Supplementary Appendix: Product Scope and Endogenous Fluctuations

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A Price elasticity of demand

This Appendix derives the demand elasticities of an intermediate good with respect to changes in its own price and the price of other goods produced by the same firm. Taking logs of equation (3) from the main paper we obtain

$$\begin{aligned}\ln y_t(i,j) &= -\gamma \ln p_t(i,j) - (\theta-\gamma)\ln P_t(i) + \theta \ln P_t + \ln Y_t \\ &\quad +[\tau(\gamma-1)-1]\ln N_t(i) + [\omega(\theta-1)-1]\ln M_t.\end{aligned}$$

From (4)

$$\frac{\partial \ln P_t(i)}{\partial \ln p_t(i,j)} = \left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1}.$$

Then from (5)

$$\frac{\partial \ln P_t}{\partial \ln p_t(i,j)} = \left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} {M_t}^{\omega(\theta-1)-1}.$$

Then the price elasticity of demand is

$$\begin{aligned}\frac{\partial \ln y_t(i,k)}{\partial \ln p_t(i,j)} &= \underbrace{-\gamma}_{\text{absent for } k\neq j} - (\theta-\gamma)\left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \\ &\quad +\theta\left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1}\left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1}.\end{aligned} \tag{A1}$$

Note that under monopolistic competition, firms are too small to influence the aggregate price index, P_t, and hence the last term in (A1) would be absent.

B Markups

This Appendix derives the optimal markups of intermediate good firms. Firm i maximizes profit (7) subject to the constraint (6):

$$\begin{aligned}\mathcal{L} &= \int_0^{N_t(i)} p_t(i,j)y_t(i,j) - w_t h_t(i,j) - r_t k_t(i,j)dj \\ &\quad +\Lambda_t \left(\int_0^{N_t(i)} \left[z_t k_t(i,j)^{\alpha} h_t(i,j)^{1-\alpha} - \phi \right] dj - \phi_f - \int_0^{N_t(i)} y_t(i,j)dj \right).\end{aligned}$$

Optimality gives

$$\frac{\partial \mathcal{L}}{\partial p_t(i,j)} = y_t(i,j) + \int_0^{N_t(i)} \left[p_t(i,j) - \Lambda_t \right] \frac{\partial y_t(i,j)}{\partial p_t(i,j)} dj = 0 \tag{A2}$$

$$\frac{\partial \mathcal{L}}{\partial h_t(i,j)} = -w_t + \Lambda_t (1-\alpha) z_t k_t(i,j)^{\alpha} h_t(i,j)^{-\alpha} = 0 \tag{A3}$$

$$\frac{\partial \mathcal{L}}{\partial k_t(i,j)} = -r_t + \Lambda_t \alpha z_t k_t(i,j)^{\alpha-1} h_t(i,j)^{1-\alpha} = 0. \tag{A4}$$

The Lagrange multiplier, Λ_t, is obtained by combining (A3) and (A4) and amounts to the marginal cost, mc_t, of producing one more variety:

$$mc_t \equiv \Lambda_t = \frac{w_t^{1-\alpha} r_t^{\alpha}}{z_t (1-\alpha)^{1-\alpha} \alpha^{\alpha}}.$$

Hence, the costs of production are

$$\int_0^{N_t(i)} w_t h_t(i,j) + r_t k_t(i,j)dj = mc_t \left(\int_0^{N_t(i)} [y_t(i,j) + \phi]dj + \phi_f \right)$$

and profits are

$$\pi_t(i) = \int_0^{N_t(i)} y_t(i,j)[p_t(i,j) - mc_t]dj - mc_t \left[N_t(i)\phi + \phi_f \right]. \tag{A5}$$

Substituting (A1) into (A2) and some algebra yields

$$y_t(i,j) - \gamma \frac{y_t(i,j)}{p_t(i,j)} \left[p_t(i,j) - mc_t\right] = \int_0^{N_t(i)} \frac{y_t(i,k)}{p_t(i,j)} \left[p_t(i,k) - mc_t\right] dk$$
$$\times \left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1} \left[\theta - \gamma + \theta \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1}\right].$$

Substituting (3) for $y_t(i,j)$, the above equation simplifies to

$$P_t Y_t \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1} \left[1 - \gamma \frac{p_t(i,j) - mc_t}{p_t(i,j)}\right] =$$
$$\int_0^{N_t(i)} y_t(i,k) \left[p_t(i,k) - mc_t\right] dk \left[\theta - \gamma - \theta \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1}\right].$$

As the second part of this equation is the same for all $j \in [0, N_t(i)]$, this implies that firm i will charge the same price for all of its varieties. Hence, $p_t(i,j) = p_t(i,k) = p_t(i)$ and the equation simplifies to

$$1 - \gamma \frac{p_t(i) - mc_t}{p_t(i)} = \tag{A6}$$
$$N_t(i)^{\tau(\gamma-1)} \left(\frac{p_t(i)}{P_t(i)}\right)^{1-\gamma} \frac{p_t(i) - mc_t}{p_t(i)} \left[\theta - \gamma - \theta \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1}\right].$$

To solve for firm i's markup, first note from (4) that $P_t(i) = N_t(i)^{-\tau} p_t(i)$. Then using this together with (1), (3) and (5), we can express firm i's market share, $\epsilon_t(i) \equiv P_t(i) Y_t(i) / (P_t Y_t)$, as

$$\epsilon_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} M_t^{\omega(\theta-1)-1} = \frac{N_t(i)^{-\tau(1-\theta)} p_t(i)^{1-\theta}}{\int_0^{M_t} N_t(i)^{-\tau(1-\theta)} p_t(i)^{1-\theta} di}. \tag{A7}$$

As long as $\tau > 0$, the price index $P_t(i)$ is decreasing in $N_t(i)$, and so increasing the product scope increases the firm's market share. Finally, the markup, $\mu_t(i) \equiv p_t(i)/mc_t$, can be found by rearranging (A6):

$$\mu_t(i) = \frac{\theta[1 - \epsilon_t(i)]}{\theta[1 - \epsilon_t(i)] - 1}. \tag{A8}$$

C Product scope

This Appendix derives the firms' optimal product scope. Substituting (3) into (A5), then using (4) and (A7), we rewrite profits as

$$\pi_t(i) = \left(\frac{p_t(i) - mc_t}{p_t(i)}\right) P_t Y_t \epsilon_t(i) - mc_t[N_t(i)\phi + \phi_f].$$

Firm i takes the number of firms and their product scopes as given and maximizes its profits with respect to $N_t(i)$ by taking account the effect of its product scope decision on its own and all other producers' pricing decisions. The first-order condition is

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = \theta P_t Y_t \left(\frac{p_t(i) - mc_t}{p_t(i)}\right)^2 \frac{\partial \epsilon_t(i)}{\partial N_t(i)} + Y_t \epsilon_t(i) \left(\frac{p_t(i) - mc_t}{p_t(i)}\right) \frac{\partial P_t}{\partial N_t(i)} - mc_t \phi = 0. \tag{A9}$$

We now calculate $\partial \epsilon_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ and then substitute in (A9) to obtain firm i's product scope. Differentiating (A7) with respect to $N_t(i)$ yields

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta - 1)\frac{\epsilon_t(i)}{N_t(i)} - (\theta - 1)\epsilon_t(i)\left[\frac{1}{p_t(i)}\frac{\partial p_t(i)}{\partial N_t(i)} - \frac{1}{P_t}\frac{\partial P_t}{\partial N_t(i)}\right]. \tag{A10}$$

Note that the second term on the right hand side of (A10) would not be present in the case of monopolistic competition. As we will see, $\partial p_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ are positive and negative, respectively; implying that firms contract their product scopes compared to the case of monopolistic competition. We rewrite the aggregate price index (5) as

$$P_t = M_t^{\frac{1}{\theta-1}-\omega}\left(\int_0^{M_t} N_t(k)^{-\tau(1-\theta)} p_t(k)^{1-\theta} dk\right)^{\frac{1}{1-\theta}}.$$

Then, after some algebra $\partial P_t/\partial N_t(i)$ can be expressed as

$$\frac{\partial P_t}{\partial N_t(i)} = P_t^{\theta} M_t^{\omega(\theta-1)-1}\left[\int_0^{M_t} N_t(k)^{-\tau(1-\theta)} p_t(k)^{-\theta}\frac{\partial p_t(k)}{\partial N_t(i)} dk - \tau N_t(i)^{-\tau(1-\theta)-1} p_t(i)^{1-\theta}\right]. \tag{A11}$$

We now show that the first term in the square brackets is equal to zero. From (A8)

$$\frac{p_t(k)}{p_t(k) - mc_t} = \theta - \theta\epsilon_t(k).$$

Then

$$\int_0^{M_t} \frac{p_t(k)}{p_t(k) - mc_t} dk = \theta M_t - \theta.$$

Differentiating with respect to $N_t(i)$ gives

$$\int_0^{M_t} -\frac{mc_t}{[p_t(k) - mc_t]^2} \frac{\partial p_t(k)}{\partial N_t(i)} dk = 0$$

which under symmetry collapses to

$$(M_t - 1)\frac{\partial p_t(k)}{\partial N_t(i)} + \frac{\partial p_t(i)}{\partial N_t(i)} = 0.$$

Replacing $\partial p_t(k)/\partial N_t(i)$ in (A11) with $-[\partial p_t(i)/\partial N_t(i)]/(M_t - 1)$ and assuming symmetry, the first term in the square brackets drops out and some rearrangement yields

$$\frac{\partial P_t}{\partial N_t(i)} = -\tau P_t \frac{\epsilon_t(i)}{N_t(i)}.$$

An increase in the product scope therefore reduces the aggregate price index. Inserting this result in (A10) gives

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta - 1)\frac{\epsilon_t(i)}{N_t(i)}[1 - \epsilon_t(i)] - (\theta - 1)\frac{\epsilon_t(i)}{p_t(i)}\frac{\partial p_t(i)}{\partial N_t(i)}. \tag{A12}$$

The next step is to compute $\partial p_t(i)/\partial N_t(i)$. From (A8) we obtain

$$\frac{\partial p_t(i)}{\partial N_t(i)} = \frac{\theta mc_t}{[1 - \theta + \theta\epsilon_t(i)]^2}\frac{\partial \epsilon_t(i)}{\partial N_t(i)}.$$

Then using this in (A12) and some simplification yields

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta - 1)\frac{\epsilon_t(i)}{N_t(i)}\frac{[1 - \epsilon_t(i)]^2(\theta[1 - \epsilon_t(i)] - 1)}{\theta(1 - \epsilon_t(i)[1 - \epsilon_t(i)]) - 1}.$$

Here, $\partial\epsilon_t(i)/\partial N_t(i) > 0$ and hence $\partial p_t(i)/\partial N_t(i) > 0$. Inserting $\partial\epsilon_t(i)/\partial N_t(i)$ and $\partial P_t/\partial N_t(i)$ into (A9), assuming symmetry where $\epsilon_t(i) = \epsilon_t = 1/M_t$, and some rearrangement gives

$$N_t = \frac{\tau P_t Y_t}{p_t \phi}\left[\frac{(\theta-1)(M_t-1)}{\theta(1-M_t)+M_t^2(\theta-1)} + \frac{1}{M_t[M_t(1-\theta)+\theta]}\right].$$

D Monopolistic competition

This Appendix shows that under monopolistic competition, markups and the product scope are constant over the business cycle. Moreover, this implies that the local dynamics and conditions for indeterminacy are identical to the mono-product model described in Pavlov and Weder (2012).

The procedure is similar to that used in the previous appendices. When firms are too small to influence the aggregate price index, P_t, the price elasticity of demand becomes

$$\frac{\partial \ln y_t(i,k)}{\partial \ln p_t(i,j)} = \underbrace{-\gamma}_{\text{absent for } k\neq j} - (\theta-\gamma)\left(\frac{p_t(i,j)}{P_t(i)}\right)^{1-\gamma} N_t(i)^{\tau(\gamma-1)-1}.$$

Substituting it in (A2) and some algebra gives a constant markup

$$\mu_t(i) = \frac{p_t(i)}{mc_t} = \frac{\theta}{\theta-1}.$$

In determining the product scope, differentiating profit by $N_t(i)$ leads to the first order condition

$$\frac{\partial \pi_t(i)}{\partial N_t(i)} = \frac{P_t Y_t}{\theta}\frac{\partial \epsilon_t(i)}{\partial N_t(i)} - mc_t\phi = 0$$

where

$$\frac{\partial \epsilon_t(i)}{\partial N_t(i)} = \tau(\theta-1)\frac{\epsilon_t(i)}{N_t(i)}$$

and hence

$$N_t = \frac{\tau P_t Y_t}{\phi p_t M_t}.$$

Substituting N_t in firm i's profits, $\pi_t(i) = 0$, and solving for M_t gives

$$M_t = \frac{P_t Y_t}{p_t \phi_f} \left[\frac{1}{\theta - 1} - \tau \right].$$

Finally, combining the last two equations yields:

$$N_t = \tau \frac{\phi_f}{\phi} \left[\frac{1}{\theta - 1} - \tau \right]^{-1}.$$

Since the markup and the product scope are constant over the business cycle, the linearized model is identical to the constant markup mono-product model presented in Pavlov and Weder (2012).