APPENDICES: An application of business cycle accounting with misspecified wedges

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In this section, we provide descriptions of our prototype model for BCA, and of the two detailed economies for our experiment.

B.1 Prototype economy

The prototype economy for BCA is almost the same as that employed by Chari, Kehoe, and McGrattan (2007a) (hereafter, CKM). The log-linearized system is as follows.

The intratemporal optimization condition[1] is

$$c_t + \frac{\ell}{1-\ell}\ell_t = y_t - \ell_t - \tau_{\ell,t}, \tag{1}$$

where c_t denotes consumption; ℓ_t, labor supply; y_t, output; and $1 - \tau_{\ell,t}$, the investment wedge.

The Euler equation is

$$\begin{aligned}
&(1+\tau_x)(\tau_{x,t} - c_t) \\
&\quad = \beta\left\{\alpha\frac{y}{k}\left(E_t y_{t+1} - k_t\right) + (1+\tau_x)(1-\delta)E_t\tau_{x+1} - \left[(1+\tau_x)(1-\delta) + \alpha\frac{y}{k}\right]E_t c_{t+1}\right\},
\end{aligned} \tag{2}$$

where k_t denotes the capital stock at the end of period t; β, the discount factor of a household; δ, the depreciation rate of capital; α, the share of capital in production; and $\frac{1}{1+\tau_{x,t}}$, the investment wedge.

The aggregate production function is

$$y_t = \alpha k_{t-1} + (1-\alpha)\ell_t + a_t, \tag{3}$$

where a_t denotes the efficiency wedge and α denotes the cost share of capital stock in

[1]We employ the period utility function $U = \log(C_t) + \gamma\log(1 - L_t)$, following CKM.

production.

The evolution of capital stock is

$$k_t = (1-\delta)k_{t-1} + \delta i_t, \tag{4}$$

where δ denotes the depreciation rate of capital.

The resource constraint is

$$\frac{c}{y}c_t + \frac{i}{y}i_t + g_t = y_t, \tag{5}$$

where g_t denotes the government wedge.

In many applications of BCA, the wedges are assumed to be exogenous and evolve according to the VAR(1) process:

$$\boldsymbol{s}_{t+1} = \boldsymbol{P}\boldsymbol{s}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{6}$$

where $\boldsymbol{P}$ denotes a constant matrix; $\boldsymbol{s}_t \equiv [a_t, \tau_{\ell,t}, \tau_{x,t}, g_t]'$, a vector of wedges; and $\boldsymbol{\varepsilon}_{t+1}$, a vector of i.i.d. shocks to wedges with mean zero.

B.2 Detailed economy: Medium-scale DSGE economy

We employ a medium-scale DSGE model for the assessment of BCA. Here, we provide a brief description of our detailed economy. Our economy is the same as that employed by Smets and Wouters (2007). Following Smets and Wouters (2007), we introduce the linearized version of their model here.

The resource constraint is

$$y_t = \frac{c}{y}c_t + \frac{i}{y}i_t + \frac{r^k k}{y}u_t + \varepsilon_t^g, \tag{7}$$

where ε_t^g is the government expenditure shock.

The consumption Euler equation is

$$
\begin{aligned}
c_t =& \frac{\lambda}{1+\lambda} c_{t-1} + \frac{1}{1+\lambda} E_t c_{t+1} \\
& + \frac{(\sigma_c - 1)\left(\frac{w^h \ell}{c}\right)}{\sigma_c (1+\lambda)} (\ell_t - E_t \ell_{t+1}) - \frac{1-\lambda}{\sigma_c (1+\lambda)} (r_t - E_t \pi_{t+1} + \varepsilon_t^b),
\end{aligned} \tag{8}
$$

where ε_t^b denotes the risk premium shock; λ, the parameter on the external habit; and σ_c, the inverse of the intertemporal elasticity of substitution.

$$
i_t = \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t i_{t+1} + \frac{1}{(1+\beta)\varphi} q_t + \varepsilon_t^i, \tag{9}
$$

where β denotes the discount factor of a household; φ, the steady state elasticity of the capital adjustment cost function; and ε_t^i, the investment-specific technology shock.

The capital Euler equation is

$$
q_t = \beta(1-\delta) E_t q_{t+1} + \Big[1 - \beta(1-\delta)\Big] E_t r_{t+1}^k - (r_t - E_t \pi_{t+1} + \varepsilon_t^b), \tag{10}
$$

where q_t denotes Tobin's q and δ denotes the depreciation rate of capital.

The aggregate production function is

$$
y_t = \phi_p \Big[\alpha k_t^s + (1-\alpha)\ell_t + \varepsilon_t^a\Big], \tag{11}
$$

where k_t^s denotes capital service; ε_t^a, total factor productivity; α, the share of capital in production; and ϕ_p, one plus the share of fixed costs in production, reflecting the presence of fixed costs in production, $\phi_p = 1 + \Phi / y$.

The capital service, k_t^s, is

$$
k_t^s = k_{t-1} + u_t, \tag{12}
$$

where k_{t-1} denotes the capital stock at the end of period $t-1$.

The utilization rate, u_t, is determined by

$$u_t = \frac{1-\psi}{\psi} r_t^k, \tag{13}$$

where ψ denotes a positive function of the elasticity of the capital utilization adjustment cost function and is normalized to be between zero and one.

The evolution of the capital stock, k_t, becomes

$$k_t = (1-\delta)k_{t-1} + \delta i_t + \delta\left[\left(1+\beta\right)\varphi\right]\varepsilon_t^i. \tag{14}$$

The definition of the price markup, μ_t^p, is

$$\mu_t^p = \alpha\left[k_t^s - \ell_t\right] + \varepsilon_t^a - w_t. \tag{15}$$

The New-Keynesian Phillips curve with partial indexation is

$$\pi_t = \frac{\iota_p}{1+\beta\iota_p}\pi_{t-1} + \frac{\beta}{1+\beta\iota_p}E_t\pi_{t+1} - \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\iota_p)\xi_p[(\phi_p-1)\varepsilon_p+1]}\mu_t^p + \varepsilon_t^p, \tag{16}$$

where ι_p denotes the degree of indexation to past inflation; ξ_p, the degree of price stickiness; ε_p, the curvature of the Kimball goods market aggregator; and ε_t^p, the price markup shock.

The rental rate of capital is

$$r_t^k = -(k_t^s - \ell_t) + w_t. \tag{17}$$

The definition of the wage markup, μ_t^w, is

$$\mu_t^w = w_t - \left[\sigma_\ell \ell_t + \frac{1}{1-\lambda}\left(c_t - \lambda c_{t-1}\right)\right], \tag{18}$$

where σ_ℓ denotes the elasticity of labor supply with respect to the real wage.

The wage curve with partial indexation is

$$w_t = \frac{1}{1+\beta} w_{t-1} + \frac{\beta}{1+\beta}(E_t w_{t+1} + E_t \pi_{t+1}) - \frac{1+\beta\iota_w}{1+\beta}\pi_t \\ + \frac{\iota_w}{1+\beta}\pi_{t-1} - \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\xi_w[(\phi_w - 1)\varepsilon_w + 1]}\mu_t^w + \varepsilon_t^w, \tag{19}$$

where ι_w denotes the degree of wage indexation; ξ_w, the degree of wage stickiness; ε_w, the curvature of the Kimball labor market aggregator; and ε_t^w, the wage markup shock.

The monetary policy reaction function is

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi \pi_t + r_y(y_t - y_t^*)] + r_{\Delta y}[(y_t - y_t^*) - (y_{t-1} - y_{t-1}^*)] + \varepsilon_t^r, \tag{20}$$

where y_t^* denotes the natural output defined in the flexible price-wage economy.

There are seven exogenous shocks in this economy. These seven driving forces are assumed to follow the following processes:

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a, \tag{21}$$

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b, \tag{22}$$

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i, \tag{23}$$

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a, \tag{24}$$

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p, \tag{25}$$

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w, \tag{26}$$

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r, \tag{27}$$

where η_t^g, η_t^b, η_t^i, η_t^a, η_t^p, η_t^w, and η_t^r are i.i.d. shocks with mean zero.

C Equivalence results

Here, we provide a brief description of the equivalence results in BCA.

The definition of the equivalence employed in this paper is as follows.

Definition 1. *A detailed model is equivalent to (covered by) a prototype model if the prototype model can achieve all realized sequences of consumption, investment, labor, output, and capital stock generated in the detailed model.*

CKM give the so-called equivalence results: the prototype model covers a large class of frictional detailed models. However, they do not specify the types of stochastic process of the wedges that are necessary for the equivalence; note that the VAR(1) specification is often employed when BCA is applied to the actual data.

The vector of wedges, $\boldsymbol{s}_t$, associated with the detailed model should be described as

$$\boldsymbol{s}_t = \boldsymbol{\Phi}\boldsymbol{x}_t, \tag{28}$$

where $\boldsymbol{\Phi}$ is a constant matrix and $\boldsymbol{x}_t$ is a vector of endogenous and exogenous state variables in the detailed model. The state vector $\boldsymbol{x}_t$ evolves according to

$$\boldsymbol{x}_{t+1} = \boldsymbol{\Psi}\boldsymbol{x}_t + \boldsymbol{\nu}_{t+1}, \tag{29}$$

where $\boldsymbol{\Phi}$ is a constant matrix and $\boldsymbol{\nu}_{t+1}$ is a vector of structural i.i.d. shocks with mean zero.

Using the above structure, Bäurle and Burren (2007) and Nutahara and Inaba (2008) investigate the necessary and sufficient condition for the equivalence in the case where the wedges of the prototype model evolve according to the conventional VAR(1) process. They find that in many DSGE economies, the equivalence results do not hold in this case.[2] The intuition of Nutahara and Inaba (2008) yields that the equivalence results do not hold if the number of independent endogenous and exogenous state variables in the detailed model is greater than the number of wedges in the prototype model.

The equivalence result does not hold between our medium-scale DSGE model and the prototype model. The number of independent state variables in our model is 12: six endogenous states (consumption c_{t-1}, capital k_{t-1}, investment i_{t-1}, wage w_{t-1}, infla-

[2]See Theorem 1 of Nutahara and Inaba (2008) for the necessary and sufficient condition for the equivalence.

tion π_{t-1}, nominal interest rate r_t) and six exogenous shocks (technology shock ε_t^a, risk premium shock ε_t^b, government purchase shock ε_t^g, price-markup shock ε_t^p, wage-markup shock ε_t^w, and investment-specific technology shock ε_t^i).[3]

In addition to this problem, there is another source of non-equivalence in our experiment. The evolution of capital stock in the prototype model, (4), is different from that in our detailed economy, (14), because of the investment-specific technology shock.

D Procedure of BCA

D.1 Parameter values of the detailed economy

The parameter values of the detailed economy are given in Table 4. All parameters are the same as those estimated by Smets and Wouters (2007). Smets and Wouters (2007) estimate these parameters based on the Bayesian method using the data of the postwar U.S. economy.

[Insert Table 4]

D.2 Definition of the true wedges

The evolution of the capital stock in the prototype model (4) is different from that in the detailed economy (14). As such, in addition to the misspecification of the stochastic process of wedges, there is a misspecification of the evolution of capital stock in BCA. In order to focus on the misspecification of the stochastic process of wedges, we define the true wedges using the capital stock generated in the prototype economy.

Letting $\hat{k}_t$ be the capital stock generated in the prototype model, the true wedges are calculated by the system that consists of (i) the equilibrium conditions of the detailed economy (7)—(27) and (ii) the equilibrium conditions of the prototype model (1), (5),

[3] We eliminate the monetary policy shock, ε_t^r, since the current nominal interest rate, r_t, contains the information on it.

and the conditions given below:

$$
\begin{aligned}
&(1+\tau_x)(\tau_{x,t}-c_t) \\
&\quad = \beta\left\{\alpha\frac{y}{k}\left(E_t y_{t+1}-\hat{k}_t\right)+(1+\tau_x)(1-\delta)E_t\tau_{x+1}-\left[(1+\tau_x)(1-\delta)+\alpha\frac{y}{k}\right]E_t c_{t+1}\right\}, \\
&y_t = \alpha\hat{k}_{t-1}+(1-\alpha)\ell_t+a_t, \\
&\hat{k}_t = (1-\delta)\hat{k}_{t-1}+\delta i_t.
\end{aligned}
$$

D.3 Wedge decomposition

Our method of wedge decomposition is the same as that employed by CKM. In wedge decompositions, the counterfactual sequences of wedges are constructed as follows. For example, to investigate the contribution of the efficiency wedge, it is assumed to be the same as the measured efficiency wedge with the other wedges being constant over time. The aggregate decision rules are computed under the specification that all wedges except the efficiency wedge are fixed constants and the only uncertainty agents face is over the realization of the efficiency wedge. This is the "theoretically-consistent methodology" that Chari, Kehoe, and McGrattan (2007b) mention.

In the case of "true," we replace the VAR(1) specification of wedges (6) with the true stochastic process of wedges (28) and (29). This true stochastic process of wedges (28) and (29) can be written as the VARMA(p,q) process, and this should be consistent with the results where we employ the prototype model with the VARMA specification of wedges.

E Robustness: Small sample bias

In the main paper, we apply BCA to artificial data with 200 observations. Here, we apply 10,000 observations in order to eliminate the small sample bias. Contrary to the main result, we apply BCA to a sample series in this case.

Table 5 reports the analogue of Table 1 in the main paper.

[Insert Table 5]

Table 6 reports the analogue of Table 2 in the main paper.

[Insert Table 6]

RMSEs are smaller than those in the main text. Then, the fit of the measurement of wedges and the output predictions by BCA improve. However, the difference is small. It is found that measured wedges by BCA can capture the property of the true wedges even under a small sample, as in the experiment in the main paper.

References

[1] Bäurle, G., Burren, D., 2007. A note on business cycle accounting, University of Bern, Discussion Paper 07–05.

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[4] Nutahara, K., Inaba, M., 2008. On equivalence results in business cycle accounting, RIETI Working Paper 09–E–18.

[5] Smets, F., Wouters, R., 2007. Shocks and frictions in U.S. business cycles: A Bayesian DSGE approach. American Economic Review 97, 586–606.

Table 4: Parameter values of the detailed economy

symbol	description	values
σ_c	relative risk aversion	1.39
σ_ℓ	elasticity of labor supply with respect to the real wage	1.92
λ	habit persistence	0.71
ξ_w	wage stickiness	0.73
ξ_p	price stickiness	0.66
ι_w	wage indexation	0.59
ι_p	inflation indexation	0.22
φ	steady state elasticity of the capital adjustment cost	5.48
ψ	elasticity of the utilization adjustment cost	0.54
Φ	fixed cost in production	1.61
ρ	Taylor rule (past interest rate)	0.81
r_π	Taylor rule (inflation)	2.03
r_y	Taylor rule (output (1))	0.08
$r_{\Delta y}$	Taylor rule (output (2))	0.22
$\bar{\pi}$	steady state quarterly inflation	0.81
$100(\beta^{-1} - 1)$	discount factor	0.16
$\bar{\ell}$	(log) steady state labor supply	-0.1
α	share of capital	0.19
δ	depreciation rate of capital	0.025
λ_w	steady state markup in labor market	1.5
ε_p	Kimball aggregators in goods market	10
ε_w	Kimball aggregators in labor market	10
g/y	steady state ratio of government spending to output	0.18
σ_a	std of technology shock	0.45
σ_b	std of risk premium shock	0.24
σ_g	std of government shock	0.52
σ_i	std of investment-specific technology shock	0.45
σ_r	std of monetary shock	0.24
σ_p	std of price markup shock	0.14
σ_w	std of wage markup shock	0.24
ρ_a	persistence of technology shock	0.95
ρ_b	persistence of risk premium shock	0.18
ρ_g	persistence of government shock	0.97
ρ_i	persistence of investment-specific technology shock	0.71
ρ_r	persistence of monetary shock	0.12
ρ_p	persistence of price markup shock	0.90
ρ_w	persistence of wage markup shock	0.97
μ_p	MA parameter of price markup	0.74
μ_p	MA parameter of wage markup	0.88
ρ_{ga}	relationship between technology and government shocks	0.52

Table 5: Cyclical behavior of true and measured investment wedges: A case with a large sample

	mean	std	autocorr.	corr w/ y_t	corr w/ true	RMSE
true	1.0010	0.0248	0.9619	-0.2007	–	–
measured	1.0017	0.0213	0.9770	-0.0040	0.8699	0.0122

Notes: The means, standard deviations, autocorrelations, correlations with current output, correlations with the true investment wedge, and RMSEs are reported. RMSE is the root mean squared error of percentage deviation between the true and measured investment wedges. The results are based on a simulation of 10,000 observations.

Table 6: Cyclical behavior of predicted output by true and measured wedges: A case with a large sample

		mean	std	autocorr.	corr. w/ true	RMSE
Efficiency wedge	true	3.2911	0.1186	0.9558	–	–
	measured	3.3045	0.1176	0.9620	0.9945	0.0056
Labor wedge	true	3.2093	0.0759	0.9563	–	–
	measured	3.2335	0.0805	0.9568	0.9791	0.0091
Investment wedge	true	3.2511	0.0377	0.8775	–	–
	measured	3.2406	0.0253	0.9327	0.7293	0.0085
Government wedge	true	3.2307	0.0360	0.9356	–	–
	measured	3.2348	0.0371	0.9399	0.9960	0.0016

Notes: The means, standard deviations, autocorrelations, correlations with the actual current output, correlations with the output predicted by the true wedges, and the RMSEs are reported. The RMSE is the root mean squared error of the percentage-deviations between the two output predictions by the true and measured wedges. The results are based on a simulation of 10,000 observations.