# Online Appendix to The dynamics of public investment under persistent electoral advantage

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## Online Appendix

## 1 Endogenous re-election probabilities

In this section, I endogenize the re-election probabilities by adding a voting stage into the model following the probabilistic voting literature.

The two groups will alternate in power based on a political institution in which "ideology" or other non-economic issues play a role. In particular, I use a "probabilistic-voting" setup following Lindbeck and Weibull in order to provide micro-foundations for political turnover: The probability of being in power next period is going to be endogenously determined via an electoral process.

A key departure from the traditional probabilistic voting model is that *parties do not have a commitment to platforms*. Therefore, announcements made during the political campaign will not be credible unless they are optimal ex-post (that is, once the party takes power).

Agents are assumed to differ not only in their preferences over the composition of expenditures but also in another dimension that is orthogonal to economic policy (religious views, charisma of the politician, etc.). Preferences over this *political dimension* imply derived preferences over candidates and will take the form of additive iid preference shocks $\omega$. The instantaneous utility of agent $j$ in region $J$ at a particular point in time is

$$\underbrace{u(c_j, n_j) + v(g^J)}_{\text{economic}} + \underbrace{\omega_j}_{\text{political}}, \tag{1}$$

**Timing**

Each period will be divided into two stages: the taxation stage and the election stage.

At the taxation stage, the incumbent chooses $\tau, g^A, g^B$, and $K_g'$ knowing the state of the economy ($K_g$) and the distribution of political shocks but not their realized values. Hence, policy is chosen under uncertainty. The probability of winning the election can be calculated by forecasting how agents vote given different realizations of the shock.

After production, consumption, and investment take place, $\omega'$ is realized. At the election stage, agents vote for the party that gives them higher expected lifetime utility. They need

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to forecast how the winner of the election chooses policy. The assumptions of rationality and perfect foresight imply that agents' predictions are correct in equilibrium.

The set of equilibrium functions to be determined in a Markov-perfect equilibrium is identical to the ones in the main body of the paper, with the addition of two new functions: The probabilities of re-election $p_i(K_g)$, which are now endogenous objects.

**Election Stage**

At this stage, agents must decide which party to vote for. The utility derived from political factors, $\omega_j$, has two components: An individual ideology bias (denoted by $\varphi^{jJ}$) and an overall popularity bias ($\psi$). In particular,

$$\omega_i = \left(\psi + \varphi^{jJ}\right) I_i,$$

where $I$ is an indicator function such that $I_B = 1$ and $I_A = 0$, since $\psi$ and the individual-specific parameter $\varphi^{jJ}$ measure voter $j$ 's ideological bias toward the candidate from party $B$. I will follow Persson and Tabellini (2000) by assuming that the distribution of $\varphi^{jJ}$ is uniform and group-specific, $\varphi^{jJ} \backsim \left[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J}\right]$, with $J = A, B$.

These shocks are *iid* over time and hence are 'candidate specific.' Each period, a given party presents a candidate and voters form expectations about the candidate's position on certain moral, ethnic or religious issues, orthogonal to the provision of public goods. Examples are attitudes toward crime (gun control or capital punishment), drugs (e.g., whether to legalize the use of marijuana), immigration policies, pro-life or pro-choice positions, same-sex marriage, etc. Since $\varphi^{jJ}$ can take positive or negative values, there are members in each group who are biased toward both candidates. Therefore, individuals belonging to the same group may vote differently.

The parameter $\psi$ represents a general bias toward party $B$ at each point in time. It measures the average relative popularity of candidates from that party relative to those from party $A$. While the realization of $\varphi^{jJ}$ is individual-specific, the value of $\psi$ is the same for all agents. This is the most essential shock, since by being common to all agents, it is the one that affects the election outcome. The role of $\varphi^{jJ}$ is to ensure the existence of equilibria by ruling out ties and is included mostly for technical reasons. The popularity shock is *iid* over time and can also take positive or negative values. It is distributed according to:

$$\psi \backsim \left[-\frac{1}{2} + \eta, \frac{1}{2} + \eta\right].$$

A positive value for $\eta$ (the expected value of $\psi$) implies that candidates from party $B$ have an average popularity advantage over those from the opposition. On the other hand, $\eta = 0$ implies that parties are symmetric, in the sense that their candidates are expected to be equally popular or charismatic. This parameter will be the main driving force behind the electoral advantage.

Finally, agents are assumed to have perfect information about the candidates, so there are no informational asymmetries in this model. At the election stage, voters compare their lifetime utility under the alternative parties. The maximization problem of voter $j$ in group $A$ is given by

$$\max\left\{V_A(K_g'),\, W_A(K_g') + \psi' + \varphi^{jA\prime}\right\},$$

where $V_A(K_g')$ denotes the welfare of this agent if a candidate representing his group wins the elections, while $W_A(K_g')$ is the value of his utility if the candidate representing group $B$ is elected. The maximization problem of an agent in group $B$ is analogously defined.

**Determination of probabilities**

Individual $j \in A$ votes for $B$ whenever the shocks are such that

$$V_A(K_g') < W_A(K_g') + \psi' + \varphi^{jA\prime}.$$

We can identify the *swing voter* in group $A$ as the voter whose value of $\varphi^{jA\prime}$ makes him indifferent between the two parties

$$\varphi^A(K_g') = V_A(K_g') - W_A(K_g') - \psi'.$$

Figure 1 illustrates this point (assuming $\psi = 0$ for simplicity). The swing voter is found where the two solid lines intersect. All voters in group $A$ with $\varphi^{jA\prime} > \varphi^A(K_g')$ also prefer party $B$ as can be seen in the graph.

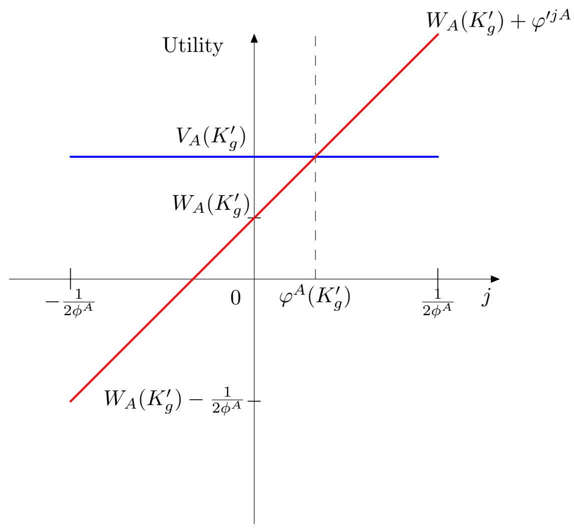


Figure 1: Utility as a function of $\varphi^{jA\prime}$

The same type of analysis can be performed for agents in group $B$, to determine the swing voter in that group.

Given the assumptions about the distributions of $\varphi^{jA}$ and $\varphi^{jB}$ the share of votes for party $B$ is:

$$\pi_B = \frac{1}{2}\left[1 - \sum_J \phi^J \varphi^J(K_g')\right].$$

Under majority voting, party $B$ wins if it can obtain more than half of the electorate; that is, if $\pi_B > \frac{1}{2}$. This occurs whenever its relative popularity is high enough. There exists a threshold for $\psi$, denoted by $\psi^*(K_g')$ such that $B$ wins for any realization $\psi > \psi^*(K_g')$. After performing some algebra using the expression above, we find that

$$\psi^*(K_g') = \frac{1}{\phi}\left(\phi^A\left[V_A(K_g') - W_A(K_g')\right] + \phi^B\left[W_B(K_g') - V_B(K_g')\right]\right), \tag{2}$$

where $\phi = \phi^A + \phi^B$.

The threshold is given by a weighted sum of the differences in the utility of the swing voter under each party. The weights depend on the dispersion in the ideology shocks and on the amount of supporters that each party has. The higher the heterogeneity within a constituency ($\phi^J$), the bigger the effect these factors have on the election outcomes. Also, the greater the number of individuals belonging to type $J$, the stronger the group in the determination of the probability. Finally, note that the threshold depends on the level of public capital, though it is not clear in which direction. In principle, this level could increase or decrease with $K'_g$.

Since $\varphi^J(K'_g)$ depends on the realized value of $\psi$, ex-ante the share of votes for party $B$ ($\pi_B$) is a random variable. $B$'s probability of winning the election is given by:

$$p_B(K'_g) \;\; = \;\; P\left(\pi_B > \frac{1}{2}\right) = P(\psi' > \psi^*(K'_g)),$$

which is equivalent to:

$$p_B(K'_g) = \frac{1}{2} + \left[\eta - \psi^*(K'_g)\right]. \tag{3}$$

$A$'s probability of winning the next election is just $p_A(K'_g) = 1 - p_B(K'_g)$.

Recall that $\eta$ represents the popularity advantage of candidates from party $B$ over those from party $A$. So in principle, $B$'s probability increases with $\eta$.

The current level of consumption in private and public capital does not affect the voting decision (i.e., no retrospective voting). Voters do not 'punish' politicians/parties for their past behavior but decide instead based on future expected policy choices.

**Taxation Stage**

The maximization problem looks exactly like the one presented in section 3, with the exception that probabilities now depend on the state variable and utility depends on ideological preference shocks. To fix ideas, consider the problem faced by an incumbent from group $B$

$$\max_{g^A, g^B, K'_g \geq 0} u(c,n) + v(g^B) + \omega_j + \beta\{p_B(K'_g)V_B(K'_g) + p_A(K'_g)W_B(K'_g) + E_B(\omega'_j; K'_g)\}$$

where consumption and labor satisfy equations (2) and (3). $E_B(\omega'_j, K'_g)$ represents the expected value of tomorrow's political shock conditional on $B$ winning the next election (recall that this shock is a relative bias toward a candidate from party $B$),

$$E_B(\omega'_j; K'_g) = \int_{\psi^*(K'_g)}^{\frac{1}{2}+\eta} z \partial z,$$

which can be shown to be equal to

$$E_B(\omega'_j; K'_g) = p_B(K'_g)\left[\frac{1}{2}p_A(K'_g) + \eta\right].$$

By changing the stock of public capital the incumbent affects not only the economic dimension but also his probability of winning and the expected value of political shocks.[1]

[1] Other papers in the literature usually ignore political shocks because they study two-period models, once the shock has been realized. Since $\omega$ is additive, focusing on net-of-shock welfare is without loss of generality. In this paper, it would not be the case because elections are held every period.

The functions $V_B(K_g)$ and $W_B(K_g)$ satisfy

$$V_B(K_g) = u\left(c_B(K_g), n_B(K_g)\right) + v(g_B^B(K_g)) + \tag{4}$$
$$\beta\{p_B(h_B(K_g))V_B(h_B(K_g)) + p_A(h_B(K_g))W_B(h_B(K_g)) + E_B(\omega_j'; h_B(K_g))\}$$

and

$$W_B(K_g) = u\left(c_A(K_g), n_A(K_g)\right) + v(g_A^B(K_g)) \tag{5}$$
$$\beta\{p_B(h_A(K_g))V_B(h_A(K_g)) + p_A(h_A(K_g))W_B(h_A(K_g)) + E_B(\omega_j'; h_A(K_g))\},$$

where $\Upsilon_i(K_g) = \{g_i^A(K_g), g_i^B(K_g), h_i(K_g)\}$ denotes the equilibrium policy functions chosen by incumbent type $i$, and where $c_i(K_g) = c(\Upsilon_i(K_g))$ and $n_i(K_g) = n(\Upsilon_i(K_g))$ are the competitive equilibrium values of consumption and labor under the political equilibrium policies.

Because the choice of expenditures is static, it is identical to the one under exogenous political turnover.

The investment decision, on the other hand, now depends on how public investment affects the probability of re-election

$$u_1(c_B(K_g), n_B(K_g)) = \beta\{p_B(K_g')V_{B1}(K_g') + p_A(K_g')W_{B1}(K_g')$$
$$+p_{B1}(K_g')\left[V_B(K_g') - W_B(K_g')\right] + E_{B2}(\omega_j'; K_g')\},$$

where $p_{B1}(K_g') = \frac{\partial p_B(K_g')}{\partial K_g'}$, and we use the fact that $p_A = 1 - p_B$.

Even though parties represent their constituencies and have no derived value of being in office, they will try to manipulate the probability of being re-elected (which allows them to implement the desired policy in the future).

A change in investment today modifies the problem faced by voters, which in turn affects the probability of being in power next period. A rational incumbent realizes this and thus takes into account the effect of expanding $K_g'$ on its likelihood of winning. It is reasonable to expect that a group is better off while in power, so $V_B(K_g') > W_B(K_g')$. However, the sign of $p_{B1}(K_g')$ is, in principle, ambiguous.

Under our functional assumptions, we can show that $p_{B1}(K_g') = 0$ in a differentiable MPE. Intuitively, if candidate $B$ proposes a higher level of investment, it will create a wedge in the marginal utilites derived from the two candidates. This margin, however, is independent of the stock of public capital in the economy. The reason is that (the natural logarithm of) capital appears to be additively separable from other arguments in all welfare functions $V_i$ and $W_i$. Inspection of equation 2 reveals that the threshold value $\psi^*$ is independent of $K_g$, and so is the re-election probability (see eq. 3). As a result, the probabilities of re-election are constant $p_i(K_g) = p_i$ for $i = A, B$. Marginal utilities, on the other hand, are affected by the marginal propensities to invest. Therefore, the probabilities of re-election are functions of these, as shown in Proposition 1.1.

**Proposition 1.1**

$$g_i(K_g) = \frac{1}{2}(1 - s_i)\tilde{A}K_g^{\bar{\theta}} - G \quad \textit{and} \quad h_i(K_g) = s_i\tilde{A}K_g^{\bar{\theta}},$$

*The marginal propensities to invest $s_i$ and the probabilities of re-election $p_i$ are jointly determined by:*

$$s_i = \bar{\theta}\beta\left[\frac{1 + p_i}{2 - \bar{\theta}\beta\,(1 - p_i)}\right]. \tag{6}$$

*The probabilities of reelection are* $p_B = \frac{1}{2} + [\eta - \psi^*]$ *and* $p_A = 1 - p_B$*, where*

$$\psi^* = \frac{3}{2}\left[\ln\left(\frac{1-s_A}{1-s_B}\right) + \frac{\bar{\theta}\beta}{1-\bar{\theta}\beta}\ln\frac{s_A}{s_B}\right]. \tag{7}$$

**Proof** Guess a constant probability $p_i(K_g) = p_i$ and a constant investment share $h_i(K_g) = s_i\tilde{A}K_g^{\bar{\theta}}$. From Proposition 4.1 we verify the guess for $h_i(K_g)$ given a constant $p_i$, where $s_B$ is defined in equation 23.

To verify that $p_i$ is a constant, note that the value functions satisfy

$$V_j(K_g) = \bar{\nu}_j + \nu_j \ln(K_g). \tag{8}$$

$$W_j(K_g) = \bar{\omega}_j + \omega_j \ln(K_g), \tag{9}$$

where

$$\nu_j = \frac{\bar{\theta}(2-\bar{\theta}\beta p_i)}{1-\bar{\theta}\beta} \quad \text{and} \quad \omega_j = \frac{\bar{\theta}(1+\bar{\theta}\beta p_j)}{1-\bar{\theta}\beta},$$

$$\bar{\nu}_j = \frac{1}{1-\beta}\left\{\beta(1-p_j)\left[\ln\left(\frac{1}{2}(1-s_i)\tilde{A}\right) + \beta[p_j\nu_j + (1-p_j)\omega_j]\ln(s_i\tilde{A})\right.\right.$$
$$\left.\left. +[1-\beta(1-p_j)]\left[2\ln\left(\frac{1}{2}(1-s_j)\tilde{A}\right) + \beta[p_j\nu_j + (1-p_j)\omega_j]\ln(s_j\tilde{A})\right]\right]\right\},$$

$$\bar{\omega}_j = \frac{1}{1-\beta(1-p_j)}\left\{\ln\left(\frac{1}{2}(1-s_i)\tilde{A}\right) + \beta\left[p_j\bar{\nu}_j + [p_j\nu_j + (1-p_j)\omega_j]\ln(s_i\tilde{A})\right]\right\}.$$

Replace eq. (8) and eq. (9) into eq. (2) to obtain the expression that determines $\psi^*(K_g)$.

Finally, we verify that probabilities are constant and that governments choose to invest a proportion of output. Notice that these rules are increasing in capital, differentiable and invertible.

*Q.E.D.*

From the proposition above, it becomes evident that electoral advantage $\xi = \eta - \psi^*$ is independent of $K_g$ in the politico-economic equilibrium. Therefore, the probabilities of reelection $p_i$ (endogenously derived here) are constant as assumed in the main text.

## 2 Numerical Appendix

A time period represents a year, so the discount factor is $\beta = 0.95$. Following Greenwood, Hercowitz and Huffman (1988) I assume that the elasticity of labor supply $\epsilon$ equals 2. The level of productivity $A$ is normalized to one. There are three non-standard parameters in this model: The elasticity of public capital $\theta$, the fixed cost of providing public goods $G$, and the popularity advantage $\xi$. I choose the three parameters so that simulated moments in the political equilibrium match three target moments in the data. The first target is mean non-defense public investment as a proportion of GDP in the US for the period 1970-2006 ($GNDI/Y$). The second target is average non-defense public consumption as a proportion of output, for the same time period ($GNDC/Y$). All figures are obtained from the NIPA tables. The third target is computed so that the equilibrium advantage of party $B$, given by $p_B - p_A$ in the model, matches the average advantage obtained by the Democrats during all congressional elections to the House of Representatives between 1970 and 2006 ($AD$). The variable is computed as follows. Let $sh_t(i) = \frac{i_t}{D_t+R_t}$ denote the share of seats obtained by party $i \in \{R, D\}$ in the House of Representatives in Congress $t \in \{91^{th}, ..., 109^{th}\}$ (that is, covering the period 1970-2006). The advantage of party $D$ at each period of time is simply $Adv_t = sh_t(D) - sh_t(R)$.

I simulated the political equilibrium for 5000 periods and discarded the first 1500 to eliminate the effects of initial conditions. Table I summarizes the value of the parameters obtained from the calibration, together with the target variables.

| Variable | Parameter | Target | Target Value |
|---|---|---|---|
| Elasticity of public investment | $\theta = 0.034$ | Public Investment/Output | $GNDI/Y = 2.56$ |
| Fixed cost of public goods | $G = 0.085$ | Public Spending/Output | $GNDC/Y = 13.12$ |
| Popularity advantage | $\xi = 0.062$ | Democrat advantage | $AD = 0.131$ |

The value of $\theta$ is in line with empirical estimates and close to the estimate used in Baxter and King (1993), who set the elasticity of public capital to 0.05.[2] While they use the same target—public investment as a ratio of output—to calibrate the model, their measure of investment includes defense expenditures, while mine excludes them. If I were to include defense expenditures as well, I would obtain a value closer to Baxter and King's. The parameter $G$ captures expenditures that have not been modeled (such as defense spending). To the best of my knowledge, this is the first time an attempt to estimate the parameter $\xi$ has been done in a calibrated political economy model. Therefore, there is no counterpart in the literature.

To test whether the mechanism highlighted in this paper is quantitatively significant, we can compare the estimated volatility of the federal government public investment to output ratio in the US with the resulting moment computed from our simulation, since this was not a targeted moment. In the data, the standard deviation of public investment to GDP throughout the period has been 0.002. The model delivers a standard deviation for this ratio of 0.001, implying that political frictions can explain up to 50 % of the observed variation in this variable. The model abstracts from productivity shocks, which would clearly affect this ratio. It would be interesting to consider both, political and economic shocks in an extension to my work.

[2] Baxter, M. and R. King (1993). "Fiscal Policy in General Equilibrium," *American Economic Review*, 83, 3: 315-34.