value of $\alpha = 0.81$ (0.1/100 versus 0.04/100 when $p_L = 0.8$).

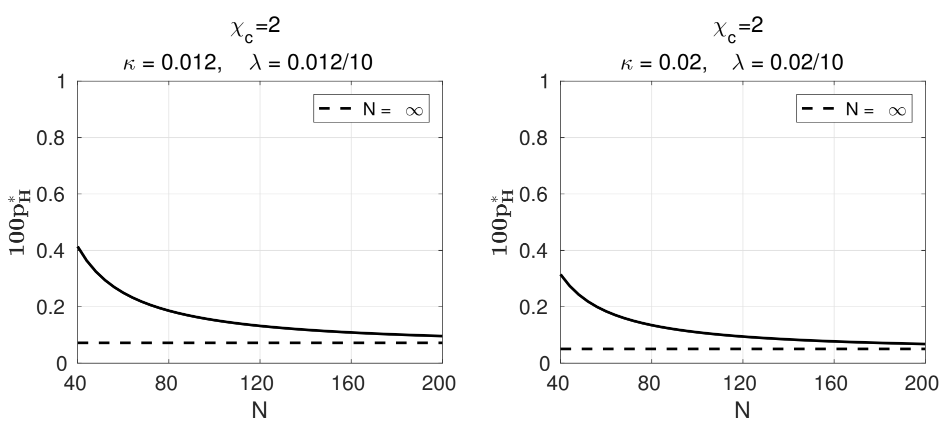
As another example, let's consider the effect of increasing χ_c in the structural parameter space. A higher χ_c is associated with a higher κ and λ. According to the sensitivity analyses conducted under the reduced-form parameter space, a higher χ_c, κ, and λ are associated with a higher, lower, and higher threshold crisis probability,

Figure K.2: Sensitivity Analysis for κ
in the Reduced-Form Parameter Space (left) versus Structural Parameter Space (right)



respectively. According to figure K.3, the threshold crisis frequency under $\chi_c = 2$ conducted in the reduced-form parameter space—shown in section 5.2 and reproduced in the left panel of figure K.3—is a bit higher than that under $\chi_c = 2$ conducted in the structural parameter space—shown in the right panel of figure K.3. Overall, the threshold crisis frequency under $\chi_c = 2$ is slightly higher than that under $\chi_c = 1$ if κ and λ are altered according to equations (17) and (18) at the same time χ_c is altered (0.05/100 versus 0.04/100 when $p_L = 0.8$).

Figure K.3: Sensitivity Analysis for χ_c
in the Reduced-Form Parameter Space (left) versus Structural Parameter Space (right)

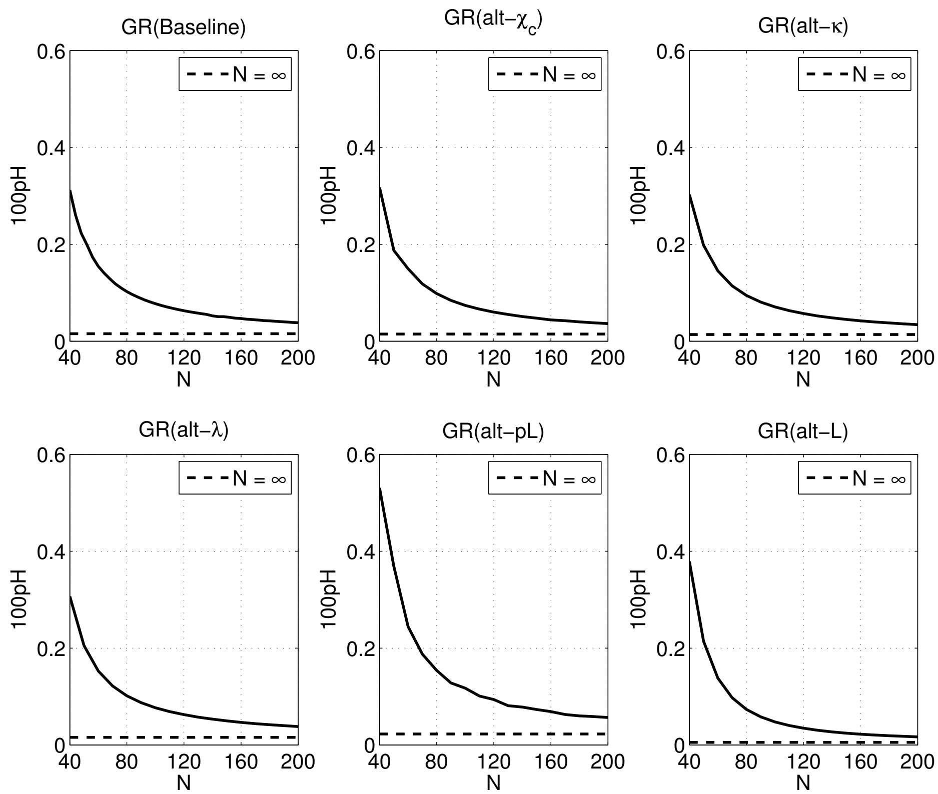


L Additional Analyses on the Revert-to-Discretion(N) Plan

How quickly the threshold crisis probability increases as the punishment duration becomes shorter depends on the parameter values used. In particular, I find that, when the crisis persistence is higher (that is, the average duration of the crisis is longer) or when the crisis is more severe, the threshold crisis probability increases more rapidly as the punishment duration shortens.

To illustrate this point, figure L.1 shows how the threshold crisis probability changes with the punishment duration for six variations of the Great Recession scenario of Denes, Eggertsson, and Gilbukh (2013). The top-left panel is their GR calibration. In next four panels, I set one of the five key parameters of the model to a value consistent with their GD calibration. For example, in the top-middle panel, χ_c is set to a value from the GD calibration (that is, 1.153), but all other parameter values are set to those under the GR calibration. Finally, in the bottom-right panel, the size of the shock is increased to obtain a 30 percent decline in consumption during the crisis, as in the GD calibration.

Figure L.1: Threshold Crisis Frequencies with Various "Modified" GR Calibrations



The figure shows that the sensitivity of the threshold crisis probability to N depends on each parameter. The top-middle, top-right, and bottom-left panels show that the use of the values consistent with the GD calibration for χ_c, κ, and λ does not affect the sensitivity much. However, the bottom-middle panel shows that, even when all other parameter values are set to the GR, the use of p_L that is consistent with the GD calibration (that is, 0.902) increases the sensitivity of the threshold crisis probability to N. Finally, the bottom-right panel shows that increasing the size of the shock to make the recession as severe as under the GD calibration also increases the sensitivity nontrivially.

These results indicate that the difference in the sensitivity of the threshold crisis probability to N depends importantly on the crisis persistence as well as the crisis severity.

M On the Discontinuity of the Threshold Crisis Frequency at $p_L = 0$

In the footnote to "Result 2" in the main body of the paper, I state that "The statements in Result 2 are more complicated than those in Result 1. The reason is that the threshold crisis frequency is discontinuous at $p_L = 0$. In particular, the threshold crisis probability drops suddenly as p_L approaches zero from above, which can be seen in figure 3. Online appendix M explains why this discontinuity arises. Note that, while the credible region is discontinuous in p_L, it is continuous in p_H; as stated in Result 1, holding all other parameters constant—including p_L—there is a threshold crisis probability above which the revert-to-discretion plan is credible and below which it is not credible."

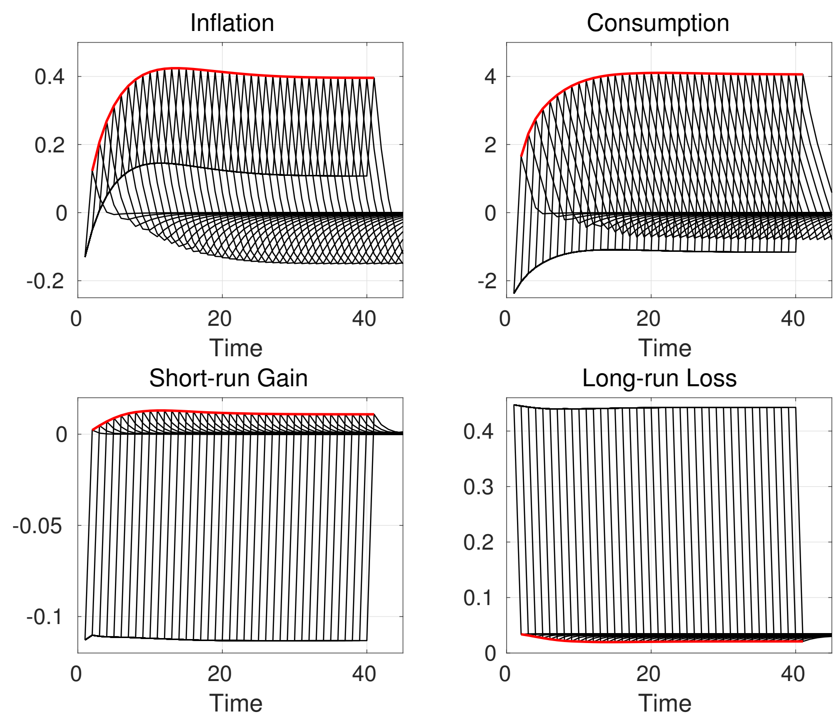
In this section, I explain why the threshold crisis frequency is discontinuous in p_L. The explanation proceeds in two steps. In the first step, I show that the incentive to deviate from the Ramsey outcome (the "short-run gain" minus the "long-run loss" in the language of online appendix I) depends on how long the crisis shock has lasted (that is, the realized shock duration). In particular, the incentive to deviate from the Ramsey outcome is typically the largest when the realized crisis duration is a few years. In the second step, I point out that, while there is a small but strictly positive probability that the crisis shock lasts for many periods when $p_L = \epsilon > 0$, the crisis shock cannot last for more than one period when $p_L = 0$. Thus, the maximum incentive to deviate from the Ramsey outcome occurs after a one-period crisis when $p_L = 0$. As the incentive to deviate is lower in the aftermath of a one-period crisis than it would be in the aftermath of a long-lasting crisis, it follows that the threshold crisis frequency does not need to be as high under $p_L = 0$ as under $p_L = \epsilon$.

Step 1:

For the revert-to-discretion plan to be credible, the incentive to deviate from the Ramsey outcome—which I define to be the (short-run) gain in today's utility flow minus the (long-run) loss in the continuation value, as they are defined in section 3—has to be negative in *all states of the world*. In particular, for the revert-to-discretion plan to be credible, it is not enough that the incentive to deviate from the Ramsey outcome is negative in the aftermath of a crisis that has lasted for one quarter; the incentive to deviate from the Ramsey outcome has to be negative in the aftermath of an N-period crisis for any positive integer N.

What is the relationship between the realized duration of the crisis and the incentive to deviate from the Ramsey outcome? One key feature of the Ramsey outcome—which is probably not well known outside the ZLB literature but is consistent with Eggertsson and Woodford (2003)—is that the magnitudes of the overshootings in inflation and consumption depend on how long the crisis shock has lasted, or the realized duration of the crisis shock.[13] As shown in figure M.1, which shows the Ramsey outcome under 40 different realizations of the crisis shock under the baseline parameterization with $p_H = 0.05/100$ and $p_L = 0.80$, the magnitude of the overshooting in inflation, shown in the top-left panel, is the largest when the realized shock duration is a few years. When the realized crisis duration increases further, the magnitude of the inflation overshooting in the aftermath of the crisis converges to a finite value. When the crisis shock lasts for only one quarter, the inflation overshooting is relatively modest. The same pattern holds true for the magnitude of the consumption overshooting, though the fact that the largest consumption overshooting occurs when the realized shock duration is a few years is a bit difficult to see from the top-right panel.

Figure M.1: IRFs under different shock realizations



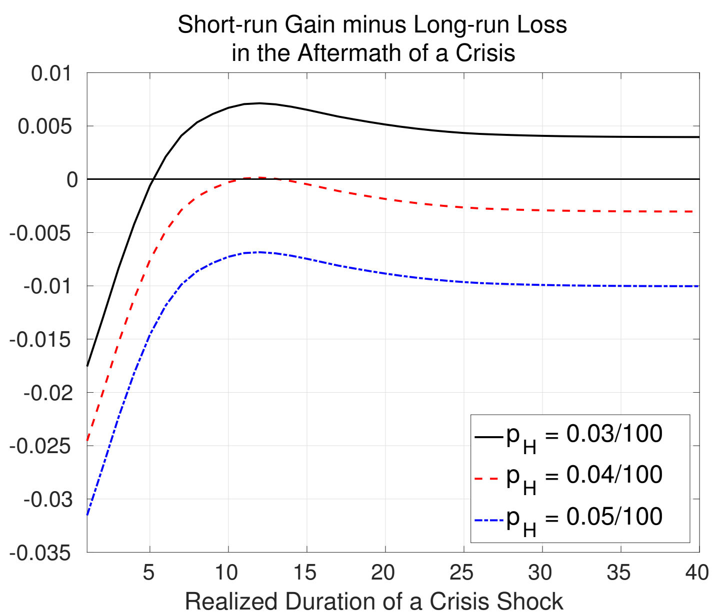
As the short-run gain from deviating from the Ramsey outcome in the aftermath of a crisis is determined by the magnitudes of the overshootings in inflation and consump-

[13]See figure 3 of Eggertsson and Woodford (2003).

tion, the short-run incentive to deviate from the Ramsey outcome also depends on the realized crisis shock duration. In particular, the short-run incentive is the largest when the realized crisis shock duration is a few years, as can be seen in the bottom-left panel of figure M.1.

The long-run incentive to deviate from the Ramsey outcome in the aftermath of a crisis also depends on the realized duration of the crisis shock. According to the bottom-right panel of figure M.1, the long-run incentive is the smallest when the realized shock duration is a few years. This pattern arises for the following reason. In the Ramsey outcome, the overshooting of inflation and consumption is followed by a moderately persistent undershooting of inflation and consumption, as can be seen in the top two panels of figure M.1. And, the magnitude and persistence of the undershooting depends on the crisis shock duration, typically increasing with the realized shock duration.

Figure M.2: The incentive to deviate is the largest in the aftermath of a crisis lasting for a few years



Putting these two pieces together—the relationship between the short-run incentive to deviate from the Ramsey outcome and the realized crisis duration on the one hand and the relationship between the long-run incentive to deviate from the Ramsey outcome and the realized crisis duration on the other hand—the overall incentive to deviate from the Ramsey outcome is the largest when the realized crisis shock duration is a few years, as shown by the dashed red lines in figure M.2. If the maximal incentive to deviate from the Ramsey outcome is negative, then the incentive to deviate from the Ramsey outcome in all states of the world and the revert-to-discretion plan is credible. Otherwise, the revert-to-discretion plan is not credible. When $p_H = 0.05/100$, the maximal incentive to deviate from the Ramsey outcome is negative, and the revert-to-discretion plan is credible. When $p_H = 0.04/100$ or $p_H = 0.03/100$, the maximal incentive to deviate from the Ramsey outcome is positive, and the revert-to-discretion plan is not credible. These results are consistent with figure 3.

Step 2:

Suppose that p_L is above zero. When we change p_L by a very small amount, the mapping shown in figure M.2—the mapping between the realized crisis shock duration and the incentive to deviate from the Ramsey outcome—changes by a very small amount, and therefore the threshold crisis frequency changes by a very small amount. As a result, the threshold crisis frequency varies continuously with p_L, as shown in figure 3 and figure J.1.

When $p_L = 0$, the determination of the threshold crisis frequency becomes qualitatively different. When $p_L = 0$, the crisis shock cannot last for more than one quarter. As a result, what matters in determining the credibility of the revert-to-discretion plan is the incentive to deviate from the Ramsey outcome in the aftermath of a one-period crisis. For the reasons described earlier, the incentive to deviate from the Ramsey outcome in the aftermath of a one-period crisis is not as high as that in the aftermath of a longer-lasting crisis. Thus, a smaller crisis frequency is enough to keep the incentive to deviate in the aftermath of a one-period crisis negative. As a result, when p_L decreases from $\epsilon > 0$—a case in which a long-lasting crisis can occur—to 0—a case in which a long-lasting crisis cannot occur—the threshold crisis frequency declines discontinuously. This discontinuous decline of the threshold crisis frequency at $p_L = 0$ can be seen in figure 3 and figure J.1.

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