Reconciling Estimates of Earnings Processes in Growth Rates and Levels*

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Abstract

The stochastic process for earnings is the key element of incomplete markets models in modern quantitative macroeconomics. It determines both the equilibrium distributions of endogenous outcomes and the design of optimal policies. Yet, there is no consensus in the literature on the relative magnitudes of the permanent and transitory innovations in earnings. When estimation is based on the earnings moments in levels, the variance of transitory shocks is found to be relatively high. When the moments in differences are used, the variance of the permanent component is relatively high instead. We show theoretically that the difference can be induced by the fact that earnings at the start or at the end of earnings spells are lower and more volatile than the observations in the interior of earnings histories. Using large administrative datasets from Denmark and Germany, we show that this property of earnings spells quantitatively accounts for the full amount of discrepancy in the estimates. Finally, we show that this property of earnings can induce a substantial upward bias in the estimate of consumption insurance against permanent shocks in the standard incomplete markets model.

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1 Introduction

The central element of many models in modern quantitative macroeconomics with heterogeneous agents is either an exogenously specified or an endogenously determined stochastic process for individual earnings. For example, in the models with incomplete insurance markets the properties of the earnings process serve as key determinants of the evolution of consumption, assets, and other economic choices over the life cycle and across individuals.[1] Following the seminal contribution by Friedman (1957), modern consumption theory recognizes that consumption should respond more to the longer lasting or permanent than to transitory innovations in earnings. This explains the keen interest in the literature in measuring the variances of these components using the variants of the permanent/transitory earnings decomposition pioneered by Friedman and Kuznets (1954) and later found to have sound empirical support in, e.g., MaCurdy (1982), Abowd and Card (1989), and Meghir and Pistaferri (2004).[2] In its basic form, such earnings process can be written as:

$$\begin{aligned} y_{it} &= \alpha_i + p_{it} + \tau_{it} \\ p_{it} &= \phi_p p_{it-1} + \xi_{it} \\ \tau_{it} &= \theta(L)\epsilon_{it}, \end{aligned} \tag{1}$$

where log-earnings y_{it} of individual i at time t consist of the permanent component p_{it}, and the transitory component, τ_{it}. If ϕ_p is close to one, the shocks ξ_{it} are highly persistent (truly permanent if ϕ_p is one), and if $\theta(L) = 1$ (where $\theta(L)$ is a moving average polynomial in the lag operator L), the shocks ϵ_{it} are completely transitory.

In addition to determining equilibrium consumption and wealth distributions, the variance and persistence of the shocks ξ_{it} and ϵ_{it} have important implications for policy design. For example, they are key for determining the optimal design of the bankruptcy code in Livshits, MacGee, and Tertilt (2007), they govern the impact of the welfare system on household savings in Hubbard, Skinner, and Zeldes (1995), stimulus effects of fiscal policy in Heathcote (2005), as well as the optimal design of the tax system in Banks and Diamond (2010) and Farhi and Werning (2012). Moreover, there is great interest in understanding whether the dramatic increase in earnings dispersion over the last few decades in the U.S. is due to the increase in the variances of persistent or transitory shocks, e.g., Gottschalk and Moffitt (1994). This is relevant for understanding why consumption inequality did not increase nearly as much,[3] e.g.,

[1] See, e.g., Deaton (1991), Carroll (1997), Castañeda, Díaz-Giménez, and Ríos-Rull (2003).

[2] A prominent alternative in the literature allows for less persistent shocks but individual-specific trends in labor income. Guvenen (2009) is a leading recent example.

[3] Attanasio, Hurst, and Pistaferri (2012) have recently challenged the evidence for this.

Krueger and Perri (2006), Blundell, Pistaferri, and Preston (2008), Heathcote, Storesletten, and Violante (2010). But even besides the implications for consumption, as important as they are, knowing the stochastic nature of earnings is essential for the design of active labor market policies. For example, Meghir and Pistaferri (2011) suggest that income maintenance policies might be an appropriate response to changes in inequality driven by transitory shocks, while training programs are potentially more relevant to counteract the effects of permanent shocks.

Unfortunately, despite their manifest importance, there is no consensus in the vast existing empirical literature on the sizes of the shocks ϵ_{it} and ξ_{it}. The key problem confronting this literature is that, on the same data, the estimates of the earnings process in equation (1) when using the moments of log-earnings in levels are dramatically different from the estimates obtained when using the moments of log-earnings in differences. This led Heathcote, Perri, and Violante (2010) to conclude that the widely used model of earnings dynamics in equation (1) is misspecified. However, the nature of this potential misspecification is unknown. Consequently, the conclusions of the models that use this income process as a primitive cannot be fully relied upon. Even if this process is used as a primitive due to the lack of a better alternative, there is no consensus on whether the parameter values estimated in levels or differences should be used. Relatedly, in the literature that endogenizes the earnings process (e.g., Huggett, Ventura, and Yaron (2011) and Postel-Vinay and Turon (2010)), it is unclear whether the implied process generated by the model should be compared to the one estimated in the data using the specification in levels or in differences, given that estimating the reduced-form process (1) on the model-generated data does not give rise to the observed discrepancy.

In this paper we uncover an important source of this misspecification. While the mechanism we describe applies to survey-based and administrative data alike, our primary focus is on understanding the source of the discrepancy in large administrative datasets that are becoming central in the literature.[4] These datasets are typically orders of magnitude larger than survey-based ones, they are free of sampling issues, they do not suffer from the typical issues of attrition, except what is due to international migration and death, they are based on administrative sources, such as tax records, and are considered highly reliable and free of issues of systematic non-response or measurement error typically plaguing survey-based data. However, despite these attractive properties, we show that these datasets have features that generically bias the estimates of earnings processes and generate the large discrepancy in the estimates based on moments in growth rates and in levels. Fortunately, we show that it is relatively easy to account for these features in estimation in order to eliminate the discrepancy and to obtain consistent estimates.

[4]Recent contributions include Blundell, Graber, and Mogstad (2015), DeBacker, Heim, Panousi, Ramnath, and Vidangos (2013), Domeij and Flodén (2010), Guvenen, Ozcan, and Song (2014), among others.

Estimation of the parameters of the earnings process in the literature is based on fitting the (entire) set of autocovariance moments for levels or differences of earnings. However, even when estimation is based on the same data, computation of the autocovariance moments in levels and differences is effectively based on different information. To clarify with an extreme example, consider an individual with a single earnings observation in the sample. This observation will contribute to the estimated variance of earnings in levels, but it will not contribute to any moment in differences. More generally, some individual contributions towards the autocovariance moments are not defined because there are no earnings observations before the start of an individual's earnings history, nor subsequent to its end, nor due to missing data in the interior of the earnings history. We show theoretically that the discrepancy in the estimates arises since individual contributions to different autocovariance moments are not defined due to missing data when earnings are taken in levels and in differences, and since earnings observations surrounding missing observations are not random. Indeed, we document that in the data the earnings at the time an individual permanently enters or exits the sample, or the earnings surrounding the missing observations, are systematically different. In particular, they are considerably lower on average and substantially more volatile. This can be expected. For example, the data on earnings are typically recorded at an annual frequency. The person, say, entering the sample for the first time is (statistically) expected to enter in the middle of the year, but may enter at any point throughout the year. Thus, earnings in that year are expected to be lower and have a larger variance than interior earnings observations from contiguous earnings histories. We will show formally below that the low mean and high variance of earnings surrounding missing observations raises the variance of transitory shocks when estimation relies on the moments in levels and the variance of permanent shocks recovered by estimation based on the moments in differences.[5]

We quantitatively assess the magnitude of these biases using large administrative datasets from Denmark and Germany and find that they fully account for the discrepancy between the estimates using data in levels and in differences. The Danish data contain complete earnings histories of each resident of Denmark from 1981 through 2006. The German data are a 2% random sample of social security numbers. For these individuals, the complete earnings history from 1975 through 2008 is available. These samples are sufficiently large to allow analysis at the level of particular age cohorts making it possible to focus on a parsimonious earnings model in (1), sidestepping the issue of modelling cohort effects. Moreover, the large size of

[5]Consistent with this conjecture, we don't find such pronounced differences in the estimated variances of permanent and transitory shocks when using the moments in levels and differences in *wages*, as much of the variability in earnings in our administrative datasets at the start and end of contiguous spells is due to the variability in hours (the results are not reported for brevity).

the data enables reliable estimation when replicating the design of samples typically used in the literature. Specifically, we consider a balanced sample spanning 25 (26) years in German (Danish) data, a sample with 9 or more consecutive observations as in e.g., Browning, Ejrnæs, and Alvarez (2010) and Meghir and Pistaferri (2004), and a sample with 20 or more not necessarily consecutive observations as in e.g., Guvenen (2009). Our smallest Danish sample is comprised of about 67,000 individuals and 1.7 million observations, while our smallest German sample contains about 10,000 individuals with more than 200,000 observations.

Using the unbalanced samples in both datasets, we find, consistently with the literature, a substantially higher estimated variance of permanent (transitory) shocks using the moments of earnings in growth rates (levels). Perhaps more surprisingly, we find that the discrepancy is nearly absent in balanced samples drawn from the two datasets. For the vast majority of individuals in the balanced sample their first year in the sample does not coincide with the first year of their earnings history. Similarly, their last year in the sample mechanically truncates earnings histories, implying that it is not the last year of the earnings spell of individuals in the sample. Thus, the mean and the variance of earnings in the first and the last sample years are similar to those of the other years. By definition, the balanced sample also does not contain missing observations. This suggests that it is the non-randomness of earnings surrounding missing observations in the unbalanced samples that drives the discrepancy between the estimates in levels and differences on the data from the unbalanced samples. However, it is still possible that the earnings processes of individuals in the unbalanced samples are fundamentally different and misspecified in some other way. To exclude this possibility, we proceed in three steps.

First, we quantify the contribution of the low mean and high variance of earnings surrounding missing observations in the unbalanced samples drawn from German and Danish data to the subset of theoretical autocovariance moments on which the identification argument in levels and differences is based, and confirm that they induce the observed discrepancy in the estimates. Second, using unbalanced samples, we drop a few observations at the start and at the end of the earnings history, as well as observations surrounding missing records. We find that estimating the earnings process in levels and in differences on the remaining data yields virtually identical estimates of the variances of permanent and transitory shocks. Third, we simulate artificial data based on these estimates of the earnings process while replicating the structure of the unbalanced samples (by design of this experiment, first and last observations as well as those surrounding missing observations are not systematically different from observations in the rest of the earnings histories). We find no discrepancy of the estimates in levels and differences in these artificial data. We then draw an additional transitory shock ("rare transitory shock") at the start and end of the earnings history and surrounding missing observations to replicate the mean and the variance of earnings in those periods in the data. We find that in this case the estimates of the variance of the permanent and transitory shocks are very different when moments in levels and differences are used, but are very close to those in the data from the corresponding unbalanced samples.

Having established that the rare transitory shocks at the start and end of earnings histories and surrounding interior missing observations are the source of discrepancy in the estimated variances of permanent and transitory shocks in our large administrative datasets using the moments for earnings growth rates or levels, we illustrate the importance of accounting for these shocks for understanding consumption responses to income shocks. We follow Blundell, Pistaferri, and Preston (2008) who use panel data on income and consumption, and estimate consumption insurance coefficients for permanent and transitory idiosyncratic income shocks, i.e., the fraction of those shocks that does not translate into movements in consumption. Using the model-generated data, we find that the presence of rare transitory shocks at the start and end of earnings histories leads to a substantial upward bias in the estimated insurance against permanent shocks. We show theoretically that the bias is driven by the same forces that cause overestimation of the variance of permanent shocks using the earnings moments in growth rates. The rare transitory (and consequently highly insurable) shocks are effectively "misinterpreted" by those moments as being permanent. Quantitatively, the size of the bias is sensitive to the details of the measurement strategy, such as the weighting matrix used and whether the degree of insurance is allowed to vary over the life cycle. Interestingly, the choice of the weighting matrix also determines whether consumption data provide any additional (to earnings data) identification power for the size of permanent and transitory shocks in earnings.

The rest of the paper is organized as follows. In Section 2 we discuss identification of the permanent-transitory decomposition of earnings, and derive theoretically the biases in the estimated variances of permanent and transitory shocks when using the moments in levels and differences constructed from an unbalanced panel. In Section 3 we describe the data and the estimation procedure. In the same section we present basic estimation results and document that earnings are typically lower and more volatile in the periods surrounding missing observations. In Section 4 we show that this property of earnings quantitatively accounts for the difference in estimates of earnings processes in levels and differences. In Section 5 we calibrate the standard incomplete markets model and study theoretically and quantitatively the bias induced by this property of earnings on the insurance coefficients against permanent and transitory shocks. We also assess under what conditions the use of consumption data, in addition to earnings data, helps identify the magnitudes of permanent and transitory shocks.

Section 6 concludes.

2 Sources of the Differences

Estimation of the parameters of the earnings process in the literature typically relies on the minimum-distance method. In particular, estimation based on the moments in levels targets the entire set of autocovariance moments in levels $E[y_{it}y_{it+\tau}]$, where $i \in [1, N]$ denotes individuals in the sample, t denotes time, and τ denotes all the leads and lags of earnings observed in the data. In differences, estimation targets the full set of autocovariance moments in differences $E[\Delta y_{it}\Delta y_{it+\tau}]$, where Δ is the difference operator between two consecutive observations, so that $\Delta y_{it} \equiv y_{it} - y_{it-1}$.

While all available autocovariance moments are used in estimation, the identification is usually established using only a subset of autocovariance moments, e.g., Blundell, Pistaferri, and Preston (2008), Hryshko (2012) and Meghir and Pistaferri (2004). For example, consider the earnings process that consists of a random walk and an iid transitory shock—this corresponds to setting $\theta(L)$ and ϕ_p to 1 in equation (1). This process was considered in Heathcote, Perri, and Violante (2010) who proposed the following sets of difference moments to identify the variances of permanent and transitory shocks at time t:

`Differences:`

$$\sigma^2_{\xi,t} = E[\Delta y_{it}\Delta y_{it-1}] + E[\Delta y_{it}\Delta y_{it}] + E[\Delta y_{it}\Delta y_{it+1}], \tag{D1}$$

$$\sigma^2_{\epsilon,t} = -E[\Delta y_{it}\Delta y_{it+1}]. \tag{D2}$$

Note that (D1) and (D2) represent linear combinations of autocovariance moments for income growth rates. For clarity, we will refer to individual autocovariance moments as simply "moments," and to a linear combination of autocovariance moments used for identification such as (D1) and (D2) as "identifying moments."

Expanding (D1) and (D2), we obtain the identifying moments for the variances of permanent and transitory shocks in levels at time t:

`Levels:`

$$\sigma^2_{\xi,t} = E[y_{it}y_{it+1}] - E[y_{it+1}y_{it-1}] - E[y_{it}y_{it-2}] + E[y_{it-1}y_{it-2}], \tag{L1}$$

$$\sigma^2_{\epsilon,t} = E[y_{it}y_{it}] - E[y_{it}y_{it+1}] - E[y_{it-1}y_{it}] + E[y_{it-1}y_{it+1}]. \tag{L2}$$

As identifying moments (D1)-(D2) and (L1)-(L2) are based on exactly the same earnings information, they are expected to deliver identical estimates of the variance of permanent and transitory shocks at time t in a sample of individuals whose earnings are non-missing for the periods $t-2$ through $t+1$.[6]

Importantly, each autocovariance moment is measured as the average across all available observations that contribute to it. Specifically, any generic moment $E[x_{it}]$ is calculated as $\frac{1}{N_s}\sum_{i=1}^{N} x_{it}$, where N is the total number of observations, N_s is the number observations with non-missing information on moment x_{it}, and x_{it} is set to zero for observations that do not contribute to it. For example, an individual with two consecutive earnings observations at t and $t+1$ will contribute to estimation of the autocovariance moments $E[y_{it}y_{it}]$, and $E[y_{it}y_{it+1}]$, but not to the construction of the autocovariance moments $E[y_{it}y_{it-1}]$, and $E[y_{it}y_{it\pm\tau}]$ for $\tau \geq 2$. Similarly, if an individual lacks an earnings observation in, say, period t, he does not contribute to the construction of any autocovariance moment involving period t. For example, for an individual who first enters the sample in period $t+1$, there is no observation on y_t and consequently this individual does not contribute to the construction of any autocovariance moment that involves his time-t earnings.

Thus, while the identifying moments (D1)-(D2) and (L1)-(L2) are based on the same information in balanced samples, the autocovariance moments used in estimation of (D1)-(D2) and (L1)-(L2) are computed using different information in unbalanced data. To take an extreme example, consider an individual who appears in the sample only once, in period t. This individual will contribute to the autocovariance moment $E[y_{it}y_{it}]$ and thus his only earnings observation will affect the identifying moment (L2) but it will not contribute to any autocovariance moment used to construct the corresponding identifying moment in differences (D2). If earnings of individuals who appear in the sample only once are systematically different, this will induce the difference between identifying moments (L2) and (D2) and lead to different estimates of the variance of transitory shocks using the moments in levels and differences.

While the preceding example seems pedagogically insightful, empirically we find that the

[6] Note that Heathcote, Perri, and Violante (2010) show that identifying moments in levels can be constructed using fewer autocovariance moments such as

$$\sigma^2_{\xi,t} = E[y_{it}y_{it+1}] - E[y_{it}y_{it-1}], \tag{L1-Short}$$
$$\sigma^2_{\epsilon,t} = E[y_{it}y_{it}] - E[y_{it}y_{it+1}]. \tag{L2-Short}$$

These identifying moments in levels do not, however, use the same information as the identifying moments (D1)-(D2) in differences. For example, the information on earnings in $t-2$ is used in (D1) but not in (L1-Short). The assessment of the sources of biases is more transparent using moments (D1)-(D2) and (L1)-(L2) which rely on the same information. Moreover, as the moments (L1)-(L2) simply represent an expansion of the moments (D1)-(D2), they are identically affected by any other potential misspecification of the earnings process. This allows us to isolate and measure the importance of the high variance and low mean of the observations at the start and end of contiguous earnings observations, which, as we show below, contribute differently to the autocovariance moments on which (D1)-(D2) and (L1)-(L2) are based.

difference between the autocovariance moments used in computing (D1)-(D2) and (L1)-(L2) is driven by the fact that in administrative data, earnings at the time an individual permanently enters or exits the sample, or earnings surrounding the missing observations, are systematically different (they are typically lower and substantially more volatile). As earnings observations at the time individuals (re-)enter and exit the sample contribute differently to the autocovariance moments on which the identifying moments (D1)-(D2) and (L1)-(L2) are based, this leads to systematic differences in estimated variances of permanent and transitory shocks using the moments in growth rates and levels. In the rest of this section we formally describe the associated biases. In subsequent sections we will show that they account for the entire difference in the estimates using the identifying moments in levels and differences.

Consecutive unbalanced samples. Let the first year of a dataset be t_0 and the last year be T. We refer to samples with uninterrupted earnings spells of duration less than or equal to $T - t_0 + 1$ as consecutive unbalanced samples. Consider first the biases arising from missing (individual contributions to the) autocovariance moments upon the start or the end of the earnings history in the sample. As mentioned above, earnings have a lower mean and are highly volatile in the first and last period of earnings history. Consider modeling this through an additional, large transitory shock in the first and last year of an individual's earnings history, that is $y_{it} = p_{it} + \epsilon_{it} + \nu_{it}$, where ν_{it} has mean μ_ν (taking a negative value) and variance σ^2_ν and is uncorrelated with permanent and transitory shocks. For ease of exposition, we also assume that the variance of individual fixed effects is zero.

In levels, the identifying moment (L1) implies that (contribution to) the variance of permanent shocks for individuals starting the earnings history at time t is $E[y_{it}y_{it+1}] = \text{var}(p_{it})$ (because earnings at time $t-1$ and $t-2$ are not available).[7] This will bias the estimated variance upwards as the difference $\text{var}(p_{it}) - \sigma^2_{\xi,t}$ is positive and equals $\text{var}(p_{it-1})$. From (L2), these individuals will contribute $E[y_{it}y_{it}] - E[y_{it}y_{it+1}] = \sigma^2_{\epsilon t} + \mu^2_\nu + \sigma^2_\nu$ to the variance of transitory shocks in levels at time t. This is larger than $\sigma^2_{\epsilon t}$ by the magnitude $\mu^2_\nu + \sigma^2_\nu$, which exclusively emerges from the squared mean and variance of the rare transitory shock at the start of the earnings history. It can be shown that rare transitory shocks do not affect the estimated variances of permanent and transitory shocks in levels for the periods $t+1$, $t+2$, etc.

In differences, individual contributions to the identifying moment (D1) one year after they are first observed in the sample will average to $E[\Delta y_{it+1}\Delta y_{it+1}] + E[\Delta y_{it+1}\Delta y_{it+2}]$, or $\sigma^2_{\xi_{t+1}} + \mu^2_\nu + \sigma^2_\nu + \sigma^2_{\epsilon_t}$. Consequently, their contributions will bias the estimated variance of

[7] This is contribution to the time-t variance of permanent shocks by individuals whose earnings spells start at time t.

the permanent shock at time $t+1$ by the magnitude $\mu_\nu^2 + \sigma_\nu^2 + \sigma_{\epsilon_t}^2$. It can be further shown that there are no biases in the estimated permanent variances at times $t+2$, and onward. Interestingly, the identifying moment (D2) implies that there are no biases in the estimated variances of transitory shocks.

Consider now individuals whose earnings histories end at time t, which differs from the last potential sample year T. In levels, their contribution to the identifying moment (L1), which measures the variance of permanent shocks at t, equals $-\sigma_{\xi_t}^2$. The identifying moment (L2) implies that their contribution to the variance of transitory shocks equals $E[y_{it}y_{it}] - E[y_{it}y_{it-1}]$, or $\sigma_{\xi_t}^2 + \sigma_{\epsilon_t}^2 + \mu_\nu^2 + \sigma_\nu^2$, which is larger than the variance of transitory shocks at t by the magnitude $\sigma_{\xi_t}^2 + \mu_\nu^2 + \sigma_\nu^2$, arising from the variation in the rare shock in the last period of an individual's earnings spell and the variance of permanent shocks at time t. One can show that there are no biases in the estimated permanent and transitory variances in levels at times $t-1$, $t-2$, etc.

In differences, those individuals contribute $E[\Delta y_{it}\Delta y_{it}] + E[\Delta y_{it}\Delta y_{it-1}]$ to the identifying moment (D1), measuring the variance of permanent shocks at t, or $\sigma_{\xi_t}^2 + \sigma_{\epsilon_t}^2 + \mu_\nu^2 + \sigma_\nu^2$, which is larger than the variance of permanent shocks by the magnitude $\sigma_{\epsilon_t}^2 + \mu_\nu^2 + \sigma_\nu^2$. The variances of transitory shocks, estimated with the identifying moments (D2), remain free of biases, and there are no further biases in the variances of permanent shocks in differences for times $t-1$, $t-2$, etc.

Non-consecutive unbalanced samples. We now consider the consequences of missing earnings in the interior points of the earnings history. We assume that individual earnings are realizations of the earnings process (1), with some observations missing in any period $t \in (t_0, T)$. We will show below that such periods are often associated in the data with high variance of earnings in periods $t-1$ and $t+1$. We model this by introducing additional rare transitory shocks with a negative mean μ_ν at the time before and after earnings is missing (ν_{it-1} and ν_{it+1}, respectively) that are assumed to be uncorrelated with permanent and transitory shocks, and each other:[8]

$$\begin{aligned}
&y_{it-1} = p_{it-1} + \epsilon_{it-1} + \nu_{it-1}, \\
&y_{it} \ \text{ missing}, \\
&y_{it+1} = p_{it+1} + \epsilon_{it+1} + \nu_{it+1}.
\end{aligned}$$

In levels, the identifying moment (L1) implies that the contribution of individuals with

[8]For the ease of exposition, we assume that the mean and variance of the rare shock one year before and after earnings are missing are the same, although they are slightly different in the data.

missing earnings at t to the variance of permanent shocks at t equals $E[y_{it-1}y_{it-2}]-E[y_{t-1}y_{it+1}] = -\sigma^2_{\xi_{t-1}}$. This differs from $\sigma^2_{\xi_t}$ by the magnitude $-(\sigma^2_{\xi_{t-1}} + \sigma^2_{\xi_t})$. One year after earnings is missing, their contribution to the identifying moment equals $E[y_{it+1}y_{it+2}] - E[y_{it+1}y_{it-1}] = \sigma^2_{\xi_t} + \sigma^2_{\xi_{t+1}}$, implying a positive bias over the variance of permanent shocks equal to $\sigma^2_{\xi_t}$. The bias equals $\sigma^2_{\xi_{t+1}}$ at time $t+2$, while there are no biases in the estimated variances of permanent shocks at times $t+3$, etc. At time $t-1$, the bias is $-\sigma^2_{\xi_{t-1}}$, while there are no biases in the estimated variances of permanent shocks for times $t-2$, $t-3$, etc.

At the time earnings is missing, from (L2), individuals contribute $E[y_{it-1}y_{it+1}] = \mathrm{var}(p_{it-1})$ to the variance of transitory shocks at t, positively biasing the variance of transitory shocks $\sigma^2_{\epsilon_t}$. One year after earnings is missing, the contribution equals $E[y_{it+1}y_{it+1}] - E[y_{it+1}y_{it+2}]$, or $\sigma^2_{\epsilon_{t+1}} + \mu^2_\nu + \sigma^2_\nu$. Similarly, the bias in the transitory shock one year before earnings is missing equals $\mu^2_\nu + \sigma^2_\nu$. There are no biases in the estimated variance of transitory shocks in levels for the periods separated by at least two years from the period of missing earnings.

In differences, two years after missing earnings, individuals contribute $E[\Delta y_{it+2}\Delta y_{it+2}] + E[\Delta y_{it+2}\Delta y_{it+3}]$ to the identifying moment (D1), or $\sigma^2_{\xi_{t+2}} + \mu^2_\nu + \sigma^2_\nu + \sigma^2_{\epsilon_{t+1}}$, while one year before missing earnings they contribute $E[\Delta y_{it-1}\Delta y_{it-1}] + E[\Delta y_{it-1}\Delta y_{it-2}]$, or $\sigma^2_{\xi_{t-1}} + \mu^2_\nu + \sigma^2_\nu + \sigma^2_{\epsilon_{t-1}}$. This will bias upward the estimated variance of permanent shocks at times $t-1$ and $t+2$. One can show that there is no bias in the estimates of the transitory shock using the identifying moments in differences and the data with missing earnings observations.

Summary. The analysis above yields two major implications. First, one may expect to recover the variance of transitory shocks in differences without any biases using the moments in growth rates. Second, the identifying moments in levels tend to produce upward-biased estimates of the variance of transitory shocks, while the identifying moments in differences produce upward-biased estimates of the variance of permanent shocks. The magnitude of the biases depends positively on the variance of the rare shocks and on the difference between their mean from the mean of the shocks in the rest of earnings histories.

3 Data, Estimation Details, and Basic Results

3.1 Data

In this section we describe the data and construction of the samples that we study. Following the literature, we focus on individuals with a strong attachment to the labor market characterized by sufficiently high earnings and time spent working.[9]

3.1.1 Danish data

Several administrative registers provided by Statistics Denmark were used to construct our samples. The tax register from 1981–2006 provides panel data on total earnings for more than 99.9 percent of Danish residents between the ages of 15 and 70. The register was merged with the Danish Integrated Database for Labor Market Research (IDA) so that additional demographic variables, such as educational status could be appended. The population consists of Danish males born in 1951 through 1955. We observe annual earnings over the period of 1980 through 2006. We first remove all individuals who were ever self-employed and drop records in which an individual is making non-positive labor market earnings. Next we drop records for those individuals who have worked less than 10 percent of the year as a full time employee.[10] Annual earnings in a particular year include all earned labor income, taken from tax records, for that calendar year. This variable is considered "high quality" by Statistics Denmark in that it very accurately captures the earnings of individuals. Earnings are expressed in 1981 monetary units (Danish kroner). We calculate the maximum number of consecutive periods in which an individual has non-missing earnings and use this information to construct two consecutive samples: a sample in which an individual's maximum spell is at least 9 consecutive periods (102,825 individuals), and a sample in which the individual's maximum spell covers the entire 26 periods, hereafter called "balanced" sample (67,008 individuals). For the sample with 9 or more consecutive observations, periods outside of the longest spell are dropped. Within

[9] These selection rules are typical of not only the literature that utilizes survey data but also of the recent literature utilizing administrative data. For example, Guvenen, Ozcan, and Song (2014) use US administrative data on individual wage and salary income and make the following sample selection: "For a statistic computed using data for not necessarily consecutive years $t_1, t_2, \dots, t_n$, an individual observation is included if the following three conditions are satisfied for all these years: the individual (i) is between the ages of 25 and 60, (ii) has annual wage/salary earnings that exceed a time-varying minimum threshold, and (iii) is not self-employed (i.e., has self-employment earnings less than the same minimum threshold). This minimum, denoted $Y_{min,t}$, is equal to one-half of the legal minimum wage times 520 hours (13 weeks at 40 hours per week), which amounts to annual earnings of approximately \$1,300 in 2005. This condition allows us to focus on workers with a reasonably strong labor market attachment and avoids issues with taking the logarithm of small numbers). It also makes our results more comparable to the income dynamics literature, where this condition is standard." Similarly, DeBacker, Heim, Panousi, Ramnath, and Vidangos (2013) "...exclude earnings (or income) observations below a minimum threshold..." and "...take the relevant threshold to be one-fourth of a full-year, full-time minimum wage." In line with our selection of consecutive samples, Blundell, Graber, and Mogstad (2015) "...restrict the sample to individuals with at least four subsequent observations with positive market income."

[10] We use the variable "erhverv" from the IDAP table provided by Statistics Denmark. This variable calculates work experience as a full time employee since 1980 based on individuals' yearly pension contributions and is available for all members of the population (with the exception of those individuals who have spent time abroad for whom the variable is reset to 0). By taking the first difference of this measure, we can calculate the percent of the year an individual has worked full time, which restricts our observation period to 1981–2006.

the longest spell, earnings outliers are defined as an individual with an increase in earnings of more than 500 percent or a fall of more than –80 percent in adjacent years. Individuals with earnings outliers within their longest spell are dropped. The third sample we consider consists of individuals who have at least 20 not necessarily consecutive periods in which they have non-missing earnings (90,668 individuals). We also drop individuals in this sample if they have earnings growth outliers. Finally, we drop individuals if their educational status has changed during the spells considered. Table 1 contains basic statistics for selected years.

3.1.2 German data

We use administrative data from the IABS, a 2% random sample of German social security records for the years 1974–2008. A detailed description of the dataset can be found in Dustmann, Ludsteck, and Schönberg (2009). We use full-time job spells for German males born in 1951–1955, dropping the spells in East Germany. We also drop annual records when an individual was in apprenticeship during any part of the year. Individual real earnings is the sum of earnings from all jobs within a year expressed in 2005 euros. We set individual education to the maximum schooling attained during the sample years, and the number of days worked to the sum of calendar days on all jobs within a year. As individual earnings are right-censored at the highest level subject to social security contributions, we impute earnings exceeding the limit assuming that daily wages in the upper tail follow a Pareto distribution, the parameters of which differ by year and an age group.[11] After 1983, earnings include one-time payments such as bonuses. To make variable definitions consistent throughout, we use only the data since 1984. We further drop individual records on annual earnings if the combined duration of job spells within a year is below 35 calendar days, and records with very low daily earnings.[12] As in the Danish data, we construct three samples—balanced, with 9 or more consecutive, and 20 or more not necessarily consecutive earnings observations—and, similarly to the Danish samples, drop individuals who have earnings growth outliers. The respective samples contain 9,452, 18,130, and 13,635 individuals with 236,300, 379,080, and 330,748 observations, respectively. Table 2 provides some descriptive details for the samples.

[11] We consider the following eight age groups: younger than 25, 6 five-year age groups (ages 25–29, 30–34, up to the group of 50–54 years old), and those who are older than 54. We use a "fixed effects" imputation, keeping a uniform draw for each individual affected by the right-censoring limit fixed, when creating a Pareto variate in different years. We also experimented with imputation based on the assumption that truncated log-wage distribution is normal, and a simpler imputation when daily wage is multiplied by the factor 1.2 if it hits the upper censoring limit—these three imputation methods have been used in Dustmann, Ludsteck, and Schönberg (2009). Our conclusions below are robust to the choice of the imputation method as well as to limiting the sample to individuals whose earnings histories are not affected by the censoring.

[12] The highest marginal part-time income threshold during the sample period was 13.15 euros a day (set for the first time in 2003), and we drop the records with daily earnings below 14 euros in 2003 prices in any year.

3.2 Estimation Details

As is standard in the literature, we estimate the earnings process in equation (1) using the method of minimum distance, fitting the data autocovariance function of log-earnings in levels and first differences to the autocovariance function implied by the model.[13] We allow for an AR(1) transitory component and an unrestricted estimation of the persistence of the permanent component, ϕ_p. We estimate five parameters in total—the persistence and the variance of permanent shocks, ϕ_p and σ_ξ^2; the persistence and the variance of transitory shocks, ϕ_τ and σ_ϵ^2; and the variance of individual fixed effects, σ_α^2.[14] The model, in reduced form, corresponds to an ARMA(2,1) process in levels.[15] The autoregressive part of the reduced form would allow one to identify autoregressive parameters of the persistent and transitory processes, while the MA part, containing two unique parameters, would allow identification of the variance of persistent and transitory shocks; see, e.g., Harvey (1989), for identification of models with unobserved components. We assume that individuals start accumulating permanent and transitory shocks at the age of 25 so that part of the estimated variance of fixed effects captures the accumulated permanent and transitory components prior to that age. We remove predictable variation in earnings by estimating cross-sectional regressions of log earnings on educational dummies, a third polynomial in age, and the interactions of the age polynomial with the educational dummies. Our measure of idiosyncratic earnings, consistent with the literature, is the residual from those regressions. Since our samples are large, we estimate the model using the optimal weighting matrix which is an inverse of the variance-covariance matrix of the data moments.

3.3 Basic Results

In this section, we present estimation results using German and Danish samples with 9 or more consecutive observations, 20 or more not necessarily consecutive observations, and for the balanced samples. We then take a closer look at the nature of unbalanced samples.

3.3.1 Samples with 9 or more consecutive observations

Columns (1)–(4) in Table 3 contain estimation results for the samples with 9 or more consecutive observations. The first two columns use German data. The permanent component is

[13] One of the recent exceptions is Browning, Ejrnæs, and Alvarez (2010) who, apart from selected moments in levels and differences, fit a variety of other data moments studied in the literature on earnings dynamics.

[14] In differences, the variance of fixed effects is not identified.

[15] If the permanent component is a random walk, the reduced form model for log earnings in first differences is an ARMA(1,1) process which is supported, for example, in U.S. data—see MaCurdy (2007).

estimated to be close to a random walk using the moments in differences, while the persistence of the permanent component is estimated to be somewhat lower using the moments in levels. The same pattern can be seen in estimations utilizing Danish data—see columns (3) and (4). Importantly, in both datasets the variance of the permanent shock is more than two times larger in the estimation that uses the moments in growth rates, while the variance of the transitory shock is about two times larger using the moments in levels. Thus, our data clearly exhibit the same large discrepancy that haunts the rest of this literature.

3.3.2 Samples with 20 or more not necessarily consecutive observations

Columns (5)–(8) in Table 3 contain the results for the samples with 20 or more not necessarily consecutive observations. Relative to the results in columns (1)–(4), the variances of persistent shocks are somewhat smaller, while the variances of transitory shocks are similar in magnitude. Importantly, we still observe that estimations using the moments in differences deliver relatively higher estimates of the variance of permanent shocks, while estimations in levels deliver relatively higher estimates of the variance of transitory shocks, once again confirming the widely documented discrepancy.

3.3.3 Balanced samples

Estimation results based on the balanced samples are reported in Table 4. The use of balanced samples results in at least a 50% reduction of the variance of permanent shocks when using the moments in differences—compare, e.g., columns (6) and (8) in Table 3, and columns (2) and (4) in Table 4. There is a similarly striking reduction of at least 50% in the variance of transitory shocks when using the moments in levels—see columns (1) and (3), and (5) and (7) in Table 3, and columns (1) and (3) in Table 4. It appears that the use of balanced samples eliminates the discrepancy between the estimates of the earnings process in levels and differences.

3.4 A Closer Look at Unbalanced Samples

The results of estimation on balanced and unbalanced samples indicate that the discrepancy between the estimates based on the moments in levels and differences is specific to unbalanced samples. One possible explanation for this finding is that individuals with shorter earnings spells are intrinsically different, and that while permanent/transitory decomposition in equation (1) is appropriate for workers in the balanced sample, it provides a fundamentally misspecified model of the earnings processes for individuals in the unbalanced samples.

Alternatively, it is possible that the decomposition is essentially valid but individuals in unbalanced panels either have higher shock variances or are simply a selection of workers who experienced a sufficiently unfavorable earnings "shocks" that pushed them out of employment. One consequence of this selection is that the earnings surrounding the missing observations are likely to belong to workers in transit into or out of employment, with a potentially large impact on earnings in those periods. As discussed in Section 2 this can induce the difference in the estimates of the earnings process in growth rates or levels.

To explore the latter possibility, we ran panel regressions of residual earnings on dummies for the first and last year an individual is observed in the sample. The results are shown in Table 5. Our logic above implies that earnings in the first and last periods in the sample are likely to differ for those individuals whose first/last year in the spell is different from the first/last sample year, while the difference of the first/last observation is not likely to be pronounced for those who are seen in the first/last sample year. The dummies "Year observed: first"−"Year observed: third" equal one if an individual's first earnings record in the sample occurs later than in 1984 in German data and 1981 in Danish data, and zero otherwise. The dummies "Year observed: second-to-last"−"Year observed: last" equal one if an individual's last earnings record is prior to 2008 in German data and 2006 in Danish data, and zero otherwise. Columns (1) and (2) explore the effects in the consecutive unbalanced samples, German and Danish, respectively, while columns (3) and (4) do the same for the non-consecutive samples.

In all samples and both datasets, earnings are more than 0.60 log points lower than an individual's average in the first record of the spell. The last earnings record is below an individual's average by about 0.40 to 0.50 log points. Earnings are still lower in the first and second years following the year of the first earnings record, reverting slightly faster to the individual's average in German data; earnings are also lower in the two years preceding the last earnings record, with more pronounced effects in Danish data. Similarly, earnings are, on average, lower in the years preceding and following a missing earnings record in the non-consecutive samples—see columns (3) and (4). Clearly, the "shock" in the first year of an individual's spell is transitory, but somewhat persistent.[16] Interestingly, the dummies for the few first and last earnings records within a spell explain 5 to 8% of the variation in residual earnings. This number is quite high taking into account that a variety of observable factors normally explain about 30% of variation in earnings.

[16] If the shock were permanent, it would elevate earnings in all periods, with no distinguishable differences of the first earnings record relative to the individual's average. We cannot make the same conclusion for the last period as one would need a history of earnings after an individual's earnings spell is interrupted to argue if the shock is temporary or persistent.

In Table 6, we explore the volatility of idiosyncratic earnings. In columns (1), (3), (5), and (7), the dependent variable is residual earnings squared. In German data, the mean of squared residuals is about 0.16 with a standard deviation of about 0.40 and 0.43 in the consecutive and non-consecutive samples; in Danish data the mean is about 0.12 with a standard deviation of 0.35 and 0.36 in the consecutive and non-consecutive samples, respectively. The results imply that earnings are not only lower on average in the (few) first and (few) last years of individual spells but are also more volatile. As an example, in German data, the mean of squared residual earnings is 0.83 in the first year which is more than a 400% larger than the typical size measured by the mean of squared residual earnings in the sample (we use the estimated coefficient in column (1) on the dummy "Year observed: first" and the mean squared residual of 0.16 for the German data to calculate this number). In the German consecutive sample, about 23% of individuals have their first earnings record after 1984, the first calendar year of the sample, and about 31% of individuals have their last record before 2008, the last year of the sample—see Table 2. The same numbers for Danish data are 18% and 22%, respectively—see Table 1. This is a non-trivial number of individuals with distinct effects on the level and volatility of residual earnings in the few first and last periods of earnings spells. In the non-consecutive samples, earnings in the periods preceding and following interior missing earnings records are also highly volatile—see columns (5) and (7). In German data, for instance, the volatility of earnings observations one year before a missing record is about 150% larger than the volatility of typical earnings observations.[17] Within shorter spells of the non-consecutive samples, the fraction of missing earnings records can be as high as 5% in German data and 14% in Danish data—see Tables 1 and 2.

The size of squared earnings is mechanically higher in the few first and last earnings records since residual earnings are more negative, on average, in those periods, as we illustrated in Table 5. To remove the influence of more negative residual earnings in those periods, we first group individual residual earnings observations by the values of the dummies, and year. We then calculate the means within these groups, remove the means from residual earnings, and square the result. In German data, the mean of squared residual earnings, calculated this way, is about 0.15, while the standard deviation is about 0.36 in the consecutive and 0.39 in the non-consecutive samples; in Danish data, the mean is 0.11 in both samples and the standard deviation is 0.30 and 0.32 in the consecutive and non-consecutive samples, respectively. Not surprisingly, this measure of volatility delivers somewhat lower variation of squared residual earnings. The results with this measure as the dependent variable are in columns (2), (4), (6), and (8) of Table 6. The volatility becomes somewhat lower in the few first and last records of

[17] The number is calculated as $100 \times (0.42/0.16 - 1)$—the estimated effect on the dummy "1 year before earnings missing" in column (5) of 0.42 relative to 0.16, the mean squared residual for the sample of column (5).

individual earnings spells but, still, the size of squared residuals in the first and last periods is substantially above the typical size of squared earnings residuals in both datasets.

Missing observations in applications using small unbalanced panels such as the PSID are typically treated as random. We can explore whether this is the case in our German and Danish samples with 20 or more not necessarily consecutive observations. In Table 7, the dependent variable is a dummy that equals 100 if individual earnings is missing, 0 otherwise. The predictive power of observables—earnings growth rates before and after missing earnings records, together with education dummies and age—on the incidence of missing earnings is small, in line with Fitzgerald, Gottschalk, and Moffitt (1998) who made a similar observation using PSID data. Importantly, missing observations signal about positive earnings growth in the periods following a missing record and negative earnings growth in the periods preceding a missing earnings record implying that individual realizations of residual earnings in the few first and last spell periods, as well as in the few periods preceding and following missing earnings records do not appear to be random draws from the earnings distribution. As pointed out by Moffitt and Gottschalk (2012), little is known about the effect of attrition on the autocovariance function of earnings and, therefore, on the estimates of the earnings process. Our results indicate that the effect can be large. In particular, lower and more volatile earnings observations surrounding missing earnings distort the autocovariance function for growth rates and levels such that the estimated variance of permanent shocks is substantially upward-biased when using the moments for growth rates, while the estimated variance of transitory shocks is substantially upward-biased when relying on the moments in levels.

For completeness, we performed the analysis of Tables 5–6, setting the dummies "Year observed: first"−"Year observed: third" to one if an individual's first earnings record in the sample occurs in 1984 in German data and 1981 in Danish data (the first sample years in German and Danish data), and to zero otherwise; and setting the dummies "Year observed: second-to-last"−"Year observed: last" to one if an individual's last earnings record in the sample happens in 2008 in German data and 2006 in Danish data (the last sample years in German and Danish data), and to zero otherwise. The results, reported in Appendix Table A-1, indicate that the first few earnings records for those individuals whose spells start in the first sample year and the few last earnings records for those individuals whose spells end in the last sample year do not differ substantially on average from earnings in the other years. Table A-2 shows that the volatility of residual earnings is lower for the first few earnings records and higher for the last few earnings records, with the pattern more pronounced in German data, reflecting the increasing life-cycle profile of the variance in earnings (as the first earnings records are likely to happen early in the life cycle, while the last earnings records are likely to happen late in the life cycle)—these effects are, however, small relative to the variance effects of the few first and last earnings records in the spells that start later than the first sample year and/or end earlier than the last sample year.

4 Quantitative Evaluation of the Mechanism

4.1 Direct Evaluation of the Biases using the Permanent-Transitory Decomposition Moments

To evaluate the contribution of outlying observations to the estimated variances of permanent and transitory shocks in levels and differences, we calculate the variances in accordance with (L1), (L2) and (D1)–(D2). As an example, we calculate an estimate of the permanent variance at time t using the identifying moment in levels (L1) as

$$\sigma^2_{\xi,l,t} = \frac{\sum_i y_{i,t}y_{i,t+1}}{\sum_i I^i_{t,t+1}} + \frac{\sum_i y_{i,t-2}y_{i,t-1}}{\sum_i I^i_{t-2,t-1}} - \frac{\sum_i y_{i,t+1}y_{i,t-1}}{\sum_i I^i_{t-1,t+1}} - \frac{\sum_i y_{i,t}y_{i,t-2}}{\sum_i I^i_{t-2,t}}, \tag{2}$$

where the subscript l indicates that we are estimating the variance using information on log-earnings in levels, and $I^i_{t,t'}$ is an indicator function taking the value of one if individual earnings observations are non-missing in both years t and t', and zero otherwise. Note that individual i's contribution to the variance of the permanent shock at time t is set to missing only if all of the earnings cross-products for that individual—$y_{it}y_{it+1}$, $y_{it-2}y_{it-1}$, $y_{it+1}y_{it-1}$, and $y_{it}y_{it-2}$—are missing.

Let I^m_{it} be an indicator function that equals one at the times $t = t_m$ when individual i's earnings residual is missing and the years t_{m+j} surrounding it ($j = \pm1, \pm2, \pm3$), and zero in all other periods $t \neq t_m$ and $t \neq t_{m+j}$. We calculate the variance of permanent shocks due to outlying observations surrounding the missing earnings records, $\sigma^2_{\xi,l,o,t}$, as

$$\sigma^2_{\xi,l,o,t} = \frac{\sum_i y_{i,t}y_{i,t+1}\cdot I^m_{it}}{\sum_i I^i_{t,t+1}\cdot I^m_{it}} + \frac{\sum_i y_{i,t-2}y_{i,t-1}\cdot I^m_{it}}{\sum_i I^i_{t-2,t-1}\cdot I^m_{it}} - \frac{\sum_i y_{i,t+1}y_{i,t-1}\cdot I^m_{it}}{\sum_i I^i_{t-1,t+1}\cdot I^m_{it}} - \frac{\sum_i y_{i,t}y_{i,t-2}\cdot I^m_{it}}{\sum_i I^i_{t-2,t}\cdot I^m_{it}}. \tag{3}$$

An estimate of the permanent variance in levels, net of the effects of outliers, $\sigma^2_{\xi,l,n,t}$, can then be calculated as

$$\begin{aligned}\sigma^2_{\xi,l,n,t} = &\frac{\sum_i y_{i,t}y_{i,t+1}\cdot(1-I^m_{it})}{\sum_i I^i_{t,t+1}\cdot(1-I^m_{it})} + \frac{\sum_i y_{i,t-2}y_{i,t-1}\cdot(1-I^m_{it})}{\sum_i I^i_{t-2,t-1}\cdot(1-I^m_{it})}\\ &- \frac{\sum_i y_{i,t+1}y_{i,t-1}\cdot(1-I^m_{it})}{\sum_i I^i_{t-1,t+1}\cdot(1-I^m_{it})} - \frac{\sum_i y_{i,t}y_{i,t-2}\cdot(1-I^m_{it})}{\sum_i I^i_{t-2,t}\cdot(1-I^m_{it})}.\end{aligned} \tag{4}$$

We can similarly define the variances of permanent and transitory shocks in levels and differences for the consecutive unbalanced panels—e.g., the permanent variance utilizing all sample information ($\sigma^2_{\xi,l,t}$ for levels and $\sigma^2_{\xi,d,t}$ for differences), the permanent variance due to outlying observations in the first few and last few periods of an individual's earnings spell ($\sigma^2_{\xi,l,o,t}$ for levels and $\sigma^2_{\xi,d,o,t}$ for differences), and the permanent variance net of outlying effects ($\sigma^2_{\xi,l,n,t}$ for levels and $\sigma^2_{\xi,d,n,t}$ for differences).

We present the estimates of those variances, averaged across all sample years, for both datasets in Table 8. For German data, in the consecutive sample, the estimates of the variance of permanent shocks in levels and differences using all sample information are 0.013 and 0.024, respectively.[18] When we drop outliers, the estimated net variances are $\hat{\sigma}^2_{\xi,l,n} = 0.010$ in levels and $\hat{\sigma}^2_{\xi,d,n} = 0.010$ in differences. The unadjusted variances of transitory shocks in levels and differences are estimated at 0.020 and 0.008, respectively, while the variances net of outliers in levels and differences are both estimated at 0.007. The results for Danish data are qualitatively similar. Clearly, the discrepancy between the estimates of permanent and transitory shocks in levels and differences is virtually eliminated when netting out the effects of outlying observations on the estimated variances.

For German data, in the non-consecutive sample, the variances of permanent shocks are $\sigma^2_{\xi,l} = 0.0096$, $\sigma^2_{\xi,l,n} = 0.0097$, $\sigma^2_{\xi,d} = 0.018$, $\sigma^2_{\xi,d,n} = 0.0097$, while the variances of transitory shocks are $\sigma^2_{\epsilon,l} = 0.018$, $\sigma^2_{\epsilon,l,n} = 0.007$, $\sigma^2_{\epsilon,d} = 0.007$, $\sigma^2_{\epsilon,d,n} = 0.007$. Netting out the influence of missing observations and the influence of the first and last records in the earnings spells eliminates most of the discrepancy between the variances of permanent and transitory shocks in differences and levels.

4.2 Restricting Unbalanced Samples

In the previous section, we found that the earnings of a few first and last observations, as well as a few observations before and after missing earnings records, are likely to be substantially lower than an individual's average, and are more volatile. In Table 9, we repeat our analysis of Table 3, dropping the first 3 observations for individuals whose earnings spells start later than 1984 in German data and later than 1981 in Danish data, and and last 3 observations for individuals whose earnings spells end earlier than 2008 in German data and 2006 in Danish data, as well as dropping 3 observations before and after a missing earnings record in the

[18] The estimates deviate from the values in Table 3 because we do not impose the exact permanent-transitory decomposition on the data in the minimum-distance estimation of Table 3; the difference between the estimated variance of permanent shocks in levels and differences is not as drastic as in Table 3 because the estimated persistence of the permanent shocks in levels is estimated to be lower than in differences in the minimum-distance estimation.

non-consecutive samples.

For the sample with 9 or more consecutive observations this barely affects the persistence of permanent shocks, while the variance of transitory (permanent) shocks is substantially reduced in estimations utilizing the moments in levels (growth rates). In German data, the variance of permanent shocks is reduced by about 70%, while the variance of transitory shocks is reduced by about 50%. Similar reductions can be seen in the Danish sample.

Dropping the first 3 and last 3 observations, as well as 3 earnings records before and after a year of missing earnings within an individual's earnings spell has a similar effect on the estimated earnings process in the non-consecutive samples—see the results in columns (5)–(8) of Table 9: the variance of the permanent (transitory) shock is reduced substantially in estimations using growth rates (levels). As a result, the estimated earnings process is virtually identical in estimations utilizing the moments for growth rates and levels in Table 9, columns (5)–(6) and (7)–(8) for German and Danish data, respectively.

A comparison between Tables 3 and 9 also indicates, consistently with the analysis in Section 2, that in both datasets the variance of the permanent component is more robustly estimated using the moments in levels while the variance of the transitory component, exclusive of the transitory variation in earnings due to rare shocks, is more robustly estimated using the moments in differences.

4.3 Simulation

Finally, we present a suggestive simulation, consistent with German data, aimed at replicating the results for the consecutive and non-consecutive samples presented above. We replicate our German unbalanced samples with 9 or more consecutive observations and 20 or not necessarily consecutive observations in terms of the number of person-year observations, and assume, consistently with Tables 5 and 6, that incomes in the spells starting (ending) in the years other than the first (last) year of the sample are, in addition, affected by a transitory shock, which has a negative mean and high variance.

For the consecutive sample, we assume that persistence of the permanent component equals 0.988, the variance of permanent shocks is 0.0056, persistence of the transitory component is 0.250, the variance of transitory shocks is 0.010, and the variance of fixed effects is 0.02. These are the estimates of the transitory component using the moments in growth rates and permanent component using the moments in levels in Table 9. We assume that the shocks and fixed effects are drawn from Student t-distributions with four degrees of freedom as our samples have high excess kurtosis.[19] We take the means and variances of the rare shocks in the

[19]Battistin, Blundell, and Lewbel (2009) document the departure of log-income from normality using survey

first and last 3 periods from columns (1) and (2) of Table 5 and 6, and assume that they are independent and normally distributed.[20] The results, averaged across 100 simulations, are in Table 10. Utilizing the full sample results in overestimation of the variance of the permanent (transitory) shock in differences (levels), and it appears that the permanent component is more robustly estimated utilizing the moments in levels while the transitory component is closer to the truth utilizing the moments in differences. Interestingly, our full-sample estimation results are similar to the data results in Table 3. Dropping the first and last 3 observations in an individual's spell aligns the results in levels and differences—see columns (3) and (4) and correctly recovers the parameters of the underlying earnings process.

For the non-consecutive sample, we assume that the persistence of the permanent component is 0.996, the variance of permanent shocks is 0.0040, the persistence of the transitory component is 0.267, the variance of transitory shocks is 0.009, and the variance of fixed effects is 0.02. This is in line with the estimated permanent component in column (5) and transitory component in column (6) of Table 9. We take the means and variances of rare shocks in the first and last 3 periods, and one year before and after missing earnings records, from columns (3) and (6) of Table 5 and 6, and assume that they are independent. The reported results are averages across 100 simulations. The full-sample estimation results in estimates close to the data estimates in Table 3, and recovers fairly well the permanent component using the moments in levels, and the transitory component using the moments in differences—columns (5) and (6) of Table 10. Dropping the first and last 3 observations in an individual's spell, as well as observations surrounding interior missing records, once again aligns the results in levels and differences as is confirmed in columns (7) and (8).

5 Implications for a Life-Cycle Model of Consumption with Incomplete Insurance Markets

In this section we study several implications of our findings for life-cycle consumption models with earnings heterogeneity and incomplete insurance markets. First, we are interested in whether the presence of additional transitory earnings deviations in the first and last observations of earnings spells, and next to the interior missing observations, affects the estimates of the insurance coefficients against permanent and transitory shocks proposed in Blundell, Pistaferri, and Preston (2008). As discussed in the Introduction, these estimates represent

data from the PSID. Assuming normal shocks instead has no impact on our findings.

[20]We assume a normal distribution rather than a t-distribution for rare shocks since the choice of a t-distribution with high variance and low degrees of freedom would sometimes result in large residual draws uncharacteristic of our empirical distribution of residuals.

state-of-the-art measures of insurance available to households in the data, providing the key benchmark for assessing the performance of incomplete markets models. Clearly, getting the amount of insurance available to the agents correctly is key for many substantive implications of these models and for assessing, e.g., welfare implications of various economic shocks and policies.

Second, we assess whether having access to consumption data can help recover the true variances of permanent and transitory shocks to earnings in the data that feature rare transitory earnings shocks at the start and end of the observed earnings spells. The approach based on combining earnings and consumption data (and sometimes the structural model) is demanding in its data requirements but is considered more capable of measuring the stochastic structure of earnings.[21]

5.1 Insurance Coefficients

We first introduce the insurance coefficients. In the standard consumption-savings model, if the Euler equation holds at equality, consumption growth, Δc_{it}, can be expressed as

$$\Delta c_{it} = \phi \xi_{it} + \psi \epsilon_{it},$$

where $1 - \phi$ is the amount of insurance of permanent shocks and $1 - \psi$ is the amount of insurance of transitory shocks. Blundell, Pistaferri, and Preston (2008) show that the age profiles of insurance of permanent and transitory shocks can be recovered using the following data moments:

$$\text{Permanent insurance:}\ \ 1 - \hat{\phi}_t = 1 - \frac{E\left[\Delta c_{it} \sum_{j=-1}^{1} \Delta y_{it+j}\right]}{E\left[\Delta y_{it} \sum_{j=-1}^{1} \Delta y_{it+j}\right]}, \tag{5}$$

$$\text{Transitory insurance:}\ \ 1 - \hat{\psi}_t = 1 - \frac{E\left[\Delta c_{it} \Delta y_{it+1}\right]}{E\left[\Delta y_{it} \Delta y_{it+1}\right]}, \tag{6}$$

where the expectation (averaging) is taken over all individuals observed at age t. Since the sample sizes utilized in the literature are typically small, and the above identifying moments may be imprecise in small samples, it is common to rely on a minimum-distance procedure for

[21] Some prominent contributions include Blundell and Preston (1998), who, assuming the permanent income hypothesis, infer the variances of permanent and transitory shocks to incomes in the U.K. using income and consumption data; Guvenen and Anthony A. Smith, Jr. (2014) who use a parameterized self-insurance model of consumption over the life cycle to estimate the income process; and Blundell, Pistaferri, and Preston (2008) who use consumption and income data in the U.S. and estimate the variances of permanent and transitory shocks to incomes without imposing any particular model of consumption on the data.

estimation of the model parameters, which utilizes all of the available autocovariance moments in the data. This is, e.g., the route taken in Blundell, Pistaferri, and Preston (2008).

5.2 The Biases in Estimating Insurance Coefficients Due to Presence of Rare Shocks

Since the denominator in equations (5)–(6) utilizes information on earnings data only, we can use our results on biases in the estimated variances of permanent and transitory shocks to earnings from Section 3 to find the biases in the estimated insurance coefficients.

For a set of individuals whose income and consumption spells start at age t, the bias in the estimated permanent insurance at age $t+1$ will equal $\lambda\phi_{t+1}$, while for a set of individuals whose income and consumption spells end at age t the bias will equal $\lambda(\phi_t - \psi_t)$, where $\lambda = \frac{\mu_\nu^2+\sigma_\nu^2+\sigma_\epsilon^2}{\mu_\nu^2+\sigma_\nu^2+\sigma_\epsilon^2+\sigma_\xi^2}$, and μ_ν and σ_ν^2 are the mean and variance of the rare shock, respectively. We assume that the insurance of the rare shock ν_{it} is the same as the insurance of the transitory shock ϵ_{it}, and equals ψ_t. For the non-consecutive unbalanced samples, the bias in the estimated permanent insurance two years after missing earnings equals $\lambda\phi_{t+2}$, while the bias in the estimated insurance one year before missing earnings equals $\lambda(\phi_{t-1} - \psi_{t-1})$. The biases are unambiguously positive and potentially large if $\phi \gg \psi$ (which is true for the self-insurance life-cycle model of consumption), and λ is large (in case the mean and/or the variance of the rare shock are large). It can be shown that the transitory insurance estimated using equations (5)–(6) is not systematically biased in our set-up as can be confirmed in Figure 1.

5.3 Quantitative Assessment of the Biases in the Estimated Insurance Coefficients Due to Presence of Rare Shocks

In this section we assess the biases in the estimated insurance coefficients due to transitory rare shocks, using consumption and income data simulated from a standard incomplete-markets model. The model and its calibration are standard and are relegated to Appendix I. We first establish a benchmark by analyzing the model with the income process in equation (1), i.e., the income process with no rare shocks. We will then analyze the consequences of adding rare transitory shocks at the beginning and the end of contiguous earnings histories. For the benchmark calibration with no rare transitory shocks we use the stochastic process for labor income in equation (1), which is parameterized as follows: the permanent component is a random walk, with the variance of permanent shocks equal to 0.004, and the transitory component is an iid shock, with a variance of 0.012—these numbers are in agreement with the estimates for unbalanced samples in German data. Shocks are drawn from a Student t-distribution with 4 degrees of freedom to make a closer correspondence to high excess kurtosis observed in the administrative earnings data. We use natural borrowing constraints in all our calibrations.[22]

We explore several ways of recovering the insurance coefficients and the income process parameters which should be equivalent in large balanced datasets. First, we follow Blundell, Pistaferri, and Preston (2008) who use a minimum-distance procedure and utilize a diagonal weighting matrix for weighting the moments. The diagonal weighting matrix is, however, not the only choice in the literature. An influential study by Altonji and Segal (1996) implies that using an identity weighting matrix might be appropriate in applications with small samples. Below, we will evaluate the age profiles of insurance of permanent and transitory shocks, and the average insurance over the life cycle, using a minimum-distance procedure with different weighting matrices (identity, diagonal, and optimal), as well as the estimates obtained by using the identifying moments in equations (5)–(6). In the spirit of minimum-distance estimation, we calculate the numerator of equation (5) as the sum of 3 cross-autocovariances, $E[\Delta c_{it}\Delta y_{it-1}]$, $E[\Delta c_{it}\Delta y_{it}]$, and $E[\Delta c_{it}\Delta y_{it+1}]$; an individual does not contribute to estimation of any particular autocovariance moment if the individual lacks observation on consumption or income growth rate needed for its calculation. The denominator is calculated as the sum of 3 income growth autocovariances. The identifying moment in equation (6) is calculated analogously. Below, we call the estimates of insurance of permanent and transitory shocks calculated this way "BPP, moments."

The results based on a "large" sample of 10,000 simulated individual histories are presented in Figure 1 and Table 11. As the appropriate choice between the weighting matrices is thought to depend on the sample size, we also present a complete set of results on a "small" sample of about 2,000 individuals in Appendix Table A-3. The design of Tables 11 and A-3 is as follows: columns correspond to various specifications of the income process and the weighting matrices used. Rows are broken up into three panels.

- In Panel A, we use simulated data from the model, and estimate variances of permanent and transitory shocks—reported in rows (1)–(2) as well as insurance coefficients for permanent and transitory shocks at each age.[23] Life-cycle averages of these insurance coefficients are in rows (3)–(4).

[22]Using zero borrowing constraints biases the insurance coefficients for permanent shocks downward, as highlighted in Kaplan and Violante (2010), but since our focus is on the potential biases due to the presence of rare transitory shocks we choose the credit environment resulting in unbiased estimates of insurance of permanent and transitory shocks under the standard income process.

[23]The model parameters are identified by matching the theoretical autocovariance function of income growth rates, consumption growth rates, and cross-covariances of income and consumption growth rates to the same moments estimated from the model-generated data.

- In Panel B, we use the same simulated data but assume in estimation that the amount of insurance of permanent and transitory shocks is constant over the life cycle. It may be difficult to estimate insurance coefficients for all ages using typically available small samples from noisy survey data. We therefore explore the consequences of the extreme version of dealing with this problem, by restricting the insurance of permanent and transitory shocks to a constant value. Rows (5)–(6) contain the resulting estimates of the variances of permanent and transitory shocks, while rows (7)–(8) contain the corresponding estimates of the insurance coefficients.

- In Panel C, we use only earnings data when estimating variances of permanent and transitory shocks relying on the moments in differences. The estimates are reported in rows (9)–(10).

To establish a benchmark, we first consider the balanced samples from the model-generated data. The results are reported in Columns (1)–(3) of Table 11 and Figures 1(a)–1(b). The income process and insurance parameters are recovered without any biases using different weighting matrices, and the estimated insurance, both for permanent and transitory shocks, lines up well with the estimates of true insurance by age. BPP moments for permanent insurance, having a somewhat noisy pattern, do not deviate from the true permanent insurance on average. The income process parameters are well recovered even if we assume that the amount of insurance of permanent and transitory shocks is constant over the life cycle, or when we only use the income moments in estimation—rows (5)–(6), and (9)–(10), respectively. Using the diagonal weighting matrix and assuming that insurance of permanent and transitory shocks is constant over the life cycle, however, results in substantial overestimation of the insurance of permanent shocks—the true insurance estimate implies that 13% of permanent shocks are insured by means of self-insurance over the life cycle, while the estimate in row (7), column (2) implies that twice as much of permanent shocks gets insured over the life cycle.

The main results of this section are reported in columns (4)–(6) Table 11 and Figures 1(c)–1(d). They are based on the unbalanced samples that mimic the properties of a sample with 9 or more consecutive observations in our German data. In particular, the model now includes additional transitory shocks with low mean and high variance at the beginning and the end of selected earnings histories in the sample. We follow a simple procedure that allows us to replicate properties of such samples in the data.[24] Specifically, the income process is modified to include a 2% probability of receiving a rare transitory shock with mean –0.50 and variance 0.30. The choice of the mean and variance is motivated by the numbers in Table 5, column (1),

[24] Other procedures to generate a sample that mimics features of the data are possible, but the substantive results should not be affected by this choice.

and Table 6, column (2). The decision rules on consumption incorporate information on the frequency and the size of these shocks. We then simulate consumption and income data, and record the number of times an individual receives a rare transitory shock during the working part of the life cycle. If the individual receives more than one rare transitory shock (about 19% of individuals), we truncate the earnings history before the first rare shock and after the last rare shock. If an individual gets exactly one rare shock over the working career (about 37% of individuals), we set incomes to missing after the rare shock happens for 50% of such individuals and before the rare shock happens for another 50% of those individuals (unless the shock appears at age 26 or 65, about 2% of individuals, in which case we keep the entire earnings histories). We also set consumption to missing when income is missing. We retain the full consumption and income histories for the individuals who never experienced rare transitory shocks (about 44% of individuals). Finally, we restrict the sample to histories that include 9 or more consecutive observations. Our choice of the frequency of rare shocks, together with the selection procedure, closely matches the proportion of observations coming from the first and last spell years (which differ from the first and last years in the sample) in the German data.

In Panel C of Table 11, we use income data only and find, consistently with the results reported above, that the estimated variance of permanent shocks is substantially larger than the true variance of 0.004, regardless of the weighting matrix used. Since, by construction of the sample, the majority of rare shocks happen either in the beginning or in the end of the earnings spell, the estimated variance of transitory shocks recovers well the true variance of continuous transitory shocks, which agrees with the arguments in Section 2. The results in Panel A indicate that using consumption and income data jointly, and the diagonal or optimal weighting matrices, nicely recovers the variance of permanent shocks, as well as the insurance of transitory shocks. However, using the diagonal matrix produces upwardly biased estimates of the insurance against permanent shocks—insurance of 25% of the permanent shock relative to the true insurance of about 14%. While this bias is fairly large, it becomes striking when the identity weighting matrix is used. In that case, the insurance of permanent shocks is estimated at about 60%. The moment-based estimates of permanent insurance are even more upwardly biased than the estimates obtained using a minimum-distance estimation with the identity weighting matrix. The insurance of transitory shocks is also overestimated when the identity weighting matrix is used—see the age profiles of the insurance coefficients for permanent and transitory shocks in Figures 1(c)–1(d). It also appears that consumption data are not informative in recovering the true variance of permanent shocks when using the identity weighting matrix—we obtain a similar estimate when using consumption and income data jointly, or using income data alone (compare panels A and C, column (4)). Assuming that the insurance of permanent and transitory shocks is constant over the life cycle, panel B, results in an additional upward bias in the estimated insurance of permanent shocks using optimal and diagonal weighting matrices, with the relatively larger bias for the diagonal weighting matrix.

In columns (4)–(6) individual consumption decisions incorporate information on rare shocks. While there is no reason to believe that these shocks in administrative data are noise, one may expect that estimation involving consumption and income data will perform much better if those shocks were pure noise as consumption data, being unresponsive to measurement error in income, will likely filter out the noise in income. To assess this conjecture, in columns (7)–(9), we construct the unbalanced samples to mimic German data following the procedure above but replace the rare transitory shocks with pure measurement error in income (with the same probability of occurrence, mean and variance as the rare transitory shocks have), to which consumption does not respond. Interestingly, all of the results are almost identical to the results for the true rare transitory shocks, to which individual consumption optimally responds. The age profiles of insurance of permanent and transitory shocks are depicted in Figures 1(e)–1(f) and are similar to the profiles for the case with rare transitory income shocks in Figures 1(c)–1(d).

While our main experiment assumes that rare transitory shocks are pure risk, to which households optimally react, part of those "shocks" might be potentially predicted by individuals in the data. The fact that consumption is typically found to be insensitive to transitory shocks makes it hard to separate, without data on individual expectations, transitory risk from the transitory variation in earnings predicted by individuals. The analysis of Section 5.2 (and the results for the measurement-error experiment) imply, however, that the bias in the estimated permanent insurance is driven by the properties of earnings which do not depend on whether the rare shocks are predicted or not.

6 Conclusion

Properties of the earnings process play an important role in various areas of macro and labor economics. Different specifications of this process have been explored in the literature, but the most widely used one is based on decomposing earnings into the sum of persistent and transitory components, where the persistent component is often assumed to follow a random walk. The parameters of such a process can be identified using the moments based on earnings growth rates (first-difference in log earnings) or the moments based on log earnings levels.

Historically, the former approach is more common in labor economics, while the latter is more common in the macroeconomics literature. Unfortunately, these two approaches lead to dramatically different estimates of the variances of permanent and transitory components. In particular, using the same data, the variance of the persistent component is typically estimated to be much higher when the moments in growth rates are used, while the variance of the transitory component is found to be much higher when the estimation is based on the moments in levels. This has important implications for substantive economic analysis. For example, the earnings process drives the heterogeneity in Bewley-type models with incomplete markets and the variances of earnings components determine not only economic choices, such as e.g., consumption, and savings but also the optimal design of policies, such as taxes and transfers. Moreover, the standard approach to estimating the amount of insurance that individuals have against permanent and transitory shocks in the data relies on the estimated variances of permanent and transitory components. The uncertainty over the size of these variances translates into uncertainty over whether the widely used Bewley models generate the right amount of insurance, and the associated uncertainty about the results of welfare analysis using those models.

In this paper we uncover the feature of the data that can quantitatively account for the large difference in the estimates based on earnings growth rates and levels in the administrative data from Denmark and Germany. In particular, we found that earnings are lower on average and more volatile at the start and end of continuous earnings spells. We have shown theoretically that these "outlying" earnings observations, that are either preceded or are followed by a missing observation, induce an upward bias in the estimates of the variance of permanent shocks based on the moments in differences and of the variance of transitory shocks when estimation is based on the moments in levels. Thus, even when working with very large administrative data sets with highly reliable information, one must remain vigilant because such natural features of the datasets as low mean and high variance of earnings at the start and end of earnings spells can induce very large biases in estimated earnings processes.

These findings have several practical implications for estimation of the earnings process. To estimate the parameters of the earnings process in equation (1), one can follow two approaches. First, consider using all non-missing earnings observations. We showed theoretically and verified empirically, that the variance of the transitory shock is estimated with no bias when estimation is based on the moments for earnings growth rates. While theoretically the estimate of the variance of the permanent shock using the moments in levels is biased, the bias is quantitatively small and can probably be ignored in many applications. One could therefore use the estimated permanent component from the moments in levels and the estimated transitory component from the moments in growth rates. An alternative way to proceed would be to estimate the earnings process in equation (1) on the data that do not include the observations surrounding the missing ones. As we have shown, this recovers the true parameters of this process quite well. One can also incorporate additional transitory shocks at the beginning and the end of contiguous earnings histories into the analysis—the mean and the variance of these shocks are readily identified from the mean and the variance of earnings in those periods.

Using a calibrated standard incomplete markets model we have shown theoretically and quantitatively that the presence of rare transitory shocks at the start and end of contiguous earnings spells induces a substantial upward bias in the exactly identified estimates of the insurance against permanent shocks. When the minimum-distance estimation approach is followed, the magnitude of the bias depends critically on the weighting matrix used. Surprisingly, despite the concerns raised in Altonji and Segal (1996), the bias is virtually eliminated when the optimal weighting matrix is used regardless of the size of the sample. Interestingly, we also find that when the identity weighting matrix is used, using consumption data provides no additional identification power for estimating the variances of permanent and transitory shocks to earnings.

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Table 1: Danish data, 1981–2006. Summary statistics for selected years.

	9 consec.	20 not nec. consec.	Balanced
Number of individuals	102,825	90,668	67,008
Number of observations	2,367,552	2,298,429	1,742,208
Education			
Less than high school	0.227	0.222	0.206
High school degree	0.032	0.031	0.029
Vocational training	0.505	0.521	0.542
Two-year university degree	0.046	0.046	0.047
Bachelors degree	0.125	0.122	0.124
Master or Ph.D.	0.065	0.059	0.051
Earnings			
1985	40,157	40,227	41,383
	(12,831)	(12,889)	(12,278)
1995	48,197	48,444	50,004
	(20,562)	(20,462)	(19,954)
2005	52,656	51,511	53,298
	(26,635)	(26,279)	(25,917)
Spell counts			
Start 1981, end 2006	67,008	80,787	67008
Start after 1981, end 2006	13,439	4,376	0
Start in 1981, end before 2006	17,723	5,210	0
Start after 1981, end before 2006	4,655	295	0
Total	102,825	90,668	67008
Number of spells with 20 or more not nec. consec. observations, by length			
[Proportion of missing observations within spell in square brackets]			
20		1,634 [0.144]	
21		2,009 [0.119]	
22		2,665 [0.096]	
23		3,296 [0.079]	
24		4,486 [0.054]	
25		9,570 [0.030]	
26		67,008 [0.00]	

Notes: Earnings are expressed in 2005 Euros; the standard deviation of earnings is given in parentheses.

Table 2: German data, 1984–2008. Summary statistics for selected years.

	9 consec.	20 not nec. consec.	Balanced
Number of individuals	18,130	13,635	9,452
Number of observations	379,080	330,748	236,300
Education			
Middle school or no degree	0.05	0.04	0.04
Vocational training	0.72	0.74	0.76
High school degree	0.06	0.05	0.05
College	0.17	0.17	0.15
Earnings			
1985	33,626	33,930	34,559
	(15,876)	(13,323)	(12,881)
1995	45,309	47,180	47,965
	(24,702)	(24,295)	(24,463)
2005	49,121	51,289	52,457
	(36,473)	(37,106)	(37,666)
Spell counts			
Start 1984, end 2008	9,452	11,179	
Start after 1984, end 2008	3,136	1,007	
Start in 1984, end before 2008	4,463	1,393	
Start after 1984, end before 2008	1,079	56	
Total	18,130	13,635	9,452
Number of spells with 20 or more not nec. consec. observations, by length			
[Proportion of missing observations within spell in square brackets]			
20		575 [0.054]	
21		509 [0.054]	
22		623 [0.05]	
23		871 [0.037]	
24		1,605 [0.027]	
25		9,452 [0.00]	

Notes: Earnings are expressed in 2005 Euros; the standard deviation of earnings is given in parentheses.

Table 3: Estimates of the earnings process in unbalanced samples.

	9 consec.				20 not nec. consec.			
	German data		Danish data		German data		Danish data	
	Levs. (1)	Diffs. (2)	Levs. (3)	Diffs. (4)	Levs. (5)	Diffs. (6)	Levs. (7)	Diffs. (8)
$\hat{\phi}_p$	0.980	0.992	0.964	0.990	0.995	0.997	0.967	0.989
	(0.001)	(0.0008)	(0.0008)	(0.0004)	(0.001)	(0.001)	(0.0007)	(0.0006)
$\hat{\sigma}^2_\xi$	0.007	0.019	0.007	0.012	0.0046	0.008	0.0066	0.0103
	(0.0002)	(0.0003)	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0001)
$\hat{\phi}_\tau$	0.173	0.173	0.289	0.285	0.158	0.316	0.184	0.355
	(0.006)	(0.014)	(0.003)	(0.004)	(0.009)	(0.012)	(0.004)	(0.004)
$\hat{\sigma}^2_\epsilon$	0.025	0.009	0.022	0.014	0.016	0.011	0.023	0.016
	(0.0004)	(0.0003)	(0.0002)	(0.0001)	(0.0003)	(0.0003)	(0.0002)	(0.0001)
$\hat{\sigma}^2_\alpha$	0.026	—	0.020	—	0.029	—	0.023	—
	(0.002)	—	(0.0004)	—	(0.002)	—	(0.0004)	—
χ^2	929.67	725.21	6166.66	4196.83	1518.17	1284.62	6637.87	4799.07
(d.f.)	320	296	346	321	320	296	346	321

Notes: The estimated earnings process is: $y_{it} = \alpha_i + p_{it} + \tau_{it}$, where $p_{it+1} = \phi_p p_{it} + \xi_{it+1}$ and $\tau_{it+1} = \phi_\tau \tau_{it} + \epsilon_{it+1}$. Models are estimated using the optimally weighted minimum distance method. Asymptotic standard errors are in parentheses. German data span the period 1984–2008, while Danish data span the period 1981–2006.

Table 4: Estimates of the earnings process. Balanced samples.

	German data		Danish data	
	Levs. (1)	Diffs. (2)	Levs. (3)	Diffs. (4)
$\hat{\phi}_p$	1	0.998	0.975	0.979
	(0.001)	(0.002)	(0.0007)	(0.0009)
$\hat{\sigma}^2_\xi$	0.0031	0.0033	0.0046	0.0045
	(0.0001)	(0.0001)	(0.0000)	(0.0001)
$\hat{\phi}_\tau$	0.278	0.258	0.311	0.317
	(0.011)	(0.012)	(0.004)	(0.004)
$\hat{\sigma}^2_\epsilon$	0.008	0.0078	0.0104	0.0106
	(0.0002)	(0.0002)	(0.0001)	(0.0001)
$\hat{\sigma}^2_\alpha$	0.024	—	0.018	—
	(0.001)	—	(0.0003)	—
χ^2	1205.84	935.52	6244.18	5094.84
(d.f.)	320	296	346	321

Notes: The estimated earnings process is: $y_{it} = \alpha_i + p_{it} + \tau_{it}$, where $p_{it+1} = \phi_p p_{it} + \xi_{it+1}$ and $\tau_{it+1} = \phi_\tau \tau_{it} + \epsilon_{it+1}$. Models are estimated using the optimally weighted minimum distance method. Asymptotic standard errors are in parentheses. German data span the period 1984–2008, while Danish data span the period 1981–2006.

Table 5: Dependent variable: residual earnings. Panel regressions.

	9 or more consec.		20 not nec. consec.	
	German data (1)	Danish data (2)	German data (3)	Danish data (4)
Year observed: first	–0.75***	–0.67***	–0.75***	–0.62***
	(–74.06)	(–144.86)	(–37.73)	(–74.14)
Year observed: second	–0.30***	–0.35***	–0.24***	–0.35***
	(–43.44)	(–93.73)	(–17.86)	(–44.27)
Year observed: third	–0.26***	–0.28***	–0.19***	–0.25***
	(–39.23)	(–81.25)	(–14.95)	(–36.41)
Year observed: second-to-last	–0.10***	–0.14***	–0.08***	–0.12***
	(–15.82)	(–47.87)	(–6.77)	(–18.58)
Year observed: next-to-last	–0.13***	–0.17***	–0.13***	–0.16***
	(–19.37)	(–55.32)	(–9.39)	(–23.54)
Year observed: last	–0.50***	–0.38***	–0.51***	–0.37***
	(–56.23)	(–95.64)	(–27.71)	(–43.71)
3 years before earn. miss., dummy			–0.12***	–0.10***
			(–12.45)	(–30.09)
2 years before earn. miss., dummy			–0.12***	–0.12***
			(–13.25)	(–38.71)
1 year before earn. miss., dummy			–0.35***	–0.38***
			(–28.76)	(–104.00)
1 year after earn. miss., dummy			–0.47***	–0.55***
			(–34.10)	(–136.97)
2 years after earn. miss., dummy			–0.21***	–0.22***
			(–21.39)	(–65.50)
3 years after earn. miss., dummy			–0.21***	–0.19***
			(–20.08)	(–51.05)
Adj. R sq.	0.068	0.055	0.053	0.081
No. obs.	379080	2367552	330748	2298429
No. indiv.	18130	102825	13635	90668

Notes: German data span the period 1984–2008, while Danish data span the period 1981–2006. The dummies "Year observed: first"–"Year observed: third" are equal to one if an individual's first earnings record is later than in 1984 in German data and 1981 in Danish data, and are zero otherwise; "Year observed: second-to-last"–"Year observed: last" are equal to one if an individual's last earnings record is earlier than in 2008 in German data and 2006 in Danish data, and are zero otherwise. Standard errors are clustered by individual; t-statistics are in parentheses. *** significant at the 1% level, ** significant at the 5% level, * significant at the 10% level.

Table 6: Dependent variable: squared residual earnings. Panel regressions.

	9 or more consec.				20 not nec. consec.			
	German data		Danish data		German data		Danish data	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year observed: first	0.83***	0.28***	0.71***	0.28***	0.84***	0.26***	0.60***	0.18***
	(42.32)	(29.30)	(83.69)	(65.60)	(21.08)	(15.89)	(41.78)	(27.39)
Year observed: second	0.14***	0.06***	0.28***	0.16***	0.10***	0.04***	0.28***	0.14***
	(14.77)	(7.71)	(48.12)	(41.37)	(5.75)	(2.89)	(22.28)	(17.76)
Year observed: third	0.10***	0.04***	0.19***	0.12***	0.05***	0.01	0.15***	0.08***
	(12.59)	(6.02)	(38.12)	(31.97)	(3.31)	(0.62)	(14.53)	(10.99)
Year observed: second-to-last	0.08***	0.08***	0.11***	0.09***	0.08***	0.07***	0.14***	0.11***
	(11.17)	(10.52)	(28.43)	(27.15)	(6.20)	(5.54)	(13.83)	(12.90)
Year observed: next-to-last	0.12***	0.10***	0.14***	0.12***	0.16***	0.11***	0.18***	0.13***
	(14.17)	(13.25)	(33.79)	(32.13)	(8.40)	(7.87)	(16.80)	(15.29)
Year observed: last	0.53***	0.29***	0.38***	0.25***	0.62***	0.32***	0.44***	0.26***
	(38.21)	(30.14)	(63.41)	(58.60)	(19.76)	(17.04)	(29.52)	(27.09)
3 years before earn. miss., dummy					0.09***	0.04***	0.10***	0.07***
					(6.27)	(4.07)	(20.85)	(20.30)
2 years before earn. miss., dummy					0.12***	0.03***	0.11***	0.07***
					(7.32)	(3.49)	(20.55)	(21.82)
1 year before earn. miss., dummy					0.42***	0.16***	0.40***	0.14***
					(17.54)	(12.95)	(57.48)	(41.43)
1 year after earn. miss., dummy					0.69***	0.23***	0.66***	0.19***
					(21.75)	(15.58)	(75.98)	(50.20)
2 years after earn. miss., dummy					0.23***	0.06***	0.21***	0.11***
					(11.76)	(6.13)	(33.14)	(30.50)
3 years after earn. miss., dummy					0.19***	0.07***	0.18***	0.07***
					(10.66)	(7.15)	(28.69)	(20.30)
Adj. R sq.	0.063	0.015	0.052	0.017	0.067	0.012	0.084	0.020
No. obs.	379080	379080	2367552	2367552	330748	330748	2298429	2298429
No. indiv.	18130	18130	102825	102825	13635	13635	90668	90668

Notes: German data span the period 1984–2008, while Danish data span the period 1981–2006. The dummies "Year observed: first"–"Year observed: third" are equal to one if an individual's first earnings record is later than in 1984 in German data and 1981 in Danish data, and are zero otherwise; "Year observed: second-to-last"–"Year observed: last" are equal to one if an individual's last earnings record is earlier than in 2008 in German data and 2006 in Danish data, and are zero otherwise. Standard errors are clustered by individual; t-statistics are in parentheses. *** significant at the 1% level, ** significant at the 5% level, * significant at the 10% level.

TABLE 7: DEPENDENT VARIABLE: THE INCIDENCE OF MISSING EARNINGS OBSERVATION; PANEL REGRESSIONS. 20 NOT NEC. CONSEC. OBSERVATIONS.

	German data					Danish data				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Earn. growth ($t-4$ to $t-3$)					–0.43***					–0.58***
					(–3.34)					(–10.65)
Earn. growth ($t-3$ to $t-2$)				–1.02***	–1.16***				–1.06***	–1.23***
				(–6.49)	(–7.04)				(–16.34)	(–18.11)
Earn. growth ($t-2$ to $t-1$)	–3.21***		–3.10***	–3.32***	–3.41***	–3.15***		–3.01***	–3.25***	–3.33***
	(–12.24)		(–12.14)	(–12.18)	(–12.20)	(–32.02)		(–31.24)	(–32.23)	(–32.57)
Earn. growth ($t+1$ to $t+2$)		3.58***	3.49***	3.88***	4.03***		4.38***	4.28***	4.77***	4.98***
		(12.67)	(12.60)	(12.80)	(12.96)		(38.88)	(38.54)	(40.48)	(41.34)
Earn. growth ($t+2$ to $t+3$)				1.32***	1.63***				1.89***	2.29***
				(7.81)	(8.80)				(24.08)	(27.41)
Earn. growth ($t+3$ to $t+4$)					0.85***					1.27***
					(6.45)					(19.03)
Adj. R sq.	0.005	0.007	0.012	0.013	0.014	0.007	0.012	0.017	0.020	0.021
No. obs.	210641	210641	210641	210641	210641	1486308	1486308	1486308	1486308	1486308
No. indiv.	13635	13635	13635	13635	13635	90584	90584	90584	90584	90584

Notes: Education dummy variables and age are also included in the regressions. Standard errors are clustered by individual; t-statistics are in parentheses. *** significant at the 1% level, ** significant at the 5% level, * significant at the 10% level.

Table 8: Variances of permanent and transitory shocks in the permanent-transitory decomposition of earnings.

	9 consec.				20 not nec. consec.			
	German data		Danish data		German data		Danish data	
	Levs. (1)	Diffs. (2)	Levs. (3)	Diffs. (4)	Levs. (5)	Diffs. (6)	Levs. (7)	Diffs. (8)
Perm. var., full sample, $\hat{\sigma}^2_{\xi}$	0.013	0.024	0.016	0.019	0.0096	0.018	0.013	0.019
Perm. var., outliers, $\hat{\sigma}^2_{\xi,o}$	0.034	0.158	0.053	0.124	–0.009	0.137	–0.004	0.133
Perm. var., net of outliers, $\hat{\sigma}^2_{\xi,n}$	0.010	0.010	0.013	0.013	0.0097	0.0097	0.013	0.013
Trans. var., full sample, $\hat{\sigma}^2_{\epsilon}$	0.020	0.008	0.014	0.009	0.018	0.007	0.019	0.009
Trans. var., outliers, $\hat{\sigma}^2_{\epsilon,o}$	0.143	0.011	0.104	0.022	0.162	0.011	0.173	0.030
Trans. var., net of outliers, $\hat{\sigma}^2_{\epsilon,n}$	0.007	0.007	0.008	0.008	0.007	0.007	0.008	0.008

Notes: The variances are calculated as in equations (2)–(4).

Table 9: Estimates of the earnings process. Drop first and last, and before/after missing earnings records.

	9 or more consec.				20 not nec. consec.			
	German data		Danish data		German data		Danish data	
	Levs. (1)	Diffs. (2)	Levs. (3)	Diffs. (4)	Levs. (5)	Diffs. (6)	Levs. (7)	Diffs. (8)
$\hat{\phi}_p$	0.988	0.998	0.967	0.986	0.996	0.999	0.971	0.985
	(0.001)	(0.001)	(0.0008)	(0.0006)	(0.001)	(0.001)	(0.0008)	(0.0007)
$\hat{\sigma}_\xi^2$	0.0056	0.005	0.0062	0.0061	0.0041	0.0042	0.0053	0.0057
	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
$\hat{\phi}_\tau$	0.305	0.250	0.338	0.313	0.317	0.267	0.339	0.318
	(0.010)	(0.009)	(0.004)	(0.004)	(0.01)	(0.011)	(0.004)	(0.004)
$\hat{\sigma}_\epsilon^2$	0.012	0.010	0.015	0.013	0.011	0.009	0.015	0.013
	(0.0003)	(0.0002)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0001)
$\hat{\sigma}_\alpha^2$	0.022	—	0.02	—	0.024	—	0.021	—
	(0.002)	—	(0.0004)	—	(0.002)	—	(0.0004)	—
χ^2	1093.77	923.01	5330.86	4844.48	1178.30	905.41	5919.64	5025.96
(d.f.)	320	296	346	321	320	296	346	321

Notes: German data span the period 1984–2008, while Danish data span the period 1981–2006. In German data, if an individual's first (last) earnings observation is not in 1984 (2008), his first (last) three observations are dropped prior to estimation. In Danish data, if an individual's first (last) earnings observation is not in 1981 (2006), his first (last) three observations are dropped prior to estimation. In columns (5)–(8), in addition, we drop three observations preceding and following a missing earnings record. The estimated earnings process is: $y_{it} = \alpha_i + p_{it} + \tau_{it}$, where $p_{it+1} = \phi_p p_{it} + \xi_{it+1}$ and $\tau_{it+1} = \phi_\tau \tau_{it} + \epsilon_{it+1}$. Models are estimated using the optimally weighted minimum distance method. Asymptotic standard errors are in parentheses.

Table 10: Estimates of the earnings process in unbalanced samples. Simulated "German" data.

	9 consec.				20 not nec. consec.			
	Full sample		Drop		Full sample		Drop	
	Levs. (1)	Diffs. (2)	Levs. (3)	Diffs. (4)	Levs. (5)	Diffs. (6)	Levs. (7)	Diffs. (8)
$\hat{\phi}_p$	0.988	0.992	0.988	0.988	0.993	0.990	0.996	0.996
	(0.001)	(0.001)	(0.0008)	(0.001)	(0.0008)	(0.001)	(0.0007)	(0.0009)
$\hat{\sigma}^2_\xi$	0.0054	0.014	0.0055	0.0055	0.0043	0.0083	0.004	0.004
	(0.0001)	(0.0009)	(0.0001)	(0.0001)	(0.0001)	(0.0003)	(0.0001)	(0.0001)
$\hat{\phi}_\tau$	0.236	0.155	0.250	0.250	0.193	0.203	0.267	0.266
	(0.007)	(0.02)	(0.005)	(0.005)	(0.009)	(0.009)	(0.005)	(0.005)
$\hat{\sigma}^2_\epsilon$	0.019	0.009	0.01	0.01	0.014	0.009	0.0087	0.0088
	(0.0008)	(0.0002)	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0001)
$\hat{\sigma}^2_\alpha$	0.02	—	0.02	—	0.018	—	0.02	—
	(0.002)	—	(0.001)	—	(0.001)	—	(0.001)	—
χ^2	3058.75	3139.04	324.92	302.79	1416.02	1343.71	331.77	308.51
(d.f.)	320	296	346	321	320	296	346	321

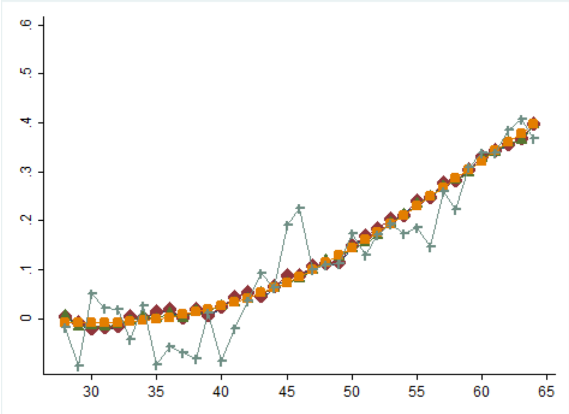
Notes: The true earnings process is: $y_{it} = \alpha_i + p_{it} + \tau_{it}$, where $p_{it+1} = \phi_p p_{it} + \xi_{it+1}$ and $\tau_{it+1} = \phi_\tau \tau_{it} + \epsilon_{it+1}$. In columns (1)–(4), $\sigma^2_\alpha = 0.02$, $\phi_p = 0.988$, $\sigma^2_\xi = 0.0056$, $\phi_\tau = 0.250$, $\sigma^2_\epsilon = 0.01$, while in columns (5)–(8), $\sigma^2_\alpha = 0.02$, $\phi_p = 0.996$, $\sigma^2_\xi = 0.0040$, $\phi_\tau = 0.267$, $\sigma^2_\epsilon = 0.009$. In columns (3)–(4) the first 3 (last 3) observations are dropped if an individual's earnings spell starts (ends) later (earlier) than in 1984 (2008); in columns (7) and (8), in addition, three observations before and after missing earnings records are dropped. The results are the averages across 100 simulations. The model is estimated using the optimal weighting minimum distance method. Standard errors, calculated as the standard deviations of the estimates across simulations, are in parentheses.

Table 11: Estimates of the earnings process. Simulated consumption and income data

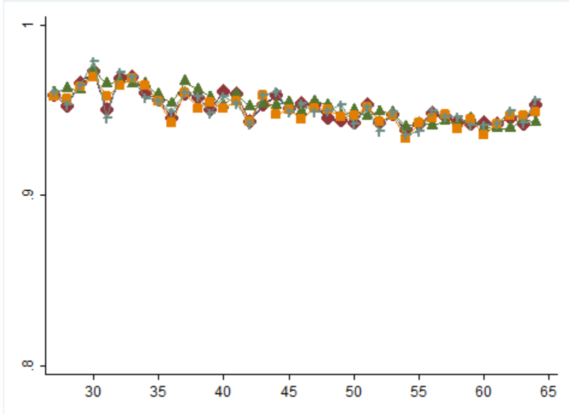
	No measurement error, no rare shocks			Rare transitory shocks			Measurement error in earnings		
	Ident.	Diag.	Opt.	Ident.	Diag.	Opt.	Ident.	Diag.	Opt.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A: Using consumption and income moments									
(1) Var. perm. shocks, $\hat{\sigma}_\xi^2$	0.004	0.004	0.004	0.015	0.005	0.004	0.015	0.005	0.004
(2) Var. trans. shocks, $\hat{\sigma}_\epsilon^2$	0.012	0.012	0.012	0.013	0.015	0.012	0.014	0.015	0.012
(3) Insurance of perm. shocks (avg.), $1-\hat{\phi}$	0.13	0.13	0.13	0.64	0.24	0.14	0.65	0.19	0.12
(4) Insurance of trans. shocks (avg.), $1-\hat{\psi}$	0.95	0.95	0.94	1.02	0.97	0.96	1.03	0.97	0.95
B: Using consumption and income moments; assume constant insurance over the life-cycle									
(5) Var. perm. shocks, $\hat{\sigma}_\xi^2$	0.004	0.004	0.003	0.015	0.006	0.004	0.015	0.005	0.004
(6) Var. trans. shocks, $\hat{\sigma}_\epsilon^2$	0.012	0.012	0.011	0.013	0.015	0.012	0.014	0.015	0.012
(7) Insurance of perm. shocks (avg.), $1-\hat{\phi}$, restr.	0.12	0.26	0.15	0.64	0.36	0.24	0.64	0.31	0.21
(8) Insurance of trans. shocks (avg.), $1-\hat{\psi}$, restr.	0.95	0.95	0.96	1.02	0.97	0.95	1.03	0.97	0.95
C: Using income moments only									
(9) Var. perm. shocks, $\hat{\sigma}_\xi^2$	0.004	0.004	0.004	0.016	0.016	0.013	0.016	0.016	0.013
(10) Var. trans. shocks, $\hat{\sigma}_\epsilon^2$	0.012	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013

Notes: The true earnings process is $y_{it} = p_{it} + \epsilon_{it}$, $p_{it} = p_{it-1} + \xi_{it}$, $\sigma_\xi^2 = 0.004$, $\sigma_\epsilon^2 = 0.012$; the shocks are iid Student-t shocks with 4 degrees of freedom. The rare transitory shock/measurement error in earnings is distributed as a Student-t shock with 4 degrees of freedom, mean –0.50, and variance 0.30.

Panel A: No Rare Shocks

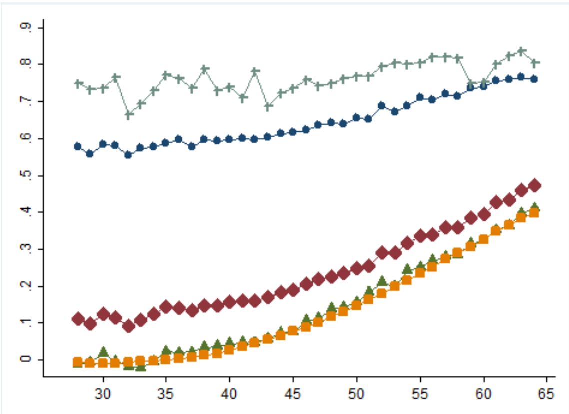


(a) Permanent shocks.

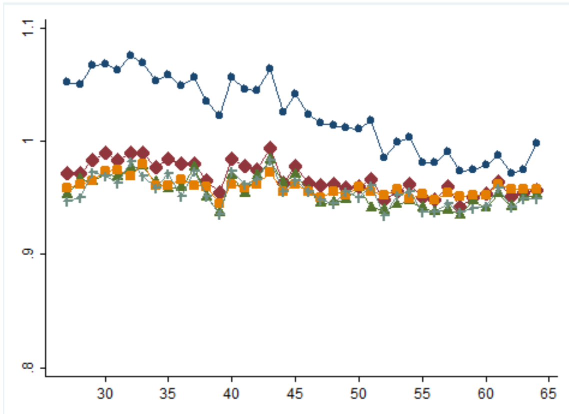


(b) Transitory shocks.

Panel B: Rare Transitory Shocks

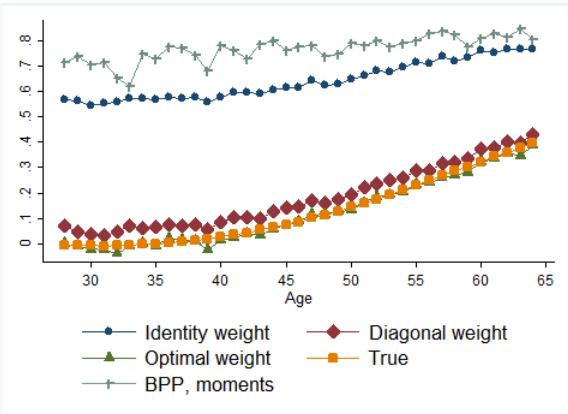


(c) Permanent shocks.

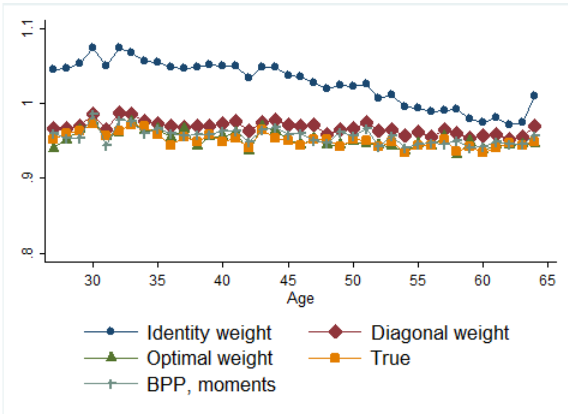


(d) Transitory shocks.

Panel C: Measurement Error



(e) Permanent shocks.



(f) Transitory shocks.

Figure 1: Insurance coefficients.

APPENDICES

I Model and Calibration

We consider a standard life cycle model with individuals heterogeneous with respect to their earnings paths, and incomplete insurance markets. Individuals value consumption, supply labor inelastically, face income uncertainty over the working part of the life cycle, and are subject to borrowing constraints. They start their working life at age 26, retire at age $R = 65$, face age-dependent mortality risk until age $T = 90$ when they die with certainty. Individual i's problem at age $t = 26$ is:

$$\max_{\{c_{it}>0\}_{t=26}^{T}} E_{i,26} \sum_{t=26}^{T} \left(\frac{1}{1+\theta}\right)^{t-26} s_t \frac{c_{it}^{1-\rho}}{1-\rho}, \tag{A1}$$

subject to the budget constraint

$$w_{it+1} = (1+r)(w_{it} + y_{it} - c_{it}), \tag{A2}$$

the liquidity constraint

$$w_{it} \geq \underline{w}_t, \text{ for } t = 26, \ldots, T, \tag{A3}$$

and (in the benchmark calibration) the stochastic process for labor income y_{it} in Equation (1).

In the above equations, c_{it} and w_{it} denote individual i's consumption and wealth at age t, respectively, r is an interest rate on a risk-free asset held between ages t and $t+1$, assumed to be constant, θ is the common time discount rate, ρ is the coefficient of relative risk aversion, s_t is the unconditional probability of surviving up to age t, and $E_{i,26}$ denotes individual i's expectation about future resources based on the information available at age 26.

An individual's wealth at age t is constrained to be above $\underline{w}_t$. We choose natural borrowing constraints, under which individuals are allowed to borrow up to the age-dependent limit, equal to the largest amount of credit they can repay in the event that they receive the lowest possible labor income realization in every period. This is the choice, e.g., in Blundell, Low, and Preston (2011). Kaplan and Violante (2010) show that the insurance of permanent and transitory shocks over the life cycle estimated using the moments proposed in Blundell, Pistaferri, and Preston (2008) are unbiased estimates of the true insurance coefficients if individuals are allowed to borrow up to the natural limit. The choice of the natural borrowing constraints, therefore, enables us to evaluate different estimators of insurance coefficients in a setting which delivers unbiased estimates of the true insurance coefficients using balanced samples of consumption and income, when consumption and income data are not subject to measurement error issues.

In calibrating the model we follow Hryshko (2014). In particular, the model period is one year. Individuals start their life cycle with zero assets. Before retirement, the unconditional probability of survival is set to 1; after retirement, individuals face an age-dependent risk of dying. The conditional probabilities of surviving up to age t provided the individual is alive at age $t-1$ for all $R < t \leq T-1$ are taken from Table A.1 in Hubbard, Skinner, and Zeldes (1994).

The age-dependent deterministic growth rate in individual disposable income is estimated using CEX data. We decompose disposable log income into cohort, time, and age effects, controlling for the effect of family size. Since age, cohort, and time effects are not separately identified, we follow Deaton (1997) to identify the age effects and restrict the time dummies to be orthogonal to a time trend and to add up to zero. We use the age effects from such a regression to construct the profile of deterministic growth in the disposable income. The stochastic process for labor income in equation (1) is parameterized as follows: the permanent component is a random walk, with the variance of permanent shocks equal to 0.004, and the transitory component is an iid shock, with a variance of 0.012—these numbers are in agreement with the estimates for unbalanced samples in German data. Shocks are drawn from a Student t-distribution with 4 degrees of freedom.

After retirement, individual income is assumed to be proportional to the permanent component of income received at age R, $Y_{it} = \kappa P_{iR}$ for ages $t = R+1, \ldots, T$, where κ is the replacement rate set to 0.60.[25]

We set the gross real interest rate to 1.03, the coefficient of relative risk aversion to 2, and calibrate the time discount factor to match an aggregate wealth-to-income ratio of 3.0.

II Appendix Tables

[25]Our modeling of pension benefits is slightly different from Kaplan and Violante (2010) who relate retirement benefits to average life-cycle earnings. As a consequence, our model implies lower insurance coefficients against permanent shocks. In Kaplan and Violante (2010), permanent shocks close to the retirement age have little effect on the permanent income implying little effect on consumption. In our model, permanent shocks at the end of the working career have a larger effect on permanent income inducing a stronger response of consumption. However, except for the magnitude of the insurance coefficients against permanent shocks, this modeling choice is inconsequential for the substantive results below.

Table A-1: Dependent variable: residual earnings. Panel regressions.

	9 or more consec.		20 not nec. consec.	
	German data (1)	Danish data (2)	German data (3)	Danish data (4)
Year observed: first	0.00	0.00**	0.00	–0.00***
	(1.07)	(2.19)	(0.14)	(–3.84)
Year observed: second	0.02***	0.02***	0.01***	0.01***
	(8.34)	(18.66)	(5.42)	(12.92)
Year observed: third	0.03***	0.03***	0.02***	0.01***
	(11.10)	(27.44)	(10.22)	(15.66)
Year observed: two years before last	0.02***	0.01***	0.02***	0.01***
	(7.50)	(12.14)	(6.60)	(12.24)
Year observed: next-to-last	0.01***	0.01***	0.01***	0.01***
	(5.01)	(7.97)	(4.54)	(9.49)
Year observed: last	0.00	0.00**	–0.00	–0.00**
	(1.04)	(2.52)	(–0.48)	(–2.09)
3 years before earn. miss., dummy			–0.12***	–0.10***
			(–12.06)	(–29.34)
2 years before earn. miss., dummy			–0.12***	–0.12***
			(–13.08)	(–39.67)
1 year before earn. miss., dummy			–0.35***	–0.39***
			(–28.86)	(–106.92)
1 year after earn. miss., dummy			–0.48***	–0.55***
			(–34.25)	(–137.21)
2 years after earn. miss., dummy			–0.21***	–0.22***
			(–21.00)	(–64.95)
3 years after earn. miss., dummy			–0.21***	–0.19***
			(–19.62)	(–50.52)
Adj. R sq.	0.000	0.000	0.032	0.068
No. obs.	379,080	2,367,552	330,748	2,298,429
No. indiv.	18,130	102,825	13,635	90,668

Notes: German data span the period 1984–2008, while Danish data span the period 1981–2006. The dummies "Year observed: first"–"Year observed: third" are equal to one if an individual's first earnings record is in 1984 in German data and 1981 in Danish data, and are zero otherwise; "Year observed: second-to-last"–"Year observed: last" are equal to one if an individual's last earnings record is in 2008 in German data and 2006 in Danish data, and are zero otherwise. Standard errors are clustered by individual; t-statistics are in parentheses. *** significant at the 1% level, ** significant at the 5% level, * significant at the 10% level.

Table A-2: Dependent variable: squared residual earnings. Panel regressions.

	9 or more consec.		20 not nec. consec.	
	German data (1)	Danish data (2)	German data (3)	Danish data (4)
Year observed: first	–0.05***	–0.01***	–0.02***	0.01***
	(–14.42)	(–10.70)	(–4.61)	(9.40)
Year observed: second	–0.07***	–0.02***	–0.04***	–0.00***
	(–23.25)	(–22.08)	(–13.88)	(–3.86)
Year observed: third	–0.07***	–0.03***	–0.05***	–0.01***
	(–26.84)	(–27.02)	(–19.22)	(–8.91)
Year observed: two years before last	0.02***	0.00	0.06***	0.02***
	(7.22)	(0.67)	(16.08)	(14.33)
Year observed: next-to-last	0.02***	0.00	0.06***	0.02***
	(7.38)	(0.67)	(17.18)	(16.64)
Year observed: last	0.04***	0.00	0.09***	0.03***
	(11.92)	(0.30)	(20.46)	(21.17)
3 years before earn. miss., dummy			0.10***	0.10***
			(6.43)	(20.80)
2 years before earn. miss., dummy			0.13***	0.11***
			(7.56)	(21.53)
1 year before earn. miss., dummy			0.43***	0.41***
			(17.88)	(59.12)
1 year after earn. miss., dummy			0.70***	0.66***
			(21.92)	(76.28)
2 years after earn. miss., dummy			0.23***	0.21***
			(11.55)	(32.84)
3 years after earn. miss., dummy			0.19***	0.18***
			(10.30)	(28.37)
Adj. R sq.	0.003	0.000	0.048	0.074
No. obs.	379,080	2,367,552	330,748	2,298,429
No. indiv.	18,130	102,825	13,635	90,668

Notes: German data span the period 1984–2008, while Danish data span the period 1981–2006. The dummies "Year observed: first"–"Year observed: third" are equal to one if an individual's first earnings record is in 1984 in German data and 1981 in Danish data, and are zero otherwise; "Year observed: second-to-last"–"Year observed: last" are equal to one if an individual's last earnings record is in 2008 in German data and 2006 in Danish data, and are zero otherwise. Standard errors are clustered by individual; t-statistics are in parentheses. *** significant at the 1% level, ** significant at the 5% level, * significant at the 10% level.

TABLE A-3: ESTIMATES OF THE EARNINGS PROCESS. SIMULATED CONSUMPTION AND INCOME DATA. SMALL SAMPLE.

	No measurement error, no rare shocks			Rare transitory shocks			Measurement error in earnings		
	Ident.	Diag.	Opt.	Ident.	Diag.	Opt.	Ident.	Diag.	Opt.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A: Using consumption and income moments									
(1) Var. perm. shocks, $\hat{\sigma}_\xi^2$	0.004	0.004	0.0038	0.015	0.005	0.004	0.015	0.005	0.0036
(2) Var. trans. shocks, $\hat{\sigma}_\epsilon^2$	0.012	0.012	0.011	0.013	0.014	0.012	0.014	0.014	0.011
(3) Insurance of perm. shocks (avg.), $1-\hat{\phi}$	0.14	0.13	0.13	0.64	0.23	0.14	0.65	0.19	0.13
(4) Insurance of trans. shocks (avg.), $1-\hat{\psi}$	0.95	0.95	0.95	1.02	0.96	0.95	1.03	0.97	0.95
B: Using consumption and income moments; assume constant insurance over the life-cycle									
(5) Var. perm. shocks, $\hat{\sigma}_\xi^2$	0.004	0.0044	0.003	0.015	0.006	0.004	0.015	0.005	0.0032
(6) Var. trans. shocks, $\hat{\sigma}_\epsilon^2$	0.012	0.012	0.011	0.013	0.014	0.012	0.014	0.014	0.012
(7) Insurance of perm. shocks (avg.), $1-\hat{\phi}$, restr.	0.13	0.25	0.17	0.64	0.35	0.22	0.64	0.30	0.18
(8) Insurance of trans. shocks (avg.), $1-\hat{\psi}$, restr.	0.95	0.95	0.94	1.02	0.96	0.94	1.03	0.97	0.94
C: Using income moments only									
(9) Var. perm. shocks, $\hat{\sigma}_\xi^2$	0.004	0.004	0.0038	0.016	0.016	0.008	0.016	0.015	0.008
(10) Var. trans. shocks, $\hat{\sigma}_\epsilon^2$	0.012	0.012	0.0115	0.013	0.013	0.012	0.013	0.013	0.011

Notes: The true earnings process is $y_{it} = p_{it} + \epsilon_{it}$, $p_{it} = p_{it-1} + \xi_{it}$, $\sigma_\xi^2 = 0.004$, $\sigma_\epsilon^2 = 0.012$; the shocks are iid Student-t shocks with 4 degrees of freedom. The rare transitory shock/measurement error in earnings is distributed as a Student-t shock with 4 degrees of freedom, mean –0.50, and variance 0.30.