Size, Trend, and Policy Implications of the Underground Economy
Technical Appendix

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Abstract

This Appendix reports the results of additional analysis and robustness exercises that are mentioned in the paper "Size, Trend, and Policy Implications of the Underground Economy".

Appendix A. Testing for the wage premium hypothesis with Italian survey data

In this appendix, we present some empirical evidence in favor of an official sector wage-premium in the Italian labor markets. Our analysis address the following question: are some earning characteristics of workers better rewarded in the official sector rather than in the underground sector? To answer this question, we use several econometric techniques with data from Italy's household survey provided by the Bank of Italy. The analysis follows the approach pursued by Pratap and Quintin (2006) for the Argentinean economy, and tests the following hypothesis:

$$H_0 : E(w^m(X, Y, \epsilon) - w^u(X, Y, \epsilon)|X, Y \in A) > 0$$

where w^m and w^u respectively denote wages in the official market and in the underground sector , as a function of individual characteristics (X), job

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Table A.1: **Average weakly earnings**

Year	**Underground earnings** (w^u)	**Market earnings** (w^m)
1998	24.93731	32.72234
2000	24.68393	32.56064
2002	26.53229	32.78491
2004	26.43856	32.57341

characteristics (Y), and an exogenous random variable ϵ; $E(\cdot)$ denotes the conditional expectation operators, whereas A is a set of characteristics.

Data

The Survey on Household Income and Wealth (SHIW) is a biannual survey collecting information on wealth and other aspects of households' economic and financial behavior. The survey is provided by the Bank of Italy and comprises about 8,000 households, distributed over about 300 Italian municipalities. For our analysis, we focus over the 1998-2004 period. To test the wage premium hypothesis, we rely on real wages, discarding earnings for individuals that are either self-employed or that declare working more than 80 hours a week. Our final sample consists in 24244 observations.

Consistently with our definition of the underground economy, we classify individual as working in the regular market or in the underground sector according on whether they pay, or their employers pay, pension contributions, even for a short period of time.[1] It is worth noting that the proportion of individuals working exclusively in the underground sector is 5.04 per cent of the overall individuals in the sample.[2]

Wage regression

Table A.1 summarizes the average real hourly earnings of individuals working in the underground sector (w^u), and of those working on the market

[1] We use the variable CONTRIB in the survey.

[2] On the basis of our selection criterium, we cannot identify those regular workers that are also involved in the underground production. For this reason, the proportion of individuals in the sample working exclusively in the underground sector (5.04%) cannot be interpreted as an estimate of the overall contribution of the work force to the underground production.

Table A.2: Wage regression. Pooled OLS over the 1998-2004 period.

Variable	**Coef.**	**(Std. Err.)**
Age	0.054**	(0.004)
Age^2	0.000**	(0.000)
Gender (1=Male, 0=Female)	-0.183**	(0.010)
Sector (1=Official, 0=Underground)	0.210**	(0.031)
Hours	-0.013**	(0.001)
Marital Status (1=Married, 0=Single)	-0.014*	(0.007)
Establishment Size (Omitted <5)		
5 to 19	0.109**	(0.019)
20 to 49	0.185**	(0.020)
50 to 99	0.196**	(0.021)
100 to 499	0.236**	(0.021)
$\geq$ 500	0.328**	(0.020)
Public Sector	0.251**	(0.024)
Education Level (Omitted No Education)		
Elementary-school	0.196**	(0.075)
Middle-school	0.367**	(0.073)
Professional secondary-school diploma	0.461**	(0.074)
High-school	0.527**	(0.073)
Short course university degree	0.611**	(0.081)
Bachelor's degree	0.745**	(0.074)
Postgraduate qualification	0.822**	(0.100)
Occupation Sector (Omitted Agriculture)		
Mining and manufactures	0.492**	(0.031)
Wholesale and retail trade	0.486**	(0.034)
Transport, warehouse and communications services	0.490**	(0.031)
Service of credits and insurance	-0.407**	(0.055)
Year Dummies		
Year Dummy 2000	-0.006	(0.012)
Year Dummy 2002	0.005	(0.012)
Year Dummy 2004	0.003	(0.012)
Intercept	-4.110**	(0.112)

sector (w^m), where hourly wages are computed by dividing annual earnings

Table A.3: **Matching estimators**

	Caliper		**Nearest neighbor**	
year	**Coefficient** (W_p)	**Std. Err.**	**Coefficient** (W_p)	**Std. Err.**
1998	0.358	(0.083)	0.206	(0.090)
2000	0.300	(0.077)	0.329	(0.102)
2002	0.255	(0.075)	0.291	(0.085)
2004	0.256	(0.078)	0.241	(0.079)

by 52 times the average number of hours worked in a week.[3] The reported results are clearly consistent with the wage premium hypothesis: in each year considered, average earnings in the regular market are higher that average earnings in the underground sector.

Table A.2 reports the results of regressing log real hourly wages on worker and employer characteristics, year dummies and a dummy variable, which we called Sector, that takes value 1 if the individual works in the official market and 0 otherwise. The independent variables used in the wage regression are the same as those used in Pratap and Quintin (2006). As the table illustrates, the coefficient on the Sector variable is positive and statistically significant. This result shows that the wage differentials we reported in Table A.1 holds true even after controlling for individual and job characteristics, thus providing evidence in favor of the hypothesis H_0. It is also interesting to note that our findings confirm the results of Pratap and Quintin (2006), showing that a positive relationship between firm size and wages also exists in Italy.

Propensity score matching

We now test the hypothesis H_0 by making use of propensity score matching estimators. Estimations based on propensity score matching techniques, allow us to relax the strong parametric assumptions of the wage regression, and at the same time, to deal with selection bias issues. As in Pratap and Quintin (2006), the empirical analysis is performed by defining the official sector premium (wp) as the average effect of treatment on the treated:

$$W_p = E(w^m|X, Y, Sector = 1) - E(w^u|X, Y, Sector = 1)$$

[3]The corresponding variables in the survey are labeled YLM for annual earnings, and ORETOT for the average number of hours worked in a week. To obtain real wages, we use the Italian consumer price index provided by ISTAT.

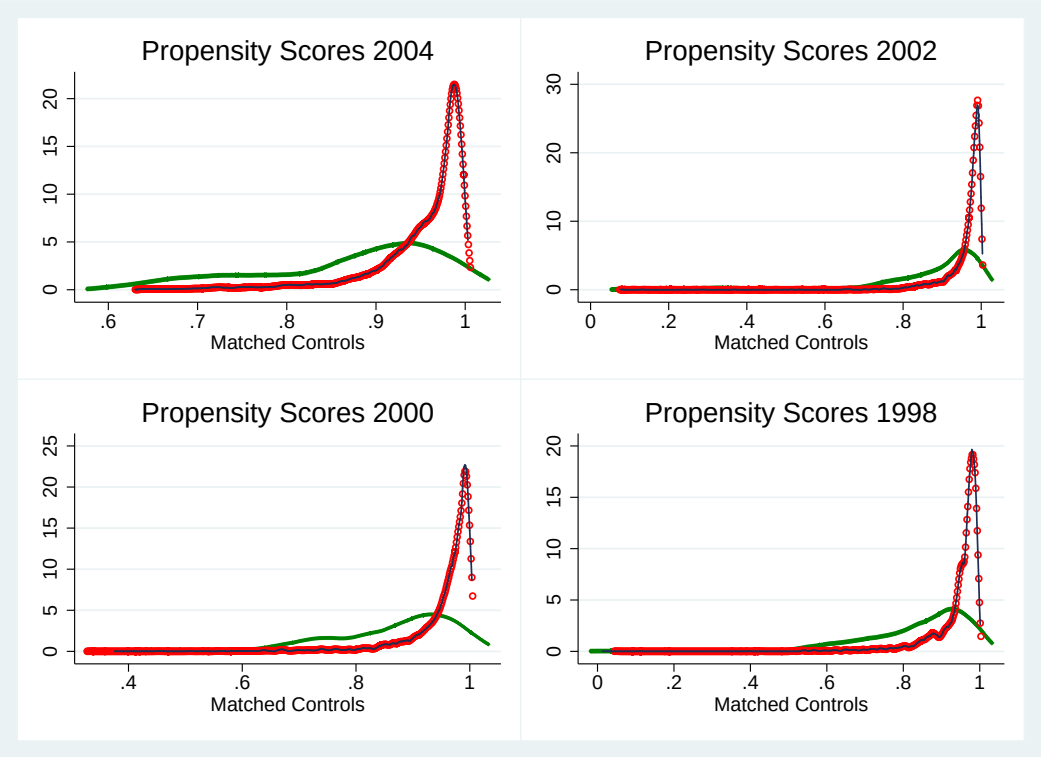


Figure A.1: **Comparison of Propensity Scores.** For each year, green line refers to the distribution of all controls; red line refers to the distribution of treated; and black line refers to the distribution of matched controls. Results are based on the nearest neighbor estimators.

Assuming that selection is driven only by characteristics X and Y, the last term in the previous formula, which is not observable, can be estimated by $E(w^u|X, Y, Sector = 0)$. This allows us to recover the wage premium W_p by making use of matching estimators that rely on propensity scores as devises to match individuals. To this end, propensity scores are estimated with a probit specification. The estimation is performed for each year separately, and using as dependent variable the Sector dummy. As concern the independent variables, we use the same set of variables as those used in Pratap and Quintin (2006) that is age, gender, firms size and education, experimenting different specifications that also include interactive and high order terms. Then, for each year, we choose the most parsimonious specification that satisfies the balanced property between official and underground sector.

In Table A.3 we report the result for two different matching estimators: the caliper estimator (with $\delta = 10^{-4}$) and the nearest neighbor estimator. As the table illustrates, the wage premium (W_p) based on both the estimators is positive and statically significant for all years. This result confirms the empirical evidence based on the wage regression, and provides evidence in favor of the presence of wage premia in Italian labor markets.

Finally, it also useful to look at the density functions of the propensity scores for the treated and the matched controls to get a sense of the overlap between them. Figure A.1 shows how the propensity score matching increases the comparability between the two groups. While prior to matching the estimated kernel densities are quite different (compare the green with the red line), after matching we can observe very similar values (compare the black with the red line).

Appendix B. On the interior solution of the firms' maximization problem

In this appendix, we show that the first order conditions for an interior solution of the firms' maximization problem represent a global maximum of the profit functions for a wide range of parameters value. To do so, we prove that under certain conditions the profit function is strictly concave, achieving a global maximum at any critical point. In addition, we also show numerically that for a wide range of fiscal parameter values, given the vector of equilibrium prices, the corner solutions of the firms' maximization problem deliver lower values of the expected profit with respect to its counterpart at the interior solution.

Let us start by analyzing the objective function of the firms. As shown in the paper, firms choose the optimal demand of productive factors by maximizing the expected profits:

$$E(\pi(y_{i,t})) = E(NR(y_{i,t})) - TC(y_{i,t}) \tag{B.1}$$

where $E(NR(y_{i,t}))$ and $TC(y_{i,t})$ respectively denote expected net revenues and total costs of producing an amount of output $y_{i,t}$. Net revenues are in turn given by the following expression:

$$NR(y_{i,t}) = \begin{cases} y^m_{i,t} - \tau^c_t \left(y^m_{i,t} - w^m_t h^m_{i,t}\right) + y^u_{i,t} - s\tau^c_t \left(y^u_{i,t} - w^u_t h^u_{i,t}\right) & \text{if detected} \\ y^m_{i,t} - \tau^c_t \left(y^m_{i,t} - w^m_t h^m_{i,t}\right) + y^u_{i,t} & \text{otherwise} \end{cases}$$

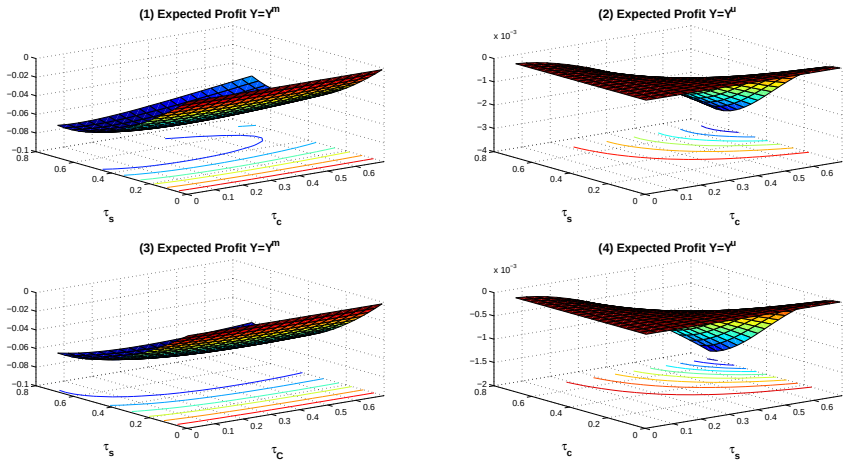


Figure B.2: **Expected profits and corner solutions**

Therefore, by direct calculation, we find that the expected net revenues can be written as follows:

$$E(NR(y_{i,t})) = (1 - \tau_t^c y_{i,t}^m + (1 - p_t \tau_t^c)\, y_{i,t}^u + p_t s \tau_t^c w_t^u h_{i,t}^u + \tau_t^c w_t^m h_{i,t}^m$$

whereas the cost function is given by:

$$TC(h_{i,t}^m, h_{i,t}^u, k_{i,t}^u, k_{i,t}^m) = (1 + \tau_t^s) w_t^m h_{i,t}^m + w_t^u h_{i,t}^u + r_t^u k_{i,t}^u + r_t^m k_{i,t}^m$$

Using the above equations with (B.1), and rearranging the terms yields:

$$\begin{aligned} E(\pi(y_{i,t})) \;\; = \;\; & (1 - \tau_t^c) y_{i,t}^m + (1 - p_t s \tau_t^c)\, y_{i,t}^u - (1 - p_t s \tau_t^c)\, w_t^u h_{i,t}^u - \\ & (1 + \tau_t^s - \tau_t^c)\, w_t^m h_{i,t}^m - r_t^m h_{i,t}^m - r_t^u h_{i,t}^u \end{aligned}$$

Now, given that y_t^u and y_t^m are Cobb-Douglass production functions, and therefore jointly strictly concave in their own arguments, it follows that the expected profit, $E(\pi(y_{i,t}))$, is a strictly concave function of $\{h_{i,t}^u, k_{i,t}^u, h_{i,t}^m, k_{i,t}^m\}$ as long as $(1 - \tau_t^c)$ and $(1 - p_t s \tau_t^c)$ are both non-negative. Hence, under the above conditions, the profit function attains a global maximum at any critical point (that is, satisfying the condition $\nabla E(\pi(h_{i,t}^u, k_{i,t}^u, h_{i,t}^m, k_{i,t}^m)) = (0, 0, 0, 0)$). This feature of the problem is also apparent in Figure B.2 where we graph the expected profit under the two corner solutions $y_{i,t} = y_{i,t}^u$ and $y_{i,t} = y_{i,t}^m$. Specifically, for alternative values of the tax rates τ_t^c and τ_t^s, we compute the interior

solutions, and then we use the resulting vector of prices $(w_t^u, w_t^m, r_t^m, r_t^u)$ to evaluate the expected profits under the corner solutions. This computation is performed for both prior parameters values (panels 1-2 of the picture) and for the posterior parameters values (panels 3-4). In both cases, the expected profits are always negative, and therefore lower than the interior solutions which in equilibrium is equal to zero.

Appendix C. Goodness-of-fit

In this appendix, we evaluate how well the estimated model fits the data. To do so, in section C1 we compare our estimates with a DSGE-VAR specification. This experiment is typically performed in the DSGE literature to evaluate the goodness-of-fit of the model. We then compare our model with underground economy with an alternative specification with home production (section C2). This experiment is useful to assess whether the inclusion of the underground sector helps improving the overall ability of the model to match the data with respect to a more standard framework. Finally, in section C3, we estimate the model with data for the US economy, and then compare the model's predictions with those obtained with the Italian data. To some extent, this exercise is useful to assess the plausibility of our inference procedure on the size and trend of the underground economy.

DSGE-VAR

In this section, we analyze how well our DSGE specification describes the dynamics of the observable variables. We address this question by considering the usual goodness-of-fit analysis based on a DSGE-VAR model as proposed in Del Negro et al. (2007), Del Negro and Schorfheide (2006) and An and Schorfheide (2007) amongst others. The main idea is to nest the linear DSGE model in a VAR model and relax the cross-coefficient restrictions imposed in the VAR by the DSGE model. It is in fact well known that a VAR specification can approximate fairly well the autocorrelation structure of a DSGE model and in particular, a DSGE parameterization is substantially equivalent to a VAR dynamics with cross-equation restrictions. It is thus of obvious interest to compare the two models, to check if relaxing the DSGE assumptions in the VAR setup provides similar dynamic patterns. A common practice is to compare the Impulse Response Functions (IRF) of the two models. In particular, if the *distance* between the two models is large, then we can conclude that the DSGE model is misspecified in some way. In case

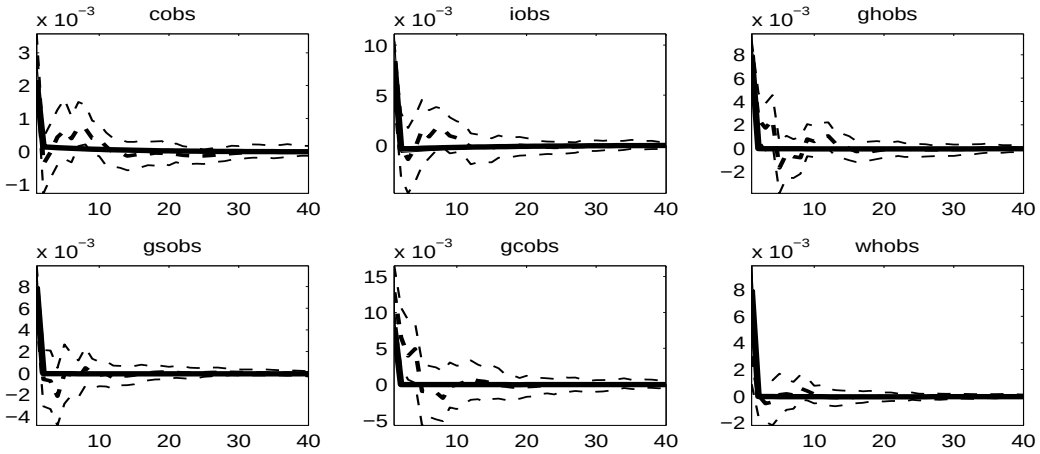


Figure C.3: IRF: Technology shock. Regular economy A_t : DSGE-VAR (dashed-dotted lines) and 95% confidence bands vs. DSGE (solid line). In particular *cobs*, *iobs*, *ghobs*, *gsobs gcobs* and *whobs* refer to the observables, i.e., consumption, investments, fiscal revenues from corporate taxation, social security contributions and personal income taxation and gross real total earnings paid in the regular market.

that the two models provide similar results, we can conclude that the DSGE model provide a reliable summary of the characteristics of the data.

The output of this analysis is displayed in Figures C.3-C.8.[4] We find that the IRF obtained through the two models are not significantly different, since the IRF obtained using the DSGE parametrization almost always fall into the IRF' confidence bands of the DSGE-VAR. This findings provide evidence that the two models describe in a similar way the reaction of the economic system with respect to the exogenous shocks of the model, and that the DSGE specification is reliable on generating realistic predictions of the effects of unanticipated changes in technology, fiscal and the other shocks.

Home production vs underground economy

In this section, we estimate an alternative version of our model where the underground sector is replaced with home production. The firms side of this model is identical of that described in section 3.1 of the paper, with the exception that firms are not allowed anymore to produce underground output. As a result, the first order conditions for the firms problem are now given

[4]The results are obtained by estimating the DSGE-VAR specification with Dynare.

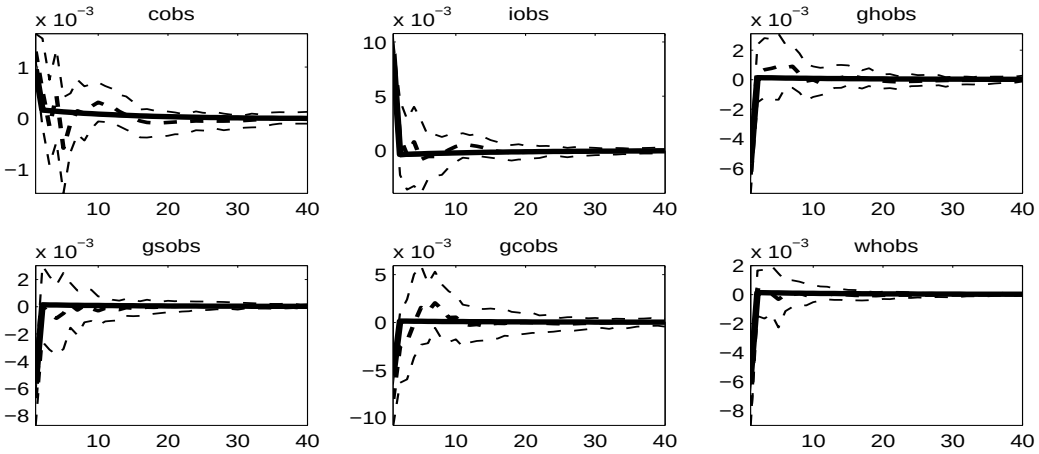


Figure C.4: IRF: Technology shock. Underground economy B_t : DSGE-VAR (dashed-dotted lines) and 95% confidence bands vs. DSGE (solid line). In particular *cobs*, *iobs*, *ghobs*, *gsobs gcobs* and *whobs* refer to the observables, i.e., consumption, investments, fiscal revenues from corporate taxation, social security contributions and personal income taxation and gross real total earnings paid in the regular market.

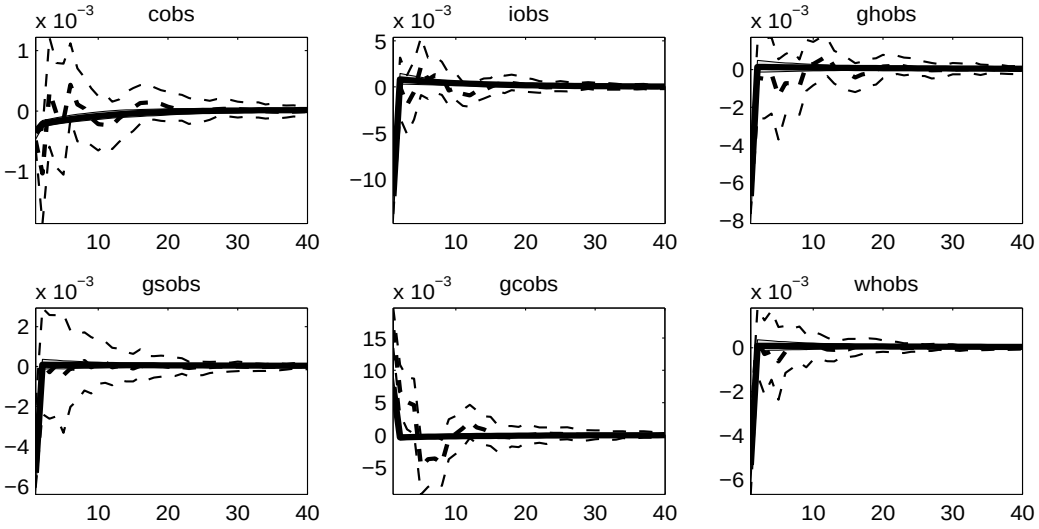


Figure C.5: IRF: Fiscal Shock. Corporate Tax Rate τ_t^c : DSGE-VAR (dashed-dotted lines) and 95% confidence bands vs. DSGE (solid line). In particular *cobs*, *iobs*, *ghobs*, *gsobs gcobs* and *whobs* refer to the observables, i.e., consumption, investments, fiscal revenues from corporate taxation, social security contributions and personal income taxation and gross real total earnings paid in the regular market.

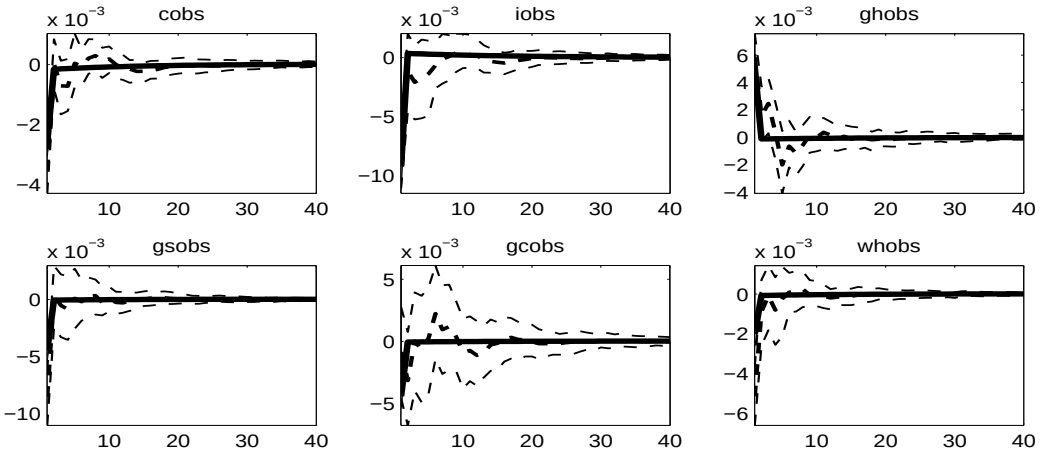


Figure C.6: IRF: Fiscal Shock. Personal Income Tax Rate τ_t^h : DSGE-VAR (dashed-dotted lines) and 95% confidence bands vs. DSGE (solid line). In particular *cobs*, *iobs*, *ghobs*, *gsobs gcobs* and *whobs* refer to the observables, i.e., consumption, investments, fiscal revenues from corporate taxation, social security contributions and personal income taxation and gross real total earnings paid in the regular market.

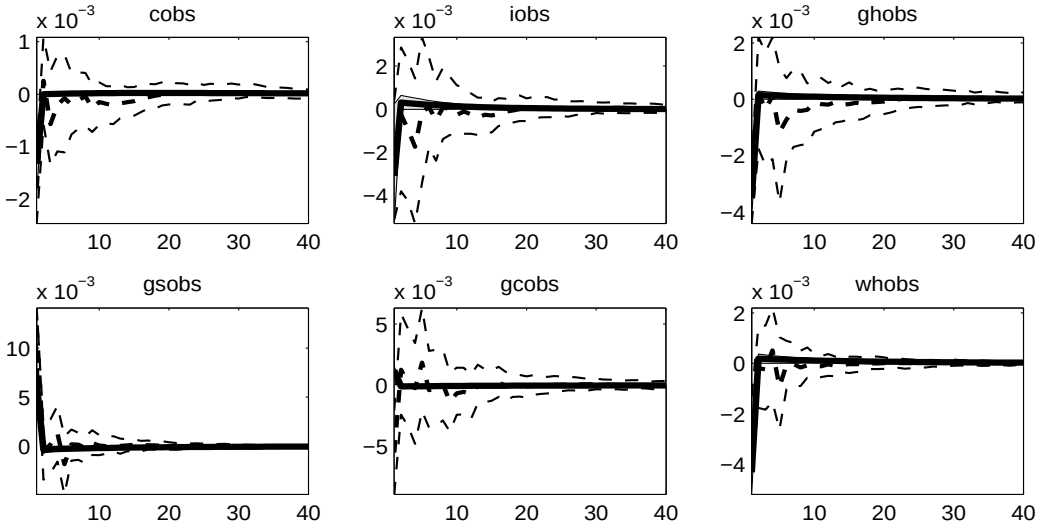


Figure C.7: IRF: Fiscal Shock. Social Security Tax Rate τ_t^s : DSGE-VAR (dashed-dotted lines) and 95% confidence bands vs. DSGE (solid line). In particular *cobs*, *iobs*, *ghobs*, *gsobs gcobs* and *whobs* refer to the observables, i.e., consumption, investments, fiscal revenues from corporate taxation, social security contributions and personal income taxation and gross real total earnings paid in the regular market.

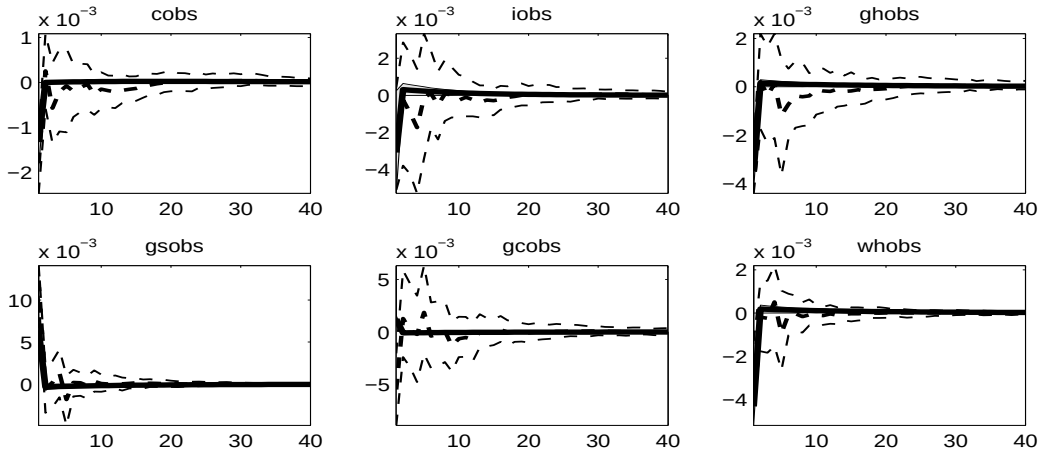


Figure C.8: IRF: Demand Shock. Investment Shock. ξ_t : DSGE-VAR (dashed-dotted lines) and 95% confidence bands vs. DSGE (solid line). In particular *cobs*, *iobs*, *ghobs*, *gsobs gcobs* and *whobs* refer to the observables, i.e., consumption, investments, fiscal revenues from corporate taxation, social security contributions and personal income taxation and gross real total earnings paid in the regular market.

by equations (4) and (5) in the paper. The households problem is somewhat different because of the presence of home production. More specially, we assume that households preferences are the same as those described in the paper, but now h_t^u denotes the amount of hours spent in home production. To maintain the structure of the model as close as possible to the original version with the underground sector, we assume that total consumption of households is given by

$$c_t = c_t^m + c_t^u$$

where c_t^m denotes the consumption of goods produced by firms, while c_t^u denotes the consumption of home produced goods. The latter are manufactured via following Cobb-Douglass technology:

$$c_t^u = B_t \left(\Gamma_t h_t^u\right)^{\alpha_u} \left(k_t^u\right)^{1-\alpha_u}$$

where k_t^u denotes capital for home production. In this environment, the households budget constraint becomes:

$$c_t^m + x_t = (1 - \tau_t^h)(w_t^m h_t^m + r_t k_t^m)$$

while the constrain given by the law of motion of capital is equal to equation (8) in the paper. Hence, the first order conditions for an interior maximum of the households problem are given by:

$$\frac{\lambda_t}{\xi_t^x} = \beta E_t \left\{ \lambda_{t+1} \left[\frac{(1-\delta_k)}{\xi_{t+1}^x} + (1-\tau_{t+1}^h) r_{t+1}^m \right] \right\}$$

$$B_0 \left(h_t^m + h_t^u\right)^{\xi} \xi_t^h = (1-\tau_t^h) w_t^m \lambda_t$$

$$B_0 \left(h_t^m + h_t^u\right)^{\xi} \xi_t^h + B_1 \left(h_t^u\right)^{\phi} = \alpha_u \left(\frac{c_t^u}{h_t^u}\right) \lambda_t$$

$$(1-\alpha_u) \left(\frac{c_t^u}{h_t^u}\right) = (1-\tau_t^h) r_t^m$$

where $\lambda_t = \Gamma_t^{(\sigma-1)} c_t^{-\sigma}$ is the Lagrange multiplier for the budget constraint. Finally, the government side of the model is the same as that of the model with underground economy, with the exception that now total revenues from corporate taxation do not include the component related to the underground production, that is

$$G_t^h = \tau_t^c \left(y_t^m - w_t^m h_t^m\right)$$

To estimate the model, we consider the following set of *measurement* equations:

$$\mathbf{y}_t \equiv \begin{bmatrix} \Delta c_t \\ \Delta x_t \\ \Delta G_t^c \\ \Delta G_t^s \\ \Delta G_t^h \\ \Delta w_t^h \end{bmatrix} = \begin{bmatrix} \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \end{bmatrix} + 100 \begin{bmatrix} \hat{c}_t^m - \hat{c}_{t-1}^m \\ \hat{x}_t - \hat{x}_{t-1} \\ \hat{G}_t^c - \hat{G}_{t-1}^c \\ \hat{G}_t^s - \hat{G}_{t-1}^s \\ \hat{G}_t^h - \hat{G}_{t-1}^h \\ \hat{w}_t^h - \hat{w}_{t-1}^h \end{bmatrix}$$

Prior distributions and calibrated parameters are the same as those described in section 4.2 of the paper. Results of the Bayesian estimations are summarized in Table C.4, where for each parameter we report prior distributions, posterior distributions, and 95% credible intervals.

To compare these results with the framework with underground economy, we re-estimate our model by excluding p_t from the observable variables, and setting the corresponding shock to its steady state value. By doing so, the two models have the same set of stochastic shocks, and are estimated by using the same dataset. This allows to perform a clearer comparison between the two models. Since our target is to understand whether the inclusion of the underground sector improves the ability of the model to match the data, we

Table C.4: DSGE with Home Production - **Posterior computation (MCMC)**

	Posterior distribution			Prior information		
	Mean	Mode	95% Cred. Int.	Mean	S.E.	Type
$p(\alpha\|\mathbf{y})$	0.6470	0.6492	[0.630,0.664]	0.650	0.010	Beta
$p(\delta_k\|\mathbf{y})$	0.0238	0.0211	[0.015,0.032]	0.025	0.005	Beta
$p(\alpha_u\|\mathbf{y})$	0.6590	0.6592	[0.646,0.675]	0.659	0.010	Beta
$p(\rho_a\|\mathbf{y})$	0.9249	0.9330	[0.890,0.953]	0.800	0.100	Beta
$p(\rho_b\|\mathbf{y})$	0.8931	0.8898	[0.753,0.993]	0.800	0.100	Beta
$p(\rho_c\|\mathbf{y})$	0.9855	0.9903	[0.975,0.998]	0.800	0.100	Beta
$p(\rho_h\|\mathbf{y})$	0.9765	0.9772	[0.962,0.990]	0.800	0.100	Beta
$p(\rho_s\|\mathbf{y})$	0.9278	0.9310	[0.910,0.949]	0.800	0.100	Beta
$p(\rho_H\|\mathbf{y})$	0.8041	0.8564	[0.659,0.963]	0.800	0.100	Beta
$p(\rho_I\|\mathbf{y})$	0.9020	0.9567	[0.768,0.994]	0.800	0.100	Beta
$p(\sigma\|\mathbf{y})$	1.1252	1.1265	[1.006,1.243]	1.000	0.100	Gamma
$p(\phi\|\mathbf{y})$	0.7236	0.7289	[0.633,0.813]	1.000	0.100	Gamma
$p(\xi\|\mathbf{y})$	1.1882	1.1331	[0.975,1.321]	1.000	0.100	Gamma
$p(B_1\|\mathbf{y})$	325.12	323.68	[307.8,340.8]	321	10	Gamma
$p(\sigma_a\|\mathbf{y})$	0.0097	0.0097	[0.008,0.011]	0.006	0.001	IG
$p(\sigma_b\|\mathbf{y})$	0.0035	0.0024	[0.002,0.005]	0.006	0.001	IG
$p(\sigma_c\|\mathbf{y})$	0.0259	0.0255	[0.023,0.029]	0.006	0.001	IG
$p(\sigma_h\|\mathbf{y})$	0.0120	0.0118	[0.011,0.014]	0.006	0.001	IG
$p(\sigma_s\|\mathbf{y})$	0.0259	0.0252	[0.023,0.029]	0.006	0.001	IG
$p(\sigma_H\|\mathbf{y})$	0.0048	0.0028	[0.002,0.009]	0.006	0.001	IG
$p(\sigma_I\|\mathbf{y})$	0.0030	0.0033	[0.002,0.004]	0.006	0.001	IG
$p(\gamma^{(Q)}\|\mathbf{y})$	0.0094	0.0095	[0.008,0.011]	0.002	0.005	Normal

compare the log data density of the two models. According to the estimated results, we find that the log data density of the model with the underground sector (1382.43) is much higher than the log data density of the model with home production (1144.72). This result indicates that the data prefers the specification with the underground sector.

Estimating the model with the US data

In this appendix, we summarize the results of estimating our model with data from the US economy. The goal is to compare the predictions of the model with different dataset to assess the plausibility of our inference procedure on the size and trend in the underground economy. To estimate the model for the US, we use the same set of observable variables as those used for Italy, with the exception of the fraction of firms inspected (i.e. the proxy of the probability of being caught) for which we did not find enough information to come out with a reliable series. As for Italy, the sample period

Table C.5: DSGE with Underground Economy US Data- **Posterior computation (MCMC)**

	Posterior distribution			Prior information		
	Mean	Mode	95% Cred. Int.	Mean	S.E.	Type
$p(\alpha\|\mathbf{y})$	0.6470	0.6492	[0.630,0.664]	0.650	0.010	Beta
$p(\delta_k\|\mathbf{y})$	0.0238	0.0211	[0.015,0.032]	0.025	0.005	Beta
$p(\alpha_u\|\mathbf{y})$	0.6590	0.6592	[0.646,0.675]	0.659	0.010	Beta
$p(\rho_a\|\mathbf{y})$	0.9249	0.9330	[0.890,0.953]	0.800	0.100	Beta
$p(\rho_b\|\mathbf{y})$	0.8931	0.8898	[0.753,0.993]	0.800	0.100	Beta
$p(\rho_c\|\mathbf{y})$	0.9855	0.9903	[0.975,0.998]	0.800	0.100	Beta
$p(\rho_h\|\mathbf{y})$	0.9765	0.9772	[0.962,0.990]	0.800	0.100	Beta
$p(\rho_s\|\mathbf{y})$	0.9278	0.9310	[0.910,0.949]	0.800	0.100	Beta
$p(\rho_H\|\mathbf{y})$	0.8041	0.8564	[0.659,0.963]	0.800	0.100	Beta
$p(\rho_I\|\mathbf{y})$	0.9020	0.9567	[0.768,0.994]	0.800	0.100	Beta
$p(\sigma\|\mathbf{y})$	1.1252	1.1265	[1.006,1.243]	1.000	0.100	Gamma
$p(\phi\|\mathbf{y})$	1.0311	1.0138	[0.996,1.064]	1.000	0.100	Gamma
$p(\xi\|\mathbf{y})$	1.2354	1.1331	[1.058,1.418]	1.000	0.100	Gamma
$p(B_1\|\mathbf{y})$	319.44	321.32	[310.9 327.1]	321.3	10.00	Gamma
$p(\sigma_a\|\mathbf{y})$	0.0028	0.0122	[0.002,0.004]	0.006	0.001	IG
$p(\sigma_b\|\mathbf{y})$	0.0221	0.0273	[0.016,0.028]	0.006	0.001	IG
$p(\sigma_c\|\mathbf{y})$	0.0586	0.0575	[0.051,0.066]	0.006	0.001	IG
$p(\sigma_h\|\mathbf{y})$	0.0426	0.0418	[0.037,0.047]	0.006	0.001	IG
$p(\sigma_s\|\mathbf{y})$	0.0103	0.0102	[0.009,0.012]	0.006	0.001	IG
$p(\sigma_H\|\mathbf{y})$	0.0409	0.0280	[0.036,0.046]	0.006	0.001	IG
$p(\sigma_I\|\mathbf{y})$	0.0050	0.0048	[0.004,0.006]	0.006	0.001	IG
$p(\gamma^{(Q)}\|\mathbf{y})$	0.0063	0.0063	[0.006,0.007]	0.005	0.002	Normal

is 1982-2006, whereas the data source is the Bureau of Economic Analysis. All of the series are deflated by the implicit GDP deflator and expressed in per-capita terms using population aged 16-65 years.

As far as model's parameters are concerned, we compute the average tax rate for corporate and personal income taxation by using fiscal revenues and tax bases (corporate profits and personal income respectively), setting $\tau^h = 0.20$ and $\tau^c = 0.13$, whereas the long-run social security tax rate, τ^s, has been set to 12.4% as established by the fiscal law in the United States. For all the other parameters, we use the same prior distributions described in section 4.2 of the paper. As a result, the two models are a priori different only in the long-run taxation rates for which we have got reliable information. Results of the Bayesian estimation are summarize in Table C.5, where for each parameter we report prior distributions, posterior distributions, and 95% credible intervals. As the table illustrates, estimates of the model parameters are substantially

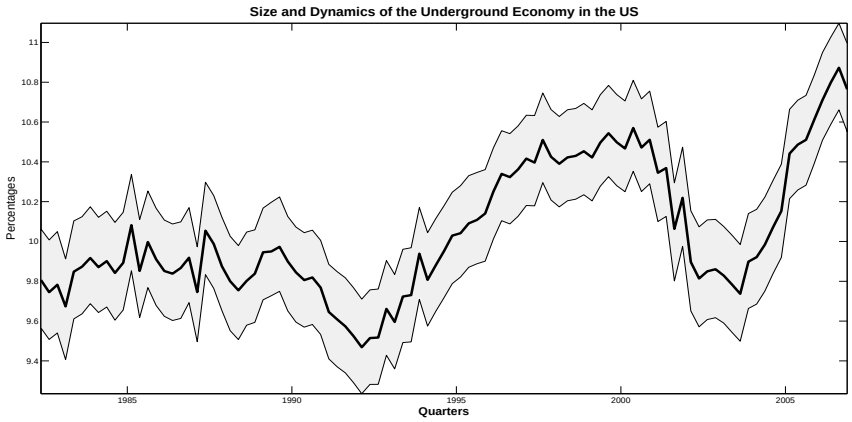


Figure C.9: **Size and trend of the underground economy in the US.**

different with respect to the estimates for the Italian economy. In particular, fiscal shocks are less persistent than their counterparts for Italian economy, possibly capturing differences in fiscal policies across these two countries. Interesting, the estimated results indicate that also in the US economy the underground labor supply is more elastic to movement in the relevant wage rate than its counterpart in the regular market.

Figure C.9 displays the smoothed estimated of the underground production GDP ratio along with the 95% credible interval. This figure summarizes how our model predicts the size and the trend of the underground economy in the US over the 1982-2006 period. Accordingly, the size of underground economy has fluctuated between 9.4% and 10.8% over the whole sample periods, with an average size of about 10%. There are two major expansions in the size of the underground economy, one in the 1993-2000 period, and another one in the 2004-2006 period. A major contraction instead occurred during the 2000-2003 period. Comparing these results with those obtained with Italian data (Panel A of Figure 1 in the paper), we see that the predictions of the two estimated models are completely different in terms of both size and dynamics of the underground economy. In particular, our framework predicts that the size of the underground economy is larger in Italy than in the US economy. For example, focusing on the 1999-2006 period, the size of underground economy in Italy is, on average, about 13 percentage points larger than it is in the US economy. This number is in line with the results of the MIMIC estimates reported in Schneider and Enste (2012), who found

that, over the 1999-2006 period, the size of underground economy in Italy is 18 percentage points larger than in the US economy (27% versus 9%). This result seems particularly interesting as shows that, although the two models are estimated with the same prior distributions, the results we obtain are nevertheless different, and in line with the other available estimates.

Appendix D. Robustness analysis

This appendix investigates the robustness of our main findings to alternative calibrations of the surcharge factor s. Jointly with the probability of being inspected p, this parameter controls for the overall effects of the tax enforcement process, and therefore is particularly important from the stand point of fiscal policy. The robustness analysis is performed by re-estimating the model under alternative calibrations of the surcharge factor, setting s to 1.25 and to 1.4. We thus compared the posterior averages of the parameters and the historical decomposition of the underground economy for the three cases (s=1.25, s = 1.30 and s=1.40).

Posterior estimates of the parameters in the three scenarios are provided in Table D.6. At a first sight, the three estimates appear substantially equivalent. However, on closer inspection, Table D.6 evidences some interesting features. In particular, we note that higher values of s correspond to smaller estimates of p. For instance, augmenting the penalty surcharge factor of 10 percentage points induces a negative average variation on p of about 8 percent (p reduces from 2.17% to 2.02%). Likely, this result hinges on the property that the overall impact of the monitoring activity depends jointly by these two parameters that enter in a multiplicative way into the equations of the model. Table D.6 evidences also a negative impact of s on the estimated growth rate parameter $\gamma^{(Q)}$. In particular $\gamma^{(Q)}$ reduces from 0.9887 to 0.9020 when s is augmented from 25% to 40%. This evidence is consistent with the results provided by the model in Section 6.2.2. In fact, as in the case of larger p, an increased surcharge factor corresponds to an higher fiscal pressure with an overall depressing effect on the economic activity, especially for consumption and investments (and their growth rate).

Figure D.10 displays the smoothed estimates of the underground economy (in log-deviation from the steady state) for the three cases, along with the historical contributions of technological, fiscal factors, controls and other shocks.

Table D.6: DSGE with Underground Economy - **Posterior averages for the parameters (for different s)**

	Posterior averages		
	Mean $s = 1.30$	Mean $s = 1.25$	Mean $s = 1.40$
$p(\alpha\|\mathbf{y})$	0.6379	0.6348	0.6367
$p(\delta_k\|\mathbf{y})$	0.0355	0.0348	0.0350
$p(\alpha_u\|\mathbf{y})$	0.6665	0.6684	0.6676
$p(\rho_a\|\mathbf{y})$	0.9954	0.9957	0.9926
$p(\rho_b\|\mathbf{y})$	0.9380	0.9297	0.9413
$p(\rho_c\|\mathbf{y})$	0.9759	0.9736	0.9719
$p(\rho_h\|\mathbf{y})$	0.9928	0.9975	0.9896
$p(\rho_s\|\mathbf{y})$	0.9450	0.9489	0.9360
$p(\rho_H\|\mathbf{y})$	0.6384	0.6096	0.5891
$p(\rho_I\|\mathbf{y})$	0.9491	0.9386	0.9420
$p(\rho_p\|\mathbf{y})$	0.9544	0.9585	0.9466
$p(p\|\mathbf{y})$	0.0217	0.0218	0.0202
$p(\sigma\|\mathbf{y})$	0.9985	1.0002	0.9880
$p(\phi\|\mathbf{y})$	0.9297	0.9179	0.9295
$p(\xi\|\mathbf{y})$	1.7333	1.6547	1.7450
$p(B_1\|\mathbf{y})$	324.43	324.2746	324.3244
$p(100\sigma_a\|\mathbf{y})$	1.3518	1.3559	1.3483
$p(100\sigma_b\|\mathbf{y})$	1.3553	1.4141	1.4097
$p(100\sigma_c\|\mathbf{y})$	1.1720	1.1876	1.1819
$p(100\sigma_h\|\mathbf{y})$	1.5205	1.5182	1.5134
$p(100\sigma_s\|\mathbf{y})$	2.0786	2.0848	2.0993
$p(100\sigma_H\|\mathbf{y})$	0.5359	0.5593	0.5784
$p(100\sigma_I\|\mathbf{y})$	0.8711	0.7903	0.8008
$p(100\sigma_p\|\mathbf{y})$	7.4399	7.3564	7.4353
$p(100\gamma^{(Q)}\|\mathbf{y})$	0.9104	0.9887	0.9020

As the picture illustrates, the results obtained in the baseline estimation (upper panel of the picture) are confirmed under the alternative calibrations of s. In all of the considered cases, the model predicts an upward trend in the size of the underground economy that is primarily explained by the fiscal components, while the technological components has been relevant mainly to explain fluctuations around the trend. Furthermore, the contributions of other shocks are negligible over the whole sample period regardless of the specific value assigned to parameter s. Summarizing, the results provided in this appendix show that both parameters estimates and model's predictions are robust to alternative calibrations of the surcharge factor s.

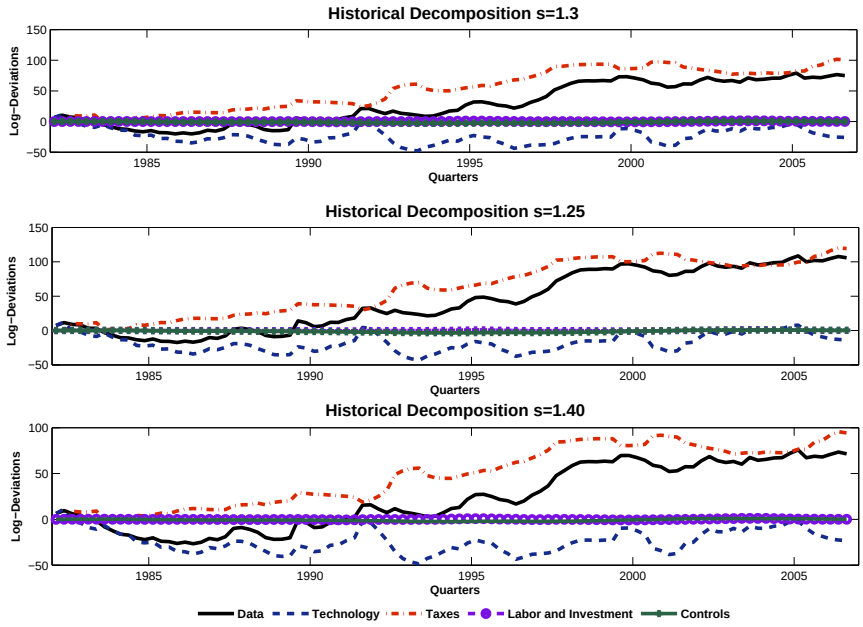


Figure D.10: Historical decomposition of the underground economy (in percentage terms) for different levels of s.

Appendix E. Convergence diagnostics

To evaluate the reliability of our estimation procedure, we also computed some convergence diagnostics. In particular Figure E.11, plots the recursive averages of the MCMC procedure. Specifically, the plot reports the evolution of $\frac{1}{i}\sum_{j=1}^{i}\boldsymbol{\theta}^{(j)}, \forall i$. Of course, it is impossible to assess the convergence properties of an MCMC algorithm through the study of only a few realizations of the chains. However a common practice is to check for the convergence of the empirical averages. (see Robert and Casella, 1999, chap. 8 for a survey on this topic). As a rule of thumb, we state that a chain converges when the evolution of the empirical averages stabilizes after a number of iterations. As Figure E.11 illustrates, it is quite evident that the running averages for the algorithm stabilize quickly enough, showing that the algorithm converges. According to our experience, it appears that the random walk algorithm implemented needs about 100,000 iterations to converge to its correct expected value, and, according to these results, we discarded the first 100,000 draws from each chain to remove the dependence from the initial conditions.

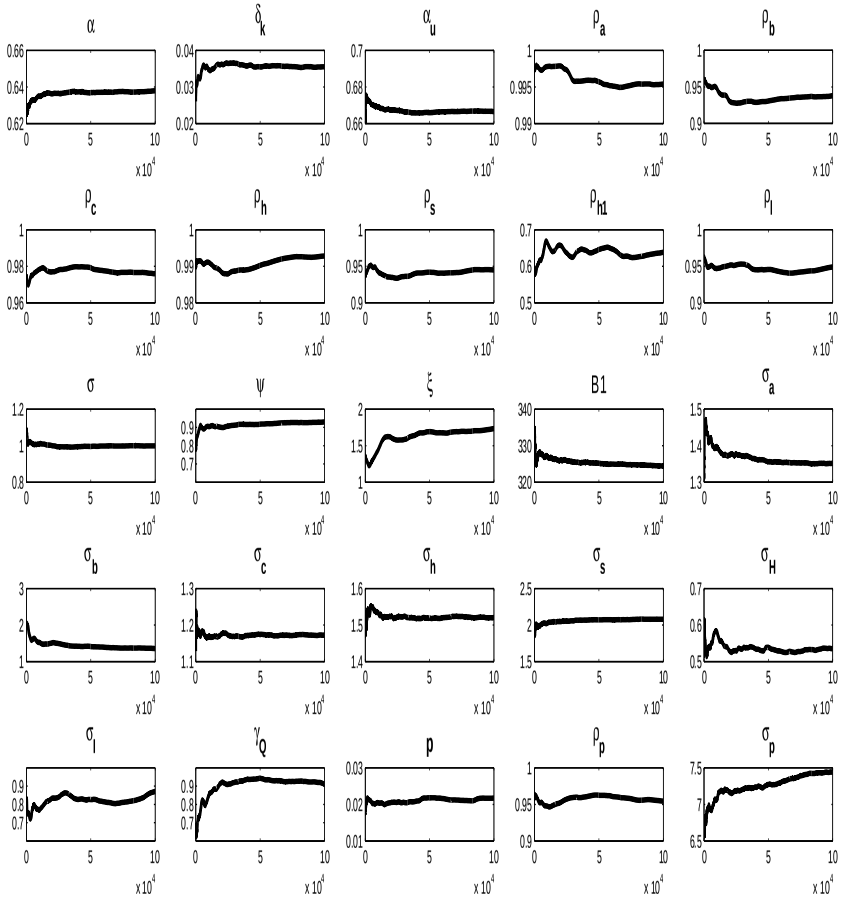


Figure E.11: Recursive MCMC averages for all the estimated parameters

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